

### Part III Practice Problems

<b>Problems</b>	<b>Pages 1- 4</b>
<b>Answers</b>	<b>Page 5</b>
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- i) What are the definitions of point estimate, the confidence interval, confidence level, the width and margin of error?
- ii) What are the conditions to use normal versus t-distribution?
- iii) What are the properties of t-distribution?
- iv) What are the critical area and critical values?
- v) Under what condition we can use normal distribution in in estimating population proportion?

1. In estimating population mean or proportion what is the width of an interval?

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2. If 25 college students out of 80 graduate in 2 years, then by using 90% confidence level find the confidence interval for the proportion of all college students who graduate in 2 years.

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3. If 50 college students out of 125 graduate in 2 years, then by using 99% confidence level find the confidence interval for the proportion of all college students who graduate in 2 years.

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4. The test scores for the for Abe's stat class from 8 randomly selected students are as such 84, 79, 95, 94, 77, 88, 85, 92. By using 90% confidence level, find the confidence interval for the average score for all Abe's stat class.

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5. Redo prob. 4 by using 99% confidence level.

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6. If the mean time to finish a refinance application for 36 applicants is 90 minutes with a standard deviation of 20, then by using 95% confidence level find the confidence interval for the mean time to finish a refinance application.

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7. If the mean time to finish a refinance application for 16 applicants is 90 minutes with a standard deviation of 20, then by using 95% confidence level find the confidence interval for the mean time to finish a refinance application.

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8. Suppose that we check for clarity in 25 locations in Lake Tahoe and discover that the average depth of clarity of the lake is 14 feet with a standard deviation of 2 feet. What can we conclude about the average clarity of the lake with a 90% confidence level?

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9. Suppose that we conduct a survey of 19 millionaires to find out what percent of their income the average millionaire donates to charity. We discover that the mean percent is 15 with a standard deviation of 5 percent. Find a 95% confidence interval for the mean percent.

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10. A survey of 100 married couples was conducted to find out how many months they dated before getting married. The sample mean was 11.41 with a sample deviation of 3.8. Find a 95% confidence interval for the true average number of months dated among all married couples

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11. As part of his class project, a Statistics student took a random sample of 50 college students and recorded how many hours a week they spent on the internet. The sample had an mean of 6.9 hrs. Calculate the 90% confidence interval for average internet usage among college students. Assume that the standard deviation of internet usage for college students is known to be 2.5 hrs/week.

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12. The current method for treating a certain disease has a 37.3% cure rate. Researchers have developed what they think is a more successful treatment. The researchers received permission from the FDA to conduct clinical trials. A random sample of 150 people suffering from the disease is given the alternative treatment. 57 people from the sample are declared cure. Setup a 95% confidence interval for the true population proportion of people cured of the disease. **(Write your answers in percentages with 2 decimal places)!!!!**

13. A recent study concluded that 445 of teenagers cite grades as their greatest source of pressure. The study was based on responses from 1,015 teenagers. What is the 99% confidence interval for percentage of teenagers that cite grades as their greatest source of pressure?

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14. Suppose that we check for clarity in 50 locations in Lake Tahoe and discover that the average depth of clarity of the lake is 14 feet with a standard deviation of 2 feet. What can we conclude about the average clarity of the lake with a 95% confidence level?

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15. How much time do students spend to prepare for a Statistics final exam? To answer this question, a random sample of 40 Statistics students was selected. The sample revealed an average of 5.5 hrs, and a standard deviation of 1.5 hrs. Construct a 95% Confidence Interval for the average number of hours that students spend preparing for a Statistics exam.

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16. A random sample of 500 points on a heated plate resulted in an average temperature of 73.54 degrees Fahrenheit with a standard deviation of 2.79 degree. Find a 99% confidence interval for the average temperature of the plate.

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17. What percentage of college students have made at least one online purchase in the last three months? To answer this question, a market researcher surveyed 200 college students. Of those surveyed, 76 said that they had made at least one online purchase. Calculate the appropriate 95% confidence interval and briefly explain what this interval means. **(Write your answers in percentages with 2 decimal places)!!!**

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18. The Burger King Corporation claims that the average weight for its pre-cooked burgers is 0.25 lb. with a St. dev. of 0.030 lb. The FDA is skeptical of this claim due to an increase in the number of complaints regarding the weight of the burgers. The FDA goes to a region and takes a random sample of 100 burgers. The average weight for the burgers is 0.248 lb. Setup a 95% confidence interval for the population mean.

