

STAT: Confidence Intervals

Calculate the confidence interval for one item representing each of the formulas. In all cases the underlying population must be normally distributed

$$n > 30$$

$\sigma =$ Unknown and $n \leq 30$

$$\text{A) } \mu = \bar{x} \pm z_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right)$$

$$\text{B) } \mu = \bar{x} \pm t_{\alpha/2, df} \left(\frac{s}{\sqrt{n}} \right)$$

$$\text{C) } P = \hat{p} \pm z_{\alpha/2} \left(\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$$

- 1) On day two of a study on body temperatures, 106 temperatures were taken. Suppose that we only have the first 10 temperatures to work with. The mean and standard deviation of these 10 scores were 98.44°F and 0.30°F, respectively. Construct a 95% confidence interval for the mean of all body temperatures. **Answer:** $98.22 < \mu < 98.65$
- 2) In a time use study 20 randomly selected managers were found to spend a mean time of 2.4 hours per day on paperwork. The standard deviation of the 20 scores was 1.30 hours. Construct a 98% confidence interval for the mean time spent on paperwork by all managers. **Answer:** $1.66 < \mu < 3.14$
- 3) A random sample of 19 women results in a mean height of 63.85 inches. Other studies have shown that women's heights are normally distributed with a standard deviation of 2.5 inches. Construct a 90% confidence interval for the mean height of all women. **Answer:** $62.86 < \mu < 64.84$
- 4) The National Center for Education Statistics surveyed 4400 college graduates about the lengths of time required to earn their bachelor's degrees. The mean was 5.15 years and the standard deviation was 1.68 years. Based on the above information, construct a 98% confidence interval for the mean time required to earn a bachelor's degree by all college students. **Answer:** $5.09 < \mu < 5.21$
- 5) A random sample of 60 female members of health clubs in Los Angeles showed that they spend on average 4 hours per week doing physical exercise with a standard deviation of .75 hours. Find a 95% confidence interval for the population mean μ . **Answer:** $3.81 < \mu < 4.19$
- 6) A random sample of 20 married women showed that the mean time spent on housework by them was 29.8 hours a week with a standard deviation of 6.7 hours. Find a 95% confidence interval for the mean time spent on housework per week by all married women. **Answer:** $26.66 < \mu < 32.94$
- 7) A fleet of 100 airplanes has an air time (time spent flying) standard deviation of 14.9 hours. A sample of 32 of these planes gave a mean air time of 49 hours. Construct a 90% confidence interval on the mean air time for this fleet. **Answer:** $44.67 < \mu < 53.33$
- 8) Automotive engineers are continually improving their products/ Suppose a new type of brake light has been developed by General Motors. As part off a product safety evaluation program General Motors' engineers wish to estimate the mean driver response time to the new brake light. Fifty drivers are selected at random and the response time (in seconds) for each driver is recorded, yielding the following results: $\bar{x} = .72$ and $s = .022$. Construct a 99% confidence interval for the mean response time. **Answer:** $.712 < \mu < .728$

