

Help can be found in class lecture, topics review or related PowerPoints

- What are the properties of a normal distribution?
- Where mean, median and mode are located in a normal distribution?
- What is the total area under normal probability distribution curve?
- Can we interchangeably use the total area under normal probability distribution curve as probability?
- What is the mean and standard deviation for standard normal probability distribution?
- What is the name of horizontal axis used in standard normal probability distribution?
- In computing the area or probability, if the upper boundary is missing, then what formula needs to be used?
- In computing the area or probability, if the lower boundary is missing, then what formula needs to be used?
- What is the formula to find the cut-off point value, and where can we look up the z-value?
- How to use **TI calculator** to compute the probability (**normalcdf**) and find the **cut-off point(invNorm)**?

Online Calculator: http://onlinestatbook.com/2/calculators/normal_dist.html

Online Calculator: http://onlinestatbook.com/2/calculators/normal_dist.html

YouTube TI Calculator: <https://www.youtube.com/watch?v=SERkAlt1Wuk> Finding the area under a SNPD curve

YouTube TI Calculator: <https://www.youtube.com/watch?v=5-AsqGuhdm0> Normalcdf and Invnorm

YouTube TI Calculator: <https://www.youtube.com/watch?v=QaNWTI78Ods> To find cut-off point for a given percentile

A) Finding Area under SNPD:

Be sure to **shade the proper region** and find the area that corresponds to the given probability.

Hint: To create **missing** lower or upper limit, use the formula (lower limit = $\mu - 5\sigma$, upper limit = $\mu + 5\sigma$)

| | | | | | | | |
|----------|-------------------------|-----------|-----------------------|-----------|-----------------------|-----------|-------------------------|
| 1 | $P(-5 < Z < 5) =$ | 2 | $P(-1.8 < Z < -.8) =$ | 3 | $P(.5 < Z < 1.5) =$ | 4 | $P(-2.11 < Z < 1.55) =$ |
| 5 | $P(-1.17 < Z < 1.34) =$ | 6 | $P(1.2 < Z < 1.6) =$ | 7 | $P(-2.0 < Z < -.5) =$ | 8 | $P(Z > -1.75)$ |
| 9 | $P(Z > 3.04) =$ | 10 | $P(Z < 1.08) =$ | 11 | $P(Z > -1.4) =$ | 12 | $P(Z < 1.57) =$ |

Answers:

| | | | | | | | |
|----------|-------|-----------|-------|-----------|-------|-----------|-------|
| 1 | 1 | 2 | .1760 | 3 | .2417 | 4 | .9220 |
| 5 | .7889 | 6 | .0603 | 7 | .2857 | 8 | .9599 |
| 9 | .0012 | 10 | .8599 | 11 | .9192 | 12 | .9418 |

B) Cut-off point practices: Use TI calculator (**invNorm**) and/or **page 3 of table** (under part two course materials) to find the Z-value that separates

- the bottom 5% **TI Instruction** 2nd, Dist, InvNorm (0.05, 0, 1) = **-1.645**
- the top 5% **TI Instruction** 2nd, Dist, InvNorm (0.95, 0, 1) = **1.645**
- the bottom 10% **Ans: -1.28** d) the top 10% **Ans: 1.28** e) the middle 80% = ± 1.28

C) If the average price for textbooks in a college university is \$75 with standard deviation of 20. Assuming that data are normally distributed then what percentage of college books is,

Hint: To create **missing** lower or upper limit, use the formula (lower limit = $\mu - 5\sigma$, upper limit = $\mu + 5\sigma$)

- Between 60 and 80 dollars **TI** 2nd, Dist, Normalcdf (60, 80, 75, 20) = **37.21 %**
- Less than 70 dollars **TI** 2nd, Dist, Normalcdf (0, 70, 75, 20) = **40.13 %**
- More than 50 dollars **TI** 2nd, Dist, Normalcdf (50, 175, 75, 20) = **89.44 %**
- Within 25 dollars of the mean = 75 ± 25 **TI** 2nd, Dist, Normalcdf (50, 100, 75, 20) = **78.88%**
- Find the price for the top 8% of expensive of textbooks. **TI** 2nd, Dist, InvNorm (0.92, 75, 20) = \$ **103.20**
- Find the price for the lowest 25% (Q1) of inexpensive of textbooks. **TI** 2nd, Dist, InvNorm (0.25, 75, 20) = \$ **61.40**
- Find the price for the top 25% (Q3) of expensive of textbooks. **TI** 2nd, Dist, InvNorm (0.75, 75, 20) = \$ **88.05**
- Find the 45th percentile (P_{45}) price for the textbooks. **TI** 2nd, Dist, InvNorm (0.45, 75, 20) = \$ **72.49**

Hint: To create **missing** lower or upper limit, use the formula (lower limit = $\mu - 5\sigma$, upper limit = $\mu + 5\sigma$)

1. The mean life of a tire is 30 000 km. The standard deviation is 2000 km.

- a) 68% of all tires will have a life between _____ km and _____ km. **You try!**
 - b) 95% of all tires will have a life between _____ km and _____ km. **You try!**
 - c) What percent of the tires will have a life that exceeds 26 000 km? **97.72%**
 - d) If a company purchased 2000 tires, how many tires would you expect to last more than 28 000 km? **1683**
-