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19. A new study based on the top 400 rental films concluded that 98% of films involve drugs, drinking, or smoking. What is the 96 % confidence interval for percentage of films that involve drugs, drinking, or smoking? Do you believe that the top 400 films represent a random sample? Explain.

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20. In a random sample of 1600 people from Sin city, 900 will support the mayor in the next election. Based on this sample, would you claim that the mayor will win a majority of the vote? Explain

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21. A poll finds that 41% of population approves of the job that the President is doing: The poll has a margin of error 4.5%. Find a 95% confidence interval for the percentage of population that approves President's performance. What was the sample size for this poll?

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22. How large a sample must we take to obtain 90% confidence interval estimate of the proportion of students who pass stat class for the first time, if the max. error of our confidence width to be .10?

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23. You want to construct a 90% confidence interval for the percent of registered voters who are planning on voting for the current governor for his second term. You want to have a margin of error of 0.03. How many registered voters should you survey?

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24. A consumer agency wants to estimate the proportion of all drivers who were seat belts while driving. Assume that a prior study has shown that 46% of drivers wear seatbelts while driving. How large the sample size be so that the 95% confidence interval for the population proportion has a maximum error of .04?

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25. How large should the sample size be if we want to estimate the true average time to finish a refinance application with 99% confidence level when previous study results with a st. dev of 20 and the error is accepted to be 4 min?

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26. How large should the sample size be if we want to estimate the true average time to finish a refinance application with 99% confidence level when previous study results with a st. dev of 20 and the maximum error is accepted to be 2 minutes? **What happened to sample size when error was cut in half?**

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27. What should be the sample size for a 95% confidence interval for  $\mu$  to have a maximum error equal to .50 and standard deviation equal to 8?

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28. What should be the sample size for a 95% confidence interval for  $\mu$  to have a maximum error equal to **1.0** and standard deviation equal to 8? What happened to sample size when error was doubled?

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29. Nationally, 2% of the population carries a venereal disease. You are interested in constructing a 95% confidence interval for the percentage of population in the Tahoe Basin who carries a venereal disease. How many people will you need to test if you want a margin of error of 1%?

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30. According to AMA, the average annual earnings of radiologists in the US are \$250,000 and those of surgeons are \$240,000. Suppose that these means are based on random samples of 400 radiologists and 500 surgeons and that the population st. dev. of the annual earnings of radiologists and surgeons are \$30,000 and \$35,000. Construct a 97% confidence interval for the difference between the annual mean earnings of radiologists and surgeons.

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31. I surveyed 50 people from a poor area of town and 70 people from an affluent area of town about their feelings towards minorities. I counted the number of negative comments made. I was interested in comparing their attitudes. The average number of negative comments in the poor area was 14 and in the affluent area was 12. The standard deviations were 5 and 4 respectively. Let's determine a 95% confidence for the difference in mean negative comments.

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32. 300 men and 400 women we asked how they felt about taxing Internet sales. 75 of the men and 60 of the women agreed with having a tax. Find a 90% confidence interval for the difference in proportions of men and women. (Write your answers in percentages with 2 decimal places)!  $< P_m - P_w <$

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33. In a sample of 40 Boston male smokers, vitamin C levels had a mean of 0.60 mg/dl and an SD of 0.32 mg/dl while in a sample of 40 Boston male nonsmokers had a mean of 0.90 mg/dl and an SD of 0.35 mg/dl. Let's determine a 90% confidence for the difference in mean vitamin C levels between smokers and nonsmokers.

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34. There are two surveys; one was carried out in East coast and another in West coast. In both surveys, random samples of 1,400 adults in a country were asked whether they were satisfied with their life. The results in East coast showed 462 were satisfied with their life and in West coast 674 were satisfied with their life. Find a 90% confidence interval for the difference in proportions of adults who are satisfied with their lives between East and West coast.

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35. Your hot sauce company rates its sauce on a scale of spiciness of 1 to 20. A sample of 50 bottles of hot sauce is taste-tested, resulting in a mean of 12 and a sample standard deviation of 2.5. Find a 95% confidence interval for the spiciness of your hot sauce.

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36. When the CEO of your hot sauce company was informed that the spiciness of the hot sauce averages only 12, he was furious and ordered instant adjustments to the recipe, threatening to fire the whole sauce division unless the average spiciness increased to above 13. Yesterday, you randomly

sampled 8 bottles of the new sauce and found an average spiciness of 13.5 with a sample standard deviation of 0.75. Compute the 95% confidence interval for the population mean. Based on the answer, can you be 95% sure that the mean spiciness of the new sauce is above 13?

