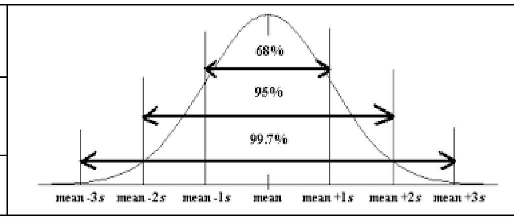
Empirical Rules: If and only if the **boxplot** or **histogram is centered** then we can apply the **three** following empirical rules.

$68\% = \bar{x} \pm s$	**68 %** of data are within 1 s of the mean ($\bar{x}$)	
$95\% = \bar{x} \pm 2s$	**95 %** of data are within 2 s of the mean ($\bar{x}$)	
$99.7\% = \bar{x} \pm 3s$	**99.7 %** of data are within 3 s of the mean ($\bar{x}$)	

Example: Find all three empirical rules for Abe Stat final exam if the average was 72 and the standard deviation was 8, assuming that boxplot was centered.

$68\% = 72 \pm 1(8) = 72 \pm 8$ $\quad$ $64 <$ **68 %** of class got scores < 80

$95\% = 72 \pm 2(8) = 72 \pm 16$ $\quad$ $56 <$ **95 %** of class got scores < 88

$99.7\% = 72 \pm 3(8) = 72 \pm 24$ $\quad$ $48 <$ **99.7 %** of class got scores < 96

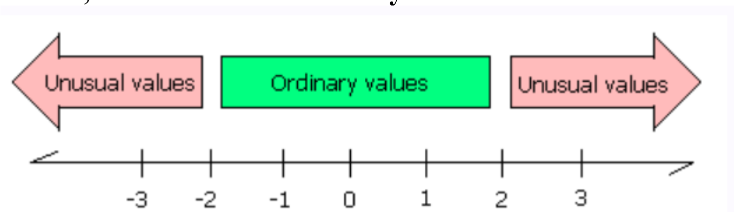
Z-score: is used to show the **relative position of a data point** with respect of the rest of data by **measuring how many standard deviation** that point is **away from the mean**. To apply the z-score the boxplot or histogram must be centered.

$$Z = \frac{x - \bar{x}}{s} \qquad \textbf{or} \qquad Z = \frac{x - \mu}{\sigma}$$

The possible range of Z-values.

If Z-value is **less than** -2 **or more than** 2**, it is called unusual that could be unusually low or unusually high.**

If Z-value is **between** -2 **and** 2**, then it is called ordinary or common.**



Example 1: Find the z-score of final exams for Tommy Yank in stat class at CSUS, if his score was 87, when the class average was 72 and the standard deviation was 8**.**

$$Z = \frac{x - \mu}{\sigma} = \frac{87 - 72}{8} = \frac{15}{8} = 1.875$$ **Ordinary** or **Unusual Value?**

So, he does relatively an ordinary performance relative to the rest of his class.

Example 2: Find the z-score of final exam for Marcy Tank in stat class at UC Davis, if his score was 82, when the class average was 71 and the standard deviation was 4**.**

$$Z = \frac{x - \mu}{\sigma} = \frac{82 - 71}{4} = \frac{11}{4} = 2.75$$ **Ordinary** or **Unusual Value?**

So, she does relatively better than the rest of her class and her z score is unusual.

Basic Probability

Probability of an event **E** $= P(E) = \frac{f}{n}$ = $\dfrac{\text{The number of } \textbf{desired} \text{ (success) } \textbf{outcomes}}{\text{The total number of } \textbf{possible outcomes}}$

$$0 \le P(E) \le 1$$

0	**.25**	**.5**	**.75**	**1**
Impossible	**unlikely**	**even chance**	**likely**	**certain**

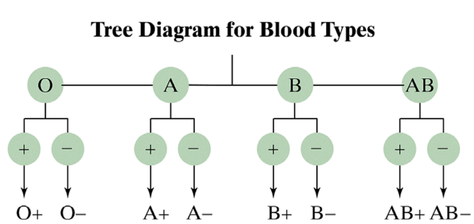
If the **probability** of occurrence of an event such as event **E** is between $0 \le P(E) < 5\%$ then its occurrence is called unusual.

