Test of Hypothesis

Worksheets

	Step 1	Step 2	Step 3	Test Statistics (<i>ts</i>)	Conclusion	Comment About SC	P- value
1	SC $\mu > 40$ OC $\mu \le 40$	H ₀ : $\mu \le 40$ H ₁ : $\mu > 40$ RTT	$\alpha = .05, n = 49$ RTT a a b 1.645 CV = 1.645	$n = 49, \overline{x} = 41.8, s = 3.8$ $z = \frac{\sqrt{49}(41.8 - 40)}{3.8} = 3.316$	<i>ts</i> falls inside $CR \implies$ Reject that $H_0: \mu \le 40$	Accept That SC : $\mu > 40$	0.00004 Lower than $\alpha = .05$
2	SC $\mu \neq 40$ OC $\mu = 40$	H ₀ : $\mu = 40$ H ₁ : $\mu \neq 40$ TTT	$\alpha = .01 n = 49$ TTT TTT -2.58 0 2.58 CV = ± 2.576	$n = 49, \overline{x} = 42.8, s = 4.8$ $z = \frac{\sqrt{49}(42.8 - 40)}{4.8} = 4.08$	ts falls inside $CR \implies$ Reject that $H_0: \mu = 40$	Accept That SC : $\mu \neq 40$	0.00004 Lower than $\alpha = .01$
3	SC $\mu < 40$ OC $\mu \ge 40$	H ₀ : $\mu \ge 40$ H ₁ : $\mu < 40$ LTT	$\alpha = .01 n = 56$ LTT -2.326 0 CV = -2.326	$n = 56, \overline{x} = 39.5, s = 1.9$ $z = \frac{\sqrt{56} (39.5 - 40)}{1.9} = -1.969$	<i>ts</i> falls not inside <i>CR</i> \Rightarrow Accept that H ₀ : $\mu \ge 40$	Reject That SC: $\mu < 40$	0.00245 Not lower than $\alpha = .01$
4	$SC \ \mu < 40$ $OC \ \mu \ge 40$	H ₀ : $\mu \ge 40$ H ₁ : $\mu < 40$ LTT	$\alpha = .05, n = 16$ LTT \underline{R} <u>A</u> -1.75 0 CV = -1.753	$n = 16, \overline{x} = 38.5, s = 2.2$ $t = \frac{\sqrt{16}(38.5 - 40)}{2.2} = -2.727$	<i>ts</i> falls inside $CR \implies$ Reject that $H_0: \mu \ge 40$	Accept SC: μ < 40	0.00779 Not lower than $\alpha = .05$
5	SC : <i>μ</i> = 15 OC <i>μ</i> ≠ 15	H ₀ : $\mu = 15$ H ₁ : $\mu \neq 15$ TTT	$\alpha = .10, n = 12$ TTT TTT $-1.796 0 1.796$ CV = ± 1.796	$n = 12, \overline{x} = 13.8, s = 2.7$ $t = \frac{\sqrt{12}(13.8 - 15)}{2.7} = -1.52$	<i>ts</i> falls not inside <i>CR</i> \Rightarrow Accept that H ₀ : $\mu = 15$	Accept That SC: $\mu = 15$	0.1519 Not lower than $\alpha = .01$
6	SC p > 0.40 OC $p \le 0.40$	H ₀ : $p \le 0.40$ H ₁ : p > 0.40 RTT	$\alpha = .01, n = 250$ RTT a a c c c c c c c c c c	n = 250, x = 120, $\hat{p} = 120 / 250 = .48$ $z = \frac{.4840}{\sqrt{\frac{.4(14)}{250}}} = 2.57$	ts falls inside $CR \implies$ Reject that $H_0: p \le 0.40$	Accept That SC : <i>p</i> > .40	0.0049 Lower than $\alpha = .01$
7	SC: $p \ge 0.40$ OC: p < 0.40	H ₀ : $p \ge 0.40$ H ₁ $p < 0.40$ LTT	$\alpha = .05, n = 360$ LTT -1.645 0 CV = -1.645	n = 360, x = 135, $\hat{p} = 135 / 360 = .375$ $z = \frac{.37540}{\sqrt{\frac{.4(14)}{360}}} =968$	<i>ts</i> falls not inside <i>CR</i> \Rightarrow Accept that H ₀ : $p \ge 0.40$	Accept That SC : $p \ge .40$	0.16646 Not lower than $\alpha = .05$