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37. Repeat problem 36, assuming the sample standard deviation was 0.58.

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38. What is the **relationship** between **error** and determining **sample size**?

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39 What is the **relationship** between **sample size** and confidence interval estimation for the **mean or proportion**?

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40. What is the **relationship** between **confidence level** and **confidence interval** estimation for  $\mu, P$ ?

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41 What assumptions are needed to use a **t-distribution**?

### Answers to Practice Problems

<b>1</b>	<b>2E(twice the error)</b>	<b>2</b>	$E = .0852$ $22.73\% < P < 39.77\%$	<b>3</b>	$E = .113$ $28.7\% < P < 51.3\%$
<b>4</b>	$E = 4.5$ $82.25 < \mu < 91.25$	<b>5</b>	$E = 8.3$ $78.45 < \mu < 95.05$	<b>6</b>	$E = 6.53$ $83.47 < \mu < 96.53$
<b>7</b>	$E = 10.65$ $79.35 < \mu < 100.65$	<b>8</b>	$E = .6844$ $13.316 < \mu < 14.684$	<b>9</b>	$E = 2.41$ $12.59 < \mu < 17.41$
<b>10</b>	$E = .75$ $10.66 < \mu < 12.15$	<b>11</b>	$E = .58$ $6.32 < \mu < 7.48$	<b>12</b>	$E = 7.76\%$ $30.23\% < P < 45.77\%$
<b>13</b>	$E = 4\%$ <b>0</b> $39.84\% < P < 47.84\%$	<b>14</b>	$E = .55$ $13.45 < \mu < 14.55$	<b>15</b>	$E = .4649$ $5.04 < \mu < 5.97$
<b>16</b>	$E = .3219$ $73.22 < \mu < 73.86$	<b>17</b>	$E = 6.73\%$ ( $31.27\% < P < 44.73\%$	<b>18</b>	$E = .0059$ $0.2441 < \mu < 0.2559$
<b>19</b>	$E = 1.44\%$ $97\% < P < 99\%$	<b>20</b>	$E = 24.3\%$ $53.82\% < P < 58.68\%$	<b>21</b>	$n = 459$
<b>22</b>	$E = .05$ $n = 271$	<b>23</b>	$n = 752$	<b>24</b>	$n = 597$
<b>25</b>	$n = 167$	<b>26</b>	$n = 666$	<b>27</b>	$n = 984$
<b>28</b>	$n = 246$	<b>29</b>	$n = 753$	<b>30</b>	$E = 4704.44$ $5295.56 < \mu_R - \mu_S < 14704.44$
<b>31</b>	$E = 1.67\%$ $0.33 < \mu_P - \mu_A < 3.67$	<b>32</b>	$E = 5.05\%$ $4.95\% < P_m - P_w < 15.05\%$	<b>33</b>	$E = 0.12$ $-0.42 < \mu_{ns} - \mu_s < -0.18$
<b>34</b>	$E = 3.02\%$ $-18.3\% < P_{EC} - P_{WC} < -12.27\%$	<b>35</b>	$E = 0.69$ $11.31 < \mu < 12.69$	<b>36</b>	$E = 0.63$ $12.87 < \mu < 14.13$
<b>37</b>	$E = 0.48$ $13.02 < \mu < 13.98$	<b>38</b>	<b>Error and sample size are squarely inverse related</b>	<b>39</b>	<b>Larger sample size the narrower interval or Smaller sample size the wider interval.</b>
<b>40</b>	<b>Higher</b> the confidence level the <b>larger</b> the interval <b>or Lower</b> the confidence level the <b>narrower</b> the interval	<b>41</b>	$n < 30$ and $\sigma$ is unknown		

**Solution****Practice Problems for Part # 3**

1. It is  $2E =$  twice the error

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2. If 25 college students out of 80 graduate in 2 years, then by using 90% confidence level find the confidence interval for the proportion of all college students who graduate in 2 years.

$$x = 25 \quad n = 80 \quad \hat{p} = 25/80 = .3125 \quad z_{.90} = 1.645 \quad E = 1.645 \sqrt{\frac{.3125(1-.3125)}{80}} = .0852,$$

$$P = \hat{p} \pm E \quad P = 31.2\% \pm 8.52\% \quad 22.73\% < P < 39.77\%$$


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3. If 50 college students out of 125 graduate in 2 years, then by using 99% confidence level find the confidence interval for the proportion of all college students who graduate in 2 years.