Definition	**Examples**		
An **experiment** is an action, or trial, through which specific results (outcomes) are obtained.	**Tossing a coin**	**Rolling a Die**	**Draw one card from deck of 52 cards**
sample space = **n** **All possible outcomes** of an experiment are called	**n** = **2** sides (H,T) **n** = **2** outcomes	**n** = **6** sides (1,2,3,4,5,6) **n** = **6** outcomes	**n** = **52** cards **n** = **52** outcomes
Out of sample space how many is/are the **desired outcome** or **outcomes**? That will be **= f** .	a tail **f = 1 tail**	an odd number (1,3,5) **f = 3 odd numbers**	an Ace **f = 4 Aces**
Probability is the measure of how likely an event to occur $= P(E) = f / n$	P(T) = 1/2 =50%	P(odd number) = 3/6 =50%	P(Ace) = 4/52 =1/13

Three Types of Probability

- **Classical**: (equally probable outcomes). Like flipping a coin, rolling a die, drawing one card from a deck of cards. In this type of probabilities, we know the probability of getting for number 5 is always 1/6.
- **Empirical**: We need data like example A on page 2. So based on **available data**, the answer may be different each time. $P(E) = \dfrac{\text{Frequency of event } E}{\text{Total frequency}} = \dfrac{f}{n}$
- **Subjective:** Guess or intuition feelings (doctor feels patient has 80% chance of recovery).

Example A:



The sample space has **eight possible outcomes**, **{O+, O– , A+, A–, B+, B–, AB+, AB–}**

Example B: (an example of **empirical** probability): the outcomes may vary from sample to sample
Frequency distribution of annual income for U.S. families

Income	Frequency (1000s)
Under \$10,000	5,216
\$10,000–\$14,999	4,507
\$15,000–\$24,999	10,040
\$25,000–\$34,999	9,828
\$35,000–\$49,999	12,841
\$50,000–\$74,999	14,204
\$75,000 & over	12,961
	69,597

Part 1: Find the probability that a randomly selected person from this group makes \$75,000 and over

1) Experiment: randomly selecting a person. **2)** Sample space $= n = 69,597$

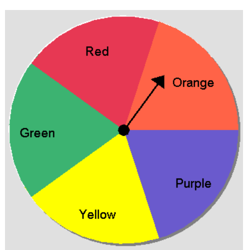
3) His/her income is \$75,000 and over: $f = 12,961$ **4)** Prob (\$75,000 and over) $= 12,961 / 69,597 = 18.63\ \%$

Part 2: Find the probability that a randomly selected person from this group makes \$24,999 or less

1) Experiment: randomly selecting a person. **2)** Sample space $= n = 69,597$

3) His/her income is \$24,999 or less: $f = 19,763$ **4)** Prob (\$24,999 or less) $= 19,763 / 69,597 = 28.40\ \%$

Example C. : (an example of **empirical** probability)



What is the probability that you spin the dial on the left spinner, and you get yellow? P(yellow)=**1/5**
What is the probability that you spin the dial on the right spinner, and you get lose turn? P(Lose Turn) =**1/12**

Example D (an example of **classical** probability)
In a deck of 52 cards there are 13 diamonds and 12 faces, and 4 aces. If one card is drawn randomly find the probability that

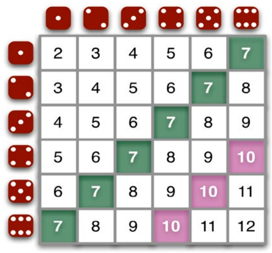
Solution:

a) P (diamond) $= 13 / 52 = 25\ \%$ b) P(face) $= 12 / 52 = 23.08\ \%$ c) P(not face) $= 40 / 52 = 76.92\ \%$

d) P(not diamond) $= 39 / 52 = 75\ \%$ e) P(diamond and face) $= 3 / 52 = 5.77\ \%$