Please complete the table

	Step 1	Step 2	Step 3	Test Statistics = ts	Conclusion	Comment	P- value
			$\alpha = .01, n = 36$		ts falls inside of	Accept	
	SC:	H ₀ :		$n = 36, \overline{x} = 42.2, \qquad s = 3.2$	<i>CR or not</i> $? \Rightarrow$	or	
1	$\mu = 41$		\frown		Reject H ₀	Reject	
					or	SC:	
	OC:	H ₁ :	0	<i>ts</i> =	Accept H ₀ ?		
			CV = ? =				
			$\alpha = 0.01$ $n = 64$		ts falls inside of	Accept	
	SC:	H ₀ :		$n = 64, \overline{x} = 56.2, \qquad s = 8.4$	<i>CR or not</i> $? \Rightarrow$	or	
2	$\mu \leq 55$		\frown			Reject	
				ts =	Reject H ₀	SC:	
	OC:	H_1 :	0		or		
		•	-		Accept H ₀ ?		
			CV = ? =		_		
			$\alpha = .05$ $n = 20$		ts falls inside of	Accept	
	SC:	H ₀ :		$n = 20, \overline{x} = 13.12, s = 3.2$	<i>CR or not</i> $? \Rightarrow$	or	
3	$\mu \neq 14$	V ·				Reject	
				ts -	Reject H ₀	SC:	
	OC:	H	0		or		
	001	111.	U		Accept H_0 ?		
			CV = ? =		1		
-			$\alpha = .025, n = 25$		ts falls inside of	Accept	
	SC:	Ho		$n = 25, \overline{x} = 380, s = 32$	<i>CR or not</i> $? \Rightarrow$	or	
4	$\mu \ge 400$		\frown	, , ,		Reject	
				te —	R eject H_0	that SC:	
	OC.	H ₁ .		13 –	or		
	0.01	111.	U		Accept H ₀ ?		
			CV = ? =		•		
			$\alpha = .01, n = 82$		ts falls inside of	Accept	
	SC:	Ho:		$n = 82, \overline{x} = 97.5, s = 17.521$	<i>CR or not</i> $? \Rightarrow$	or	
5	$\mu < 102$	0.	\frown			Reject	
				ts -	Reject H ₀	SC:	
	OC:	H ₁ :	0		or		
		1.	Ŭ		Accept H ₀ ?		
			CV = ? =		-		
			$\alpha = .05, n = 400$		ts falls inside of	Accept	
	SC:	H ₀ :		$n = 400, x = 64 \hat{p} =$	<i>CR or not</i> ? \Rightarrow	or	
6	$p \neq 0.13$			-		Reject	
				ts =	Reject H ₀	SC	
	OC:	H ₁ :	0		or		
		_			Accept H ₀ ?		
			CV = ? =				
			$\alpha = .01, n = 200$		ts falls inside of	Accept	
	SC:	H ₀ :		$n = 200, x = 92, \hat{p} =$	$CR \text{ or not } ? \Rightarrow$	or	
7	p > 0.44					Reject	
				ts =	Reject H ₀	SC:	
	OC:	H ₁ :	0		or		
					Accept H ₀ ?		
			CV = ? =				