$$x = 50 \quad n = 125 \quad \hat{p} = 50/125 = 0.4 \quad z_{.99} = 2.58 \quad E = 2.58 \sqrt{\frac{0.4(1-0.4)}{125}} = .1130$$

$$P = \hat{p} \pm E \quad P = 40\% \pm 11.30\% \quad 28.70\% < P < 51.3\%$$


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4. The test scores for Abe's stat class from 8 randomly selected students are as such 84, 79, 95, 94, 77, 88, 85, 92. By using 90% confidence level, find the confidence interval for the average score for all Abe's stat class.

$$n = 8 \quad \bar{x} = 86.75 \quad s = 6.71 \quad df = 7 \quad t_{.90} = 1.895 \quad E = 1.895 \frac{6.71}{\sqrt{8}} = 4.5$$

$$\mu = \bar{x} \pm E \quad \mu = 86.75 \pm 4.5 \quad 82.25 < \mu < 91.25$$


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5. Redo prob. 4 by using 99% confidence level.

$$n = 8 \quad \bar{x} = 86.75 \quad s = 6.71 \quad df = 7 \quad t_{.99} = 3.499 \quad E = 3.499 \frac{6.71}{\sqrt{8}} = 8.3$$

$$\mu = \bar{x} \pm E \quad \mu = 86.75 \pm 8.3 \quad 78.45 < \mu < 95.05$$


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6. If the mean time to finish a refinance application for 36 applicants is 90 minutes with a standard deviation of 20, then by using 95% confidence level find the confidence interval for the mean time to finish a refinance application.

$$n = 36 \quad \bar{x} = 90 \quad s = 20 \quad z_{.95} = 1.96 \quad E = 1.96 \frac{20}{\sqrt{36}} = 6.53$$

$$\mu = \bar{x} \pm E \quad \mu = 90 \pm 6.53 \quad 83.47 < \mu < 96.53$$


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7. If the mean time to finish a refinance application for 16 applicants is 90 minutes with a standard deviation of 20, then by using 95% confidence level find the confidence interval for the mean time to finish a refinance application.

$$n = 16 \quad \bar{x} = 90 \quad s = 20 \quad df = 15 \quad t_{.95} = 2.131 \quad E = 2.131 \frac{20}{\sqrt{16}} = 10.65$$

$$\mu = \bar{x} \pm E \quad \mu = 90 \pm 10.65 \quad 79.35 < \mu < 100.65$$


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8. Suppose that we check for clarity in 25 locations in Lake Tahoe and discover that the average depth of clarity of the lake is 14 feet with a standard deviation of 2 feet. What can we conclude about the average clarity of the lake with a 90% confidence level?

$$n = 25 \quad \bar{x} = 14 \quad s = 2 \quad df = 24 \quad t_{.90} = 1.711 \quad E = 1.711 \frac{2}{\sqrt{25}} = .6844$$

$$\mu = \bar{x} \pm E \quad \mu = 14 \pm .6844 \quad 13.316 < \mu < 14.684$$


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9. Suppose that we conduct a survey of 19 millionaires to find out what percent of their income the average millionaire donates to charity. We discover that the mean percent is 15 with a standard deviation of 5 percent. Find a 95% confidence interval for the mean percent.

$$n = 19 \quad \bar{x} = 15 \quad s = 5 \quad df = 18 \quad t_{.95} = 2.101 \quad E = 2.101 \frac{5}{\sqrt{19}} = 2.41$$

$$\mu = \bar{x} \pm E \quad \mu = 15 \pm 2.41 \quad 12.59 < \mu < 17.41$$

10. A survey of 100 married couples was conducted to find out how many months they dated before getting married. The sample mean was 11.41 with a sample deviation of 3.8. Find a 95% confidence interval for the true average number of months dated among all married couples.

$$n = 100 \quad \bar{x} = 11.41 \quad s = 3.8 \quad z_{.95} = 1.96 \quad E = 1.96 \frac{3.8}{\sqrt{100}} = .75$$

$$\mu = \bar{x} \pm E \quad \mu = 11.41 \pm .75 \quad 10.66 < \mu < 12.15$$

11. As part of his class project, a Statistics student took a random sample of 50 college students and recorded how many hours a week they spent on the Internet. The sample reveals an average of 6.9 hrs. Calculate the 90% confidence interval for average internet usage among college students. Assume that the standard deviation of internet usage for college students is known to be 2.5hrs/week.