Example E: (an example of **classical** probability)

If we roll 2 dice, then there are 36 possible outcomes meaning that the **sample space is 36** or $= n = 36$



Solution:

a) find the probability that the sum of rolling two dice is 10
Event or desired outcomes: a sum of 10 $\Rightarrow \{(4,6),(5,5),(6,4)\} \Rightarrow f = 3$
Prob (a sum of 10) $= 3/36 = 1/12 = 8.33\,\%$

b) find the probability that the sum of rolling two dice is 7
Event or desired outcomes: a sum of 7 $\Rightarrow \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\} \Rightarrow f = 6$
Prob (a sum of 7) $= 6/36 = 1/6 = 16.66\,\%$

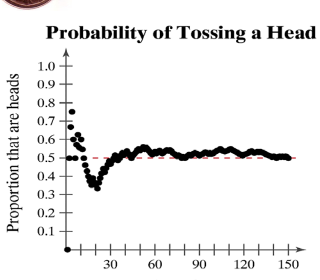
c) find the probability that the sum of rolling two dice is not 7
Event or desired outcomes: a sum of not 7 $\Rightarrow \neq \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\} \Rightarrow f = 30$
Prob (sum is not 7) $= 30/36 = 5/6 = 83.33\,\%$

d) find the probability that the sum of rolling is10 or more
Event or desired outcomes: to get a sum 10 or more $\Rightarrow \{(4,6),(5,5),(5,6),(6,4),(6,5),(6,6),\} \Rightarrow f = 6$
Prob (a sum of 10 or more) $= 6/36 = 1/6 = 16.67\,\%$

e) find the probability that their sum is 5
Event or desired outcomes: to get a sum of 5 $\Rightarrow \{(1,4),(2,3),(3,2),(4,1)\} \Rightarrow f = 4$
Prob (a sum of 5) $= 4/36 = 1/9 = 11.11\,\%$

Law of Large Numbers

- As an experiment is repeated over and over, the empirical probability of an event approaches the theoretical (actual) probability of the event.



Multiplication Rule (Keywords: and, both, all)

$$P(A \text{ and } B \text{ and } C \text{ and} \ldots) = P(A)P(B)P(C)\ldots$$

We use multiplication rule to find the probability that events A, B, C happen together or one after each other.
Hint:
When you make a selection out of a group by using multiplication rule be aware of **with** or **w/o** replacement effect.

In a deck of 52 cards there are 13 diamonds and 12 faces, and 4 aces.

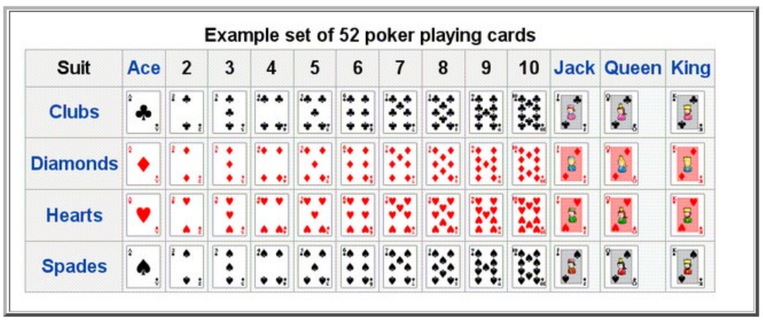
If 2 cards are randomly drawn **w/o replacement,** what is the probability that both are diamonds?

P(both diamond) $= \frac{13}{52} \bullet \frac{12}{51} = 5.88\,\%$

If 2 cards are randomly drawn **with replacement,** what is the probability that both are diamonds?

P(both diamond) $= \frac{13}{52} \bullet \frac{13}{52} = 6.25\,\%$

There are 13 diamonds and 12faces, and 4 aces in a deck of 52 cards.