	Step 1	Step 2	Step 3	Test Statistics = ts	Conclusion	Comment	P -value
1	SC: $\mu = 41$ OC: $\mu \neq 41$	H ₀ : $\mu = 41$ H ₁ : $\mu \neq 41$	$\alpha = .01, n = 36$ $\boxed{R} A R$ 0 $CV = \pm 2.576$	$n = 36, \overline{x} = 42.2, s = 3.2$ $z = \frac{\sqrt{36} (42.2 - 41)}{3.2} = 2.25$	<i>ts</i> falls not inside of <i>CR</i> \Rightarrow A ccept H ₀ $\mu = 41$	Accept that SC: $\mu = 41$	0.024 Not lower than $\alpha = .05$
2	SC: $\mu \le 55$ OC: $\mu > 55$	H ₀ : $\mu \le 55$ H ₁ : $\mu > 55$	$\alpha = 0.01 n = 64$ $A = R$ 0 $CV = 2.326$	$n = 64, \overline{x} = 56.2, s = 8.4$ $z = \frac{\sqrt{64} (56.2 - 55)}{8.4} = 1.1429$	<i>ts</i> falls not inside of <i>CR</i> \Rightarrow Accept H ₀ $\mu \le 55$	Accept that SC: $\mu \le 55$	0.127 Not lower than $\alpha = .01$
3	SC: $\mu \neq 14$ OC: $\mu = 14$	H ₀ : $\mu = 14$ H ₁ : $\mu \neq 14$	$\alpha = .05 n = 20$ $\overrightarrow{R} \overrightarrow{A} \overrightarrow{R}$ 0 $CV = \pm 2.093$	$n = 20, \overline{x} = 13.12, s = 3.2$ $t = \frac{\sqrt{20} (13.12 - 14)}{3.2} = -1.229$	<i>ts</i> falls not inside of <i>CR</i> \Rightarrow Accept H ₀ $\mu = 14$	Reject that SC: $\mu \neq 14$	0.2338 Not lower than $\alpha = .05$
4	SC: $\mu \ge 400$ OC: $\mu < 400$	H0: $μ ≥ 400H1:μ < 400$	$\alpha = .025, n = 25$ $R = \frac{A}{0}$ CV = -2.064	$n = 25, \overline{x} = 380, s = 32$ $t = \frac{\sqrt{25} (380 - 400)}{32} = -3.125$	<i>ts</i> falls inside of $CR \Rightarrow$ R eject H ₀ $\mu \ge 400$	Reject that SC: $\mu \ge 400$	0.0023 Lower than $\alpha = .025$
5	SC: $\mu < 102$ OC: $\mu \ge 102$	H ₀ : $\mu \ge 102$ H ₁ : $\mu < 102$	$\alpha = .01, n = 82$	$n = 82, \overline{x} = 97.5, s = 17.521$ $z = \frac{\sqrt{82}(97.5 - 102)}{17.521} = -2.326$	ts falls on the border line of $CR \Rightarrow$ Inconclusive	SC: Inconclusive	0.0100 Same as $\alpha = .01$
6	SC: $p \neq 0.13$ OC: p = 0.13	H ₀ : p = 0.13 H ₁ : $p \neq 0.13$	$\alpha = .05, n = 400$ $R \qquad R$ 0 $CV = \pm 1.96$	$n = 400, x = 64$ $\hat{p} = 64 / 400 = .16$ $z = \frac{.1613}{\sqrt{\frac{.13(113)}{400}}} = 1.784$	ts falls <i>not</i> inside of <i>CR</i> \Rightarrow Accept H ₀	Reject that SC: $p \neq 0.13$	0.0744 Not lower than $\alpha = .05$
7	SC: p > 0.44 OC: $p \le 0.44$	H ₀ : $p \le 0.44$ H ₁ : p > 0.44	$\alpha = .01, n = 200$	n = 200, x = 92, $\hat{p} = 92 / 200 = .46$ $z = \frac{.4644}{\sqrt{\frac{.44(144)}{200}}} = 0.5698$	ts falls not inside of CR \Rightarrow Accept H ₀	Reject that SC: p > 0.44	0.2844 Not lower than $\alpha = .01$



t -Distribution for small sample n < 30 and σ Unknown

df = n-1 < alpha α>							>	
2-Tailed	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.005
1-Tailed	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.0025
Conf. Levl.	60%	70%	80%	90%	95%	98%	99%	99.5%
1	1.376	1.963	3.078	6.314	12.706	31.821	63.656	127.321
2	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.089
3	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453
4	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598
5	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773
6	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317
7	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029
8	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833
9	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690
10	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581
11	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497
12	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428
13	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372
14	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326
15	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286
16	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252
17	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222
18	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197
19	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174
20	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153
21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135
22	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119
23	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104
24	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091
25	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078
26	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067
27	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057
28	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047
29	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038
30	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030
$n>30 \Rightarrow Z$	0.842	1.036	1.282	1.645	1.96	2.326	2.576	2.807
2-T	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.005
1-T	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.0025
Conf. Levl.	60%	70%	80%	90%	95%	98%	99%	99.5%

n > 30

Hypothesis Testing on the TI-83/84

Written by Jeff O'Connell – joconnell@ohlone.edu Ohlone College http://www2.ohlone.edu/people2/joconnell/ti/ - A video tutorial can be found at this site

Stat vs. Data – Throughout this section the calculator will ask you if you have [Data] or [Stats]. *Stats* is when you just have the statistics about the data such as the mean and standard deviation. *Data* is when you have the actual data. In the case where you have Data, you will enter the data into a list and tell the calculator which list the data is in. Both types of examples are shown in this section.

p-values – The Calculator does hypothesis testing by finding the p-value. Recall that the p-value is the area of the tail(s) that the test statistic cuts off. If the p-value is less than the level of significance then we reject the null hypothesis, if the p-value is more that the level of significance then we fail to reject the null hypothesis.

All Confidence intervals and Hypothesis testing can be found by pressing STAT and scrolling to [TESTS]

The Population Mean

Example 1: A sample of 38 items is chosen from a normally distributed population with a sample mean of 12.5 and a population standard deviation of 2.8. At the 0.05 level of significance test the null hypothesis that the population mean is 14, that is H_0 : $\mu = 14$, H_1 : $\mu \neq 14$, with $\alpha = 0.05$.

Solution: We choose [1:Z-TEST...] since we are using a z-distribution. Enter the information as shown in screen 1 below, highlight [Calculate] and press ENTER to get screen 2 or [Draw] to get screen 3.