$$n = 50 \quad \bar{x} = 6.9 \quad s = 2.5 \quad z_{.90} = 1.645 \quad E = 1.645 \frac{2.5}{\sqrt{50}} = .58$$

$$\mu = \bar{x} \pm E \quad \mu = 6.9 \pm 0.58 \quad 16.32 < \mu < 17.48$$

12. The current method for treating a certain disease has a 37.3% cure rate. Researchers have developed what they think is a more successful treatment. The researchers received permission from the FDA to conduct clinical trials. A random sample of 150 people suffering from the disease is given the alternative treatment. 57 people from the sample are declared cure. Setup a 95% confidence interval for the true population proportion of people cured of the disease.

$$x = 57 \quad n = 150 \quad \hat{p} = 57/150 = .38 \quad z_{.95} = 1.96 \quad E = 1.96 \sqrt{\frac{.38(1-.38)}{150}} = .0777$$

$$P = \hat{p} \pm E \quad P = 38\% \pm 7.77\% \quad 30.23\% < P < 45.77\%$$

13. A recent study concluded that 445 of teenagers cite grades as their greatest source of pressure. The study was based on responses from 1,015 teenagers. What is the 99% confidence interval for percentage of teenagers that cite grades as their greatest source of pressure?

$$x = 445 \quad n = 1015 \quad \hat{p} = 445/1015 = .4384 = 43.84\% \quad z_{.99} = 2.58 \quad E = 2.58 \sqrt{\frac{.4384(1-.4384)}{1015}} = .04$$

$$P = \hat{p} \pm E \quad P = 43.84\% \pm 4\% \quad 39.84\% < P < 47.84\%$$

14. Suppose that we check for clarity in 50 locations in Lake Tahoe and discover that the average depth of clarity of the lake is 14 feet with a standard deviation of 2 feet. What can we conclude about the average clarity of the lake with a 95% confidence level?

$$n = 50 \quad \bar{x} = 14 \quad s = 2 \quad z_{.95} = 1.96 \quad E = 1.96 \frac{2}{\sqrt{50}} = 0.55$$

$$\mu = \bar{x} \pm E \quad \mu = 14 \pm .55 \quad 13.45 < \mu < 14.55$$

15. How much time do students spend to prepare for a Statistics final exam? To answer this question, a random sample of 40 Statistics students was selected. The sample revealed an average of 5.5 hrs, and a standard deviation of 1.5 hrs. Construct a 95% Confidence Interval for the average number of hours that students spend preparing for a Statistics exam.

$$n = 40 \quad \bar{x} = 5.5 \quad s = 1.5 \quad z_{.95} = 1.96 \quad E = 1.96 \frac{1.5}{\sqrt{40}} = .4649$$

$$\mu = \bar{x} \pm E \quad \mu = 5.5 \pm 0.4649 \quad 5.04 < \mu < 5.97$$

16. A random sample of 500 points on a heated plate resulted in an average temperature of 73.54 degrees Fahrenheit with a standard deviation of 2.79 degree. Find a 99% confidence interval for the average temperature of the plate.

$$n = 500 \quad \bar{x} = 73.56 \quad s = 2.79 \quad z_{.99} = 2.58 \quad E = 2.58 \frac{2.79}{\sqrt{500}} = .3219$$

$$\mu = \bar{x} \pm E \quad \mu = 73.54 \pm 0.3219 \quad 73.22 < \mu < 73.86$$

17. What percentage of college students have made at least one online purchase in the last three months? To answer this question, a market researcher surveyed 200 college students. Of those surveyed, 76 said that they had made at least one online purchase. Calculate the appropriate 95% confidence interval and briefly explain what this intervals

$$x = 76 \quad n = 200 \quad \hat{p} = 76 / 200 = .38 \quad z_{.95} = 1.96 \quad E = 1.96 \sqrt{\frac{.38(1-.38)}{200}} = .0673 = 6.73\%$$

$$P = \hat{p} \pm E$$

$$P = 38\% \pm 6.73\%$$

$$31.27\% < P < 44.73\%$$

18. The Burger King Corporation claims that the average weight for its pre-cooked burgers is 0.25 lb. with a st. dev. of 0.030 lb. The FDA is skeptical of this claim due to an increase in the number of complaints regarding the weight of the burgers. The FDA goes to a region and takes a random sample of 100 burgers. The average weight for the burgers is 0.248 lb. Setup a 95% confidence interval for the population mean.