If 4 cards are randomly drawn **w/o replacement** then,

a) What is the probability that all 4 are diamond and how likelihood is this?

13 Diamo 39 Others		12 Diamo 39 Others		11 Diamo 39 Others		10 Diamo 39 Others
52	⇒	51	⇒	50	⇒	49

$\frac{13}{52} \bullet \frac{12}{51} \bullet \frac{11}{50} \bullet \frac{10}{49} = .26\%$ very unlikely

b) What is the probability that all 4 are aces and how likelihood is this?

4 Aces 48 Others		3 Aces 48 Others		2 Aces 48 Others		1 Aces 48 Others
52	⇒	51	⇒	50	⇒	49

$\frac{4}{52} \bullet \frac{3}{51} \bullet \frac{2}{50} \bullet \frac{1}{49} = .000369\ \%$ very much unlikely

c) What is the probability that all 4 are faces and how likelihood is this?

12 Faces 40 Others		11 Faces 40 Others		10 Faces 40 Others		9 Faces 40 Others
52	⇒	51	⇒	50	⇒	49

$\frac{12}{52} \bullet \frac{11}{51} \bullet \frac{10}{50} \bullet \frac{9}{49} = .001828 = .1828\%$ much unlikely

d) What is the probability that all 4 are non faces and how likelihood is this?

12 faces 40 non-faces		12 faces 39 non-faces		12 faces 38 non-faces		12 faces 37 non-faces
52	⇒	51	⇒	50	⇒	49

$\frac{40}{52} \bullet \frac{39}{51} \bullet \frac{38}{50} \bullet \frac{37}{49} = .337575 = 33.76\ \%$ It is likely

A. If we have a group of 4 men and 6 women, and we select two at random, without replacement, then

1. Find the probability that both are women. **P(both W) = P(W and W) =** $\frac{6}{10} \bullet \frac{5}{9} = \frac{30}{90} = 0.33$

2. Find the probability that one of each gender is selected. That means **one man one woman** or **one woman one man**

P(MW) = $\frac{4}{10} \bullet \frac{6}{9} = \frac{24}{90} = \frac{8}{30} = 0.267$ or **P(WM) =** $\frac{6}{10} \bullet \frac{4}{9} = \frac{24}{90} = \frac{8}{30} = 0.267$

Then **you need to add these probabilities**. $0.267 + 0.267 = 0.533 = 53.33\%$

B. In a bag there are 3 red, 4 blue and 5 green marbles. If we draw **3** marbles at random (**without replacement)** then,

Find the probability that all

1) **All red** **P(RRR) =** $\frac{3}{12} \bullet \frac{2}{11} \bullet \frac{1}{10} = \frac{1}{220} = 0.0004545$

2) **Non red** $\frac{9}{12} \bullet \frac{8}{11} \bullet \frac{7}{10} = \frac{21}{55} = 0.38181$

3) **All blue** $\frac{4}{12} \bullet \frac{3}{11} \bullet \frac{2}{10} = \frac{1}{55} = 0.018181$

4) **None blue** $\frac{8}{12} \bullet \frac{7}{11} \bullet \frac{6}{10} = \frac{14}{55} = 0.2545$

C. In a bag there are 3 red, 4 blue and 5 green marbles. If we draw 3 marbles at random (**with replacement**, then,

Find the probability that all

1) **All red** **P(RRR) =** $\frac{3}{12} \bullet \frac{3}{12} \bullet \frac{3}{12} = \frac{1}{64} = 0.0156$

2) **Non red** $\frac{9}{12} \bullet \frac{9}{12} \bullet \frac{9}{12} = \frac{27}{64} = 0.4219$

3) **All blue** $\frac{4}{12} \bullet \frac{4}{12} \bullet \frac{4}{12} = \frac{1}{27} = 0.037$

4) **None blue** $\frac{8}{12} \bullet \frac{8}{12} \bullet \frac{8}{12} = \frac{8}{27} = 0.2963$