2. A line for tickets to a local concert had an average waiting time of 20 min. and a $\sigma = 4$ min.

What percentage of the people in line waited for more than 28 minutes? **2.27%**

- a) What is the Z-Score for people who waited 16 minutes? **-1**
 - b) If 2000 ticket buyers were in line, how many of them would expect to wait for less than 16 minutes? **317**
 - c) What is the Z -score for people who waited 15 minutes? **You try!**
 - d) What is the probability that a person waited at least 15 minutes? **89.43%**
-

3. In an Oreo factory, the mean mass of a cookie is given as 40g. For quality control, the σ is 2 g.

- a) If 10 000 cookies were produced, how many cookies are within 2 g of the mean? **(.68268)=6827**
 - b) Cookies are rejected if they weigh more than 44 g or less than 36 g. How many cookies would you expect to be rejected in a sample of 10 000 cookies? **455**
 - c) How many standard deviations away from the mean is an Oreo that weighs 49 g? **4.5**
 - d) What is the probability that a randomly selected cookie weighs between 36.3 g and the 50th percentile? **46.78%**
-

4. A grading scale is set up for 1000 students' test scores. It is assumed that the scores are normally distributed with a mean score of 75 and a standard deviation of 15

- a) If 60 is the lowest passing score, how many students are expected to pass the test? **(.7936)=794**
 - b) What score would a student have to score to be in the 64th percentile? **Score of 80.37**
 - c) What score would a student have to make to be in the top 25% of the class? **Score of 85**
-

5. Women's heights are normally distributed with $\mu = 63.6$ inches and $\sigma = 2.5$ inches. Suppose that a modeling agency will only accept the tallest 10% of women. What is the cutoff height that the agency uses (ie, how tall would a woman have to be to be hired by the agency)? **Minimum of 66.80 inch**

6. At two years of age, sardines inhabiting Japanese waters have a length distribution that is approximately normal with mean 22.80 cm and standard deviation 0.65 cm.

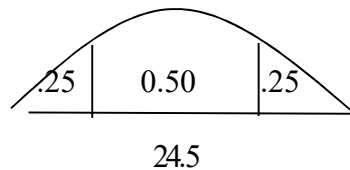
- a. How long are the longest 13% of all these sardines? **23.53 cm**
- b. How short are the shortest 13% of all these sardines? **22.07 cm**
- c. In what range do the middle 74% of all sardines fall? **You try!**

7. Porphyrin is a pigment in blood protoplasm and other body fluids that is significant in body energy and storage. In healthy Alaskan brown bears, the amount of porphyrin in the bloodstream (in mg/dL) has approximate normal distribution with mean 47.5 and standard deviation 10.2.
- What proportion of these bears have between 31.5 and 63.5 mg/dL porphyrin in their bloodstream? (That's within 16 of the mean amount of 47.5.) **88.33%**
 - How low are the porphyrin levels for the lowest 5% of all the bears? **30.72 mg/Dl**

8. Healthy 10-week-old domesticated kittens have average weight 24.5 oz. with a standard deviation of 5.5 oz. The distribution is approximately normal.
- A kitten is designated as dangerously underweight when, at 10 weeks, it weighs less than 10.25 oz. What proportion of healthy kittens will be designated as dangerously underweight? **0.48%**
 - What is the median weight of the kittens? **You try!**
 - What are the first and third quartiles of the kitten weights? (25% of the kittens weigh less than Q_1 ; 75% weigh more. 75% of the kittens weigh less than Q_3 ; 25% weigh more.) What is the interquartile range (IQR) for the kitten weights? **Between 20.79 and 28.21**

$$\mu = 24.5$$

$$\sigma = 5.5$$



TI Instruction

Left Boundary 2nd, Dist, InvNorm (.25, 24.5, 5.5) = 20.79

Right Boundary 2nd, Dist, InvNorm (.75, 24.5, 5.5) = 28.21

9. I.Q. scores are normally distributed with a mean of 100 and a standard deviation of 15.
- People are considered "intellectually very superior" if their score is above 130. What percentage of people fall into that category? **2.27%**
 - If we redefine the category of "intellectually very superior" to be scores in the top 1%, what does the minimum score become? **135**

10. The lengths of human pregnancies are approximately normally distributed with a mean of 266 days and a standard deviation of 16 days.
- a wife claimed to have given birth 308 days after a brief visit from her husband, who was serving in the Navy. Find the probability of a pregnancy lasting more than 308 days. What does the result suggest? **0.43%**
 - If we stipulate that a baby is premature if the length of the pregnancy is in the lowest 4%, find the length (in days) that separates premature babies from those who are not premature. **238 days**

11. Scores on the SAT form a normal distribution with $\mu = 500$ and $\sigma = 100$.
- What is the minimum score necessary to be in the top 15% of the SAT distribution? **604**
 - Find the range of values that defines the middle 80% of the distribution of SAT scores (372 and 628). Find the z-scores **-1.28, 1.28**

12. For a normal distribution, find the z-score that separates the distribution as follows:
- Separate the highest 30% from the rest of the distribution. **0.52**
 - Separate the lowest 40% from the rest of the distribution. **0.25**
 - Separate the highest 75% from the rest of the distribution. **-0.67**
-
13. A patient recently diagnosed with Alzheimer's disease takes a cognitive abilities test and scores a 45. The mean on this test is 52 and the standard deviation is 5. What is the patient's percentile rank? **8.1%**
-
14. A fifth grader takes a standardized achievement test ($\mu = 125$, $\sigma = 15$) and scores a 148. What is the child's percentile rank? **94%**