$$n = 100 \quad \bar{x} = .25 \quad s = .03 \quad z_{.95} = 1.96 \quad E = 1.96 \frac{.03}{\sqrt{100}} = .0059$$

$$\mu = \bar{x} \pm E$$

$$\mu = 0.25 \pm 0.0059$$

$$0.2441 < \mu < 0.2559$$

19. A new study based on the top 400 rental films concluded that 98% of films involve drugs, drinking, or smoking. What is the 96% confidence interval for percentage of films that involve drugs, drinking, or smoking? Do you believe that the top 400 films represent a random sample? Explain.

$$x = \text{not needed} \quad n = 400 \quad \hat{p} = .98 = 98\% \quad z_{.96} = 2.05 \quad E = 2.05 \sqrt{\frac{.98(1-.98)}{400}} = .0144 = 1.44\%$$

$$P = \hat{p} \pm E$$

$$P = 98\% \pm 1.44\%$$

$$97\% < P < 99\%$$

20. In a random sample of 1600 people from a large city, it is found that 900 support the mayor in the upcoming election. Based on this sample, would you claim that the mayor will win a majority of the vote? Explain

$$x = 900 \quad n = 1600 \quad \hat{p} = 900 / 1600 = .5625 = 56.25\% \quad z_{.95} = 1.96 \quad E = 1.96 \sqrt{\frac{.5625(1-.5625)}{1600}} = .0243 = 2.43\%$$

$$P = \hat{p} \pm E$$

$$P = 56.25\% \pm 2.43\%$$

$$53.82\% < P < 58.68\%$$

21. A poll finds that 41% of population approves of the job that the President is doing. The poll has a margin of error 4.5%. Find a 95% confidence interval for the percentage of population that approves President's performance. What was the sample size for this poll?

$$x = \quad n = \quad \hat{p} = .41 = \quad z_{.95} = 1.96 \quad E = .045 \quad P = \hat{p} \pm E \quad , E = z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} =$$

$$P = \hat{p} \pm E = 41\% \pm 4.5\% \quad 36.5\% < P < 45.5\% \quad n = (z/E)^2 \hat{p}(1-\hat{p}) = n = (1.96/.045)^2 .41(1-.41) = 459$$

22. How large a sample must we take to obtain 90% confidence interval estimate of the proportion of students who pass stat class for the first time, if the maximum error of our confidence width to be .10?

$$\text{The width is .10 that means } \pm E = .10 \rightarrow E = .05 \quad n = (z/E)^2 \hat{p}(1-\hat{p}) = (1.645/.05)^2 .5(1-.5) = 271$$

23. You want to construct a confidence a 90% interval for the percent of registered voters who are planning on voting for the current governor for his second term. You want to have a margin of error of 0.03. How many registered voters should you survey

$$n = (z/E)^2 \hat{p}(1-\hat{p}) = (1.645/.03)^2 .5(1-.5) = 752$$

24. A consumer agency wants to estimate the proportion of all drivers who were seat belts while driving. Assume that a prior study has shown that 46% of drivers wear seatbelts while driving. How large the sample size be so that the 95% confidence interval for the population proportion has a maximum error of .04?

$$n = (z/E)^2 \hat{p}(1-\hat{p}) = (1.96/.04)^2 .46(1-.46) = 597$$

25. How large should the sample size be if we want to estimate the true average time to finish a refinance application with 99% confidence level when previous study results with a st. dev of 20 and the error is 4 min?



$$n = (s z / E)^2 = (20 \times 2.58 / 4)^2 = 167$$

26. How large should the sample size be if we want to estimate the true average time to finish a refinance application with 99% confidence level when previous study results with a st. dev of 20 and the maximum error is accepted to be 2 minutes? **What happened to sample size when error was cut in half?**  $n = (s z / E)^2 = (20 \times 2.58 / 2)^2 = 666$  It became four times larger
27. What should be the sample size for a 95% confidence interval for  $\mu$  to have a maximum error equal to .50 and standard deviation equal to 8?  $n = (s z / E)^2 = (8 \times 1.96 / .5)^2 = 984$

28. What should be the sample size for a 95% confidence interval for  $\mu$  to have a maximum error equal to and standard deviation equal to 8? **What happened to sample size when error was doubled?**  
 $n = (s z / E)^2 = (8 \times 1.96 / 1)^2 = 246$  The sample size became 4 times less.