The p-value is $0.0082 < \alpha$ so we Reject H₀.

Example 2: A sample of 7 items is chosen from a normal distribution with the following results: {1, 5, 6, 8, 12, 16, 18}. Test the claim that $\mu < 10$, that is H_0 : $\mu = 10$, H_1 : $\mu < 10$, with $\alpha = 0.01$.

Solution: Here we are given the actual data from the sample. We can have the calculator do all of the work on the sample by entering the data into a list, say L1. We choose [2:T-TEST...]. Enter the information as shown in screen 4 below, highlight [Calculate] and press ENTER to get screen 5 or [Draw] to get screen 6.





NOTE: Freq stands for Frequency which may be used if you have data where a lot of the data points are repeated. For example, if your data consists of 1, 1, 1, 2, 2, 3, 4 you can enter all of the distinct the data points in L1 and the frequencies in L2. So $L1 = \{1, 2, 3, 4\}$ and $L2 = \{3, 2, 1, 1\}$. We can enter L1 as the *List* and L2 as the *Freq*. It will most often be the case that we will use 1 as the Freq but this option is available.

The population proportion

Example 3: For x = 14, n = 35 test the claim that p > 0.3, that is H_0 : p = 0.3, H_1 : p > 0.3, with $\alpha = 0.05$.

Solution: We choose [5:1–PropZTest...]. Enter the information as shown in screen 7 below, highlight [Calculate] and press ENTER to get screen 8 or [Draw] to get screen 9.



The p-value is $0.0984 > \alpha$ so we Fail to Reject H₀.

NOTE: *x* and *n* must be an integers.

Comparing two population proportions

<u>Example 4</u>: For $x_1 = 14$, $n_1 = 40$, $x_2 = 17$, and $n_2 = 50$ test the claim that $p_1 > p_2$, that is H_0 : $p_1 = p_2$, H_1 : $p_1 > p_2$, with $\alpha = 0.1$.

Solution: We choose [6:2–PropZTest...]. Enter the information as shown in screen 10 below, highlight [Calculate] and press ENTER to get screen 11 or [Draw] to get screen 12.



The p-value is $0.4605 > \alpha$ so we Fail to Reject H₀.

Hypothesis testing for two population means.

Example 5: The following samples were taken from normal distributions. Test the claim that $\mu_1 \neq \mu_2$, that is H₀: $\mu_1 = \mu_2$, H₁: $\mu_1 \neq \mu_2$, with $\alpha = 0.05$.

 $\begin{array}{l} \overline{x_1} = 78.5 \\ \overline{x_1} = 78.5 \\ \overline{x_2} = 75.3 \\ \overline{\sigma_1} = 12.8 \\ n_1 = 40 \\ n_2 = 50 \end{array}$

Solution: Select [3:2–SampZtest...] and enter the information shown in screen 13, highlight [Calculate] press ENTER to get the results shown in screen 14 or [Draw] to get the results in screen 15.





<u>Example 6</u>: For the sample information taken from normal distributions shown in the screen to the right with L1 being sample from population 1 and L2 from population 2 test the claim that $\mu_1 > \mu_2$, that is $H_0: \mu_1 = \mu_2$, $H_1: \mu_1 > \mu_2$, with $\alpha = 0.05$.



Solution: After entering the sample data into L1 and L2 as shown, we must determine if the variances are significantly different, that is, test the claim $H_0: \sigma_1^2 = \sigma_2^2$ against $H_1: \sigma_1^2 \neq \sigma_2^2$. Select [D:2–SampFTest...] and enter the information shown in screen 16, highlight [Calculate] press ENTER to get the results shown in screen 17 or [Draw] to get the results in screen 18.



The large p-value (bigger than $\alpha = 0.05$) indicated that we must "pool" the variances. If the p-value were smaller than α we would not pool the variances. Select [4:2–SampTTest...] and enter the information shown in screen 19, highlight [Calculate] press ENTER to get the results shown in screen 20 or [Draw] to get the results in screen 21.



The p-value is $0.5 > \alpha$ so we Fail to Reject H₀.

ANOVA

<u>Example 7</u>: Consider the samples taken from three normally distributed populations shown in screen 22. Test the claim that the populations all have the same mean, that is $H_0: \mu_1 = \mu_2 = \mu_3$, $H_1: Not all populations have the same mean, with <math>\alpha = 0.05$.

Solution: After entering the data as shown, select [F:ANOVA(], enter the information shown in screen 23, press ENTER to get the results shown in screen 24.



Screen 22

Screen 23



The p-value is $0.2488 > \alpha$ so we Fail to Reject H₀.

NOTE: To do the ANOVA test on the TI-83/84 you must have the data, not the statistics for the data.