29. Nationally, 2% of the population carries a venereal disease. You are interested in constructing a 95% confidence interval for the percentage of population in the Tahoe Basin who carries a venereal disease. How many people will you need to test if you want a margin of error of 1%?

$$n = (z / E)^2 \hat{p}(1 - \hat{p}) = (1.96 / .01)^2 .02(1 - .02) = 753$$

30. According to AMA, the average annual earnings of radiologists in the US are \$250,000 and those of surgeons are \$240,000. Suppose that these means are based on random samples of 400 radiologists and 500 surgeons and that the population st. dev. of the annual earnings of radiologists and surgeons are \$30,000 and \$35,000. Construct a 97 % confidence interval for the difference between the mean annual earnings of radiologists and surgeons  $\mu_R - \mu_S = ?$

	Radiologist	surgeons
n	400	500
$\bar{x}$	250,000	240,000
s	30,000	35,000

Point estimate  $= (\bar{x}_R - \bar{x}_S) = (250,000 - 240,000) = 10,000$   $E = z \sqrt{\frac{s_R^2}{n_R} + \frac{s_S^2}{n_S}} = 2.17 \sqrt{\frac{30,000^2}{400} + \frac{35,000^2}{500}} = 4704.44$

$$\mu_R - \mu_S = (\bar{x}_R - \bar{x}_S) \pm E = 10,000 \pm 4704.44 \quad 5295.56 < \mu_R - \mu_S < 14704.44$$

31. I surveyed 50 people from a poor area of town and 70 people from an affluent area of town about their feelings towards minorities. I counted the number of negative comments made. I was interested in comparing their attitudes. The average number of negative comments in the poor area was 14 and in the affluent area were 12. The standard deviations were 5 and 4 respectively. Let's determine a 95% confidence interval for the difference in mean negative comments.  $\mu_P - \mu_A = ?$

	Poor	Affluent
n	50	70
$\bar{x}$	14	12
s	5	4

Point estimate  $= (\bar{x}_P - \bar{x}_A) = (14 - 12) = 2$   $E = z \sqrt{\frac{s_P^2}{n_P} + \frac{s_A^2}{n_A}} = 1.96 \sqrt{\frac{5^2}{50} + \frac{4^2}{70}} = 1.67$

$$\mu_P - \mu_A = (\bar{x}_P - \bar{x}_A) \pm E = 2 \pm 1.67 \quad 0.33 < \mu_P - \mu_A < 3.67$$

32. 300 men and 400 women we asked how they felt about taxing Internet sales. 75 of the men and 60 of the women agreed with having a tax. Find a 90% confidence interval for the difference in proportions of men and women. **(Write your answers in percentages with 2 decimal places)!!!**  $< P_m - P_w <$

	Men	Women
n	300	400
X	75	60
$\hat{p}$	$75/300 = .25=25\%$	$60/400 = .15=15\%$

**Point estimate**  $(\hat{p}_m - \hat{p}_w) = .25 - .15 = .10 = 10\%$

$$E = Z \sqrt{\frac{\hat{p}_m(1-\hat{p}_m)}{n_m} + \frac{\hat{p}_w(1-\hat{p}_w)}{n_w}} = 1.96 \sqrt{\frac{.25(1-.25)}{300} + \frac{.15(1-.15)}{400}} = 1.645 \sqrt{.000625 + .00031875} = .0505 = 5.05\%$$

$$P_m - P_w = (\hat{p}_m - \hat{p}_w) \pm E = 10 \pm 5.05\% \qquad 4.95\% < P_m - P_w < 15.05\%$$

It appears that there were between 4.95% and 15.05% men more than women agreed with having a tax

**33.** In a sample of 40 Boston male smokers, vitamin C levels had a mean of 0.60 mg/dl and an SD of 0.32 mg/dl while in a sample of 40 Boston male nonsmokers had a mean of 0.90 mg/dl and an SD of 0.35 mg/dl. Let's determine a 99% confidence for the difference in mean vitamin C levels between smokers and nonsmokers

	smokers	nonsmokers
n	40	40
$\bar{x}$	.6	.90
s	0.32	0.35

**Point estimate**  $(\bar{x}_{ns} - \bar{x}_s) = 0.6 - 0.9 = -0.3$        $E = z \sqrt{\frac{s_{ns}^2}{n_{ns}} + \frac{s_s^2}{n_s}} = 1.645 \sqrt{\frac{0.35^2}{40} + \frac{0.32^2}{40}} = 0.12$

$$\mu_s - \mu_{sn} = (\bar{x}_s - \bar{x}_{sn}) \pm E = -0.3 \pm 0.12 \qquad -0.42 < \mu_s - \mu_{sn} < -0.18$$

Both sides are negative; it means that nonsmokers have a higher mean of vitamin C than smokers that ranges between 0.18 and 0.42.

**34** There are two surveys, one was carried out in East and another in West coast. In both surveys, random samples of 1,400 adults in a country were asked whether they were satisfied with their life. The results in East coast showed 462 were satisfied with their life and in West coast 674 were satisfied with their life. Find a 90% confidence interval for the difference in proportions of adults who are satisfied with their lives between East and West coast.

	East coast	West coast
n	1400	1400
X	462	674
$\hat{p}$	462/1400 = 0.33 = 33%	674/1400 = .48 = 48%

**Point estimate**  $(\hat{p}_{EC} - \hat{p}_{WC}) = .33 - .48 = -.15 = -15\%$

$$E = Z \sqrt{\frac{\hat{p}_{EC}(1-\hat{p}_{EC})}{n_{EC}} + \frac{\hat{p}_{WC}(1-\hat{p}_{WC})}{n_{WC}}} = 1.645 \sqrt{\frac{.33(1-.33)}{1400} + \frac{.48(1-.48)}{1400}} = .0302 = 3.02\%$$

$$P_{EC} - P_{WC} = (\hat{p}_{EC} - \hat{p}_{WC}) \pm E = -15\% \pm 3.02\% \qquad -18.3\% < P_{EC} - P_{WC} < -12.27\%$$

Both sides are negative, it means that there were people in west coast are between 12.27% and 18.3% are more satisfied with their life than people in East Coast

**35.** Your hot sauce company rates its sauce on a scale of spiciness of 1 to 20. A sample of 50 bottles of hot sauce is taste-tested, resulting in a mean of 12 and a sample standard deviation of 2.5. Find a 95% confidence interval for the spiciness of your hot sauce.

$$n = 50 \qquad \bar{x} = 12 \qquad s = 2.5 \qquad z_{.95} = 1.96 \qquad E = 1.96 \frac{2.5}{\sqrt{50}} = .69 \qquad \mu = \bar{x} \pm E$$

$$\mu = 12 \pm 0.69 \qquad 11.31 < \mu < 12.69$$

It says that, if you repeatedly test 50-bottle random samples of hot sauce and compute the confidence intervals each time, the confidence intervals you get will include the population mean 95% of the time. In that sense, there is a 95% chance that any specific confidence interval (such as the one above) actually contains the population mean. So, you can be 95% "certain" that the mean spiciness of your hot sauce is somewhere between 11.31 and 12.69.

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36. When the CEO of your hot sauce company was informed that the spiciness of the hot sauce averages only 12, he was furious and ordered instant adjustments to the recipe, threatening to fire the whole sauce division unless the average spiciness increased to above 13. Yesterday, you randomly sampled 8 bottles of the new sauce and found an average spiciness of 13.5 with a sample standard deviation of 0.75. Compute the 95% confidence interval for the population mean. Based on the answer, can you be 95% sure that the mean spiciness of the new sauce is

above 13?  $n = 8$        $\bar{x} = 13.5$        $s = .75$        $t_{.95} = 2.365$        $E = 2.365 \frac{.75}{\sqrt{8}} = .63$        $\mu = \bar{x} \pm E$

$\mu = 13.5 \pm 0.63$        $12.87 < \mu < 14.13$       No, because the lower boundary of our estimation is lower than 13.

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37. Repeat prob. 36 assuming the sample standard deviation was 0.58.

$n = 8$        $\bar{x} = 13.5$        $s = .58$        $t_{.95} = 2.365$        $E = 2.365 \frac{.58}{\sqrt{8}} = .48$        $\mu = \bar{x} \pm E$

$\mu = 13.5 \pm 0.48$        $13.02 < \mu < 13.98$

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The calculation is almost identical to the one above, except for the value  $s = 0.58$ , which gives the new confidence interval [13.02, 13.98]. Since this interval does not contain 13, we can be 95% certain that the mean spiciness of all the sauce is above 13.

38. What is the **relationship** between **error** and determining **sample size**?

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39. What is the **relationship** between **sample size** and confidence interval estimation for the **mean or proportion**?

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40. What is the **relationship** between **confidence level** and confidence interval estimation for  $\mu$ ,  $P$ ?

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41. What assumptions are needed to use a **t-distribution**?  $n < 30$  and  $\sigma$  is unknown