- 9) A random sample of 45 life insurance policy holders showed that the average premiums paid on their life insurance policies was \$340 per year with a standard deviation of \$62. Construct a 90% confidence interval for the population mean. **Answer:** $324.8 < \mu < 355.2$
- 10) A process has been developed that can transform ordinary iron into a kind of super iron called *metallic glass*. Metallic glass is three to four times stronger than the toughest steel alloys. To estimate the mean temperature, μ , at which a particular type of metallic glass becomes brittle, 25 pieces of this metallic glass were randomly sampled from a recent production run. Each piece was subjected to higher and higher temperatures until it became brittle. The temperature at which brittleness first appeared was recorded for each piece in the sample. The following results were obtained: $\bar{x} = 480^\circ\text{F}$ and $s = 11^\circ\text{F}$. Construct a 95% confidence interval to estimate μ . **Answer:** $475.46 < \mu < 484.54$
- 11) Health insurers and the federal government are both putting pressure on hospitals to shorten the average length of stay (LOS) of their patients. A random sample of 27 hospitals in one state had a mean LOS in 1998 of 3.8 days and a standard deviation of 1.2 days. Construct a 98% confidence interval to estimate the population mean of the LOS for the state's hospitals in 1998. **Answer:** $3.23 < \mu < 4.37$
- 12) A random sample of 50, 8 ounce cups of black "Early Riser" coffee dispensed by a new machine gave a mean of 11.0 mg. of caffeine. It is known from previous studies that the standard deviation for 8 oz. cups of black "Early Riser" coffee dispensed by this machine was 1.2 mg. Construct a 90% confidence interval for μ , the mean caffeine content for cups dispensed by this machine. **Answer:** $10.72 < \mu < 11.28$
- 13) The U.S Bureau of the Census conducted a survey of 5000 people and found that the mean income for a person with a bachelor's degree was \$38,973. It is known from previous studies nationwide that the standard deviation in income for a person with a bachelor's degree is \$6,340. Construct a 98% confidence interval for μ , the mean income nationwide for persons with a bachelor's degree. **Answer:** $38764.42 < \mu < 39181.58$
- 14) In a Roper poll of 3000 working men, 56% said "they feel guilty that they don't spend more time with their families." Construct a 98% confidence interval for the proportion of all working men who hold this view. **Answer:** $0.5389 < P < 0.5811$
- 15) A bank took a sample of 100 of its delinquent credit card accounts and found that the mean owed on these accounts was \$2,130. It is known that the standard deviation for all delinquent credit card accounts at this bank is \$578. Give a 97% confidence interval for the mean amount owed on all delinquent credit card accounts for this bank. $3.23 < \mu < 4.37$
- 16) A random sample of 100 movie theaters showed that the mean price of a movie was \$7.00 with a standard deviation of \$.80. Construct a 99% confidence interval for the population mean μ . **Answer:** $2004.57 < \mu < 2255.43$
- 17) In a *Time/CNN* telephone poll of 1012 adult Americans, 11% of the respondents said that Ronald Regan was a great president. Give a 98% confidence interval for the proportion of all adult Americans who think that Regan was a great president. **Answer:** $0.0871 < P < 0.1329$
- 18) Find n : A researcher wants to determine the 99% confidence interval for the mean number of hours per week that adults spend doing community service. How large of a sample should the researcher select so that the estimate will be within 1 hour of the population mean? Assume that the standard deviation for hours spent per week by adults doing community service is 3. **Answer:** $n = 35$

Worksheet: Confidence Intervals for Proportions

1. The paralyzed Veterans of America is a philanthropic organization that relies on contributions. They send free mailing labels and greeting cards to potential donors on their list and ask for voluntary contribution. To test a new campaign they recently sent letters to a random sample of 100,000 potential donors and received 4781 donations.
 - a) Give a 95% confidence interval for the true proportion of those from their entire mailing list who may donate.
 - b) A staff member thinks that the true rate is 5%. Given the confidence interval you found, do you find that percentage plausible?

2. A national health organization warns that 30% of the middle school students nationwide have been drunk. Concerned, a local health agency randomly and anonymously surveys 110 of the middle 1212 middle school students in its city. Only 21 of them report having been drunk.
 - a) What proportion of the sample reported having been drunk?
 - b) Does this mean that this city's youth are not drinking as much as the national data would indicate?
 - c) Create a 95% confidence interval for the proportion of the city's middle school students who have been drunk.
 - d) Is there any reason to believe that the national level of 30% is not true of the middle school students in the city?
 - e) To keep the margin of error at most 5%, how many middle school students do we need to survey?

3. In a poll taken in March of 2007, Gallup asked 1006 national adults whether they were baseball fans. 36% said they were. A year previously 37% of a smaller size sample had reported being baseball fans.
 - a) Find the margin of error for the 2007 poll if we want 90% confidence in our estimate of the percent of national adults who are baseball fans.
 - b) Explain what the margin of error means.

 - c) If we wanted to be 99% confident, would the margin of error be larger or smaller?
 - d) Find the margin of error for 99% confidence level.

- e) In general, all other aspects of the situation remain the same; will smaller margins of error produce greater or less confidence in the interval?
 - f) Do you think there's been a change from 2006 to 2007 in the real proportion of national adults who are baseball fans?
4. Several factors are involved in the creation of a confidence interval. Among them are the sample size, the level of confidence, and the margin of error. Which statements are true?
- a) For a given sample size, higher confidence means a smaller margin of error.
 - b) For a specified confidence level, larger samples provide smaller margins of error.
 - c) For a fixed margin of error, larger samples provide a greater confidence.
 - d) For a given confidence level, halving the margin of error requires a sample twice as large.
 - e) For a given sample size reducing the margin of error will mean lower confidence.
 - f) For a certain confidence level, you can get a smaller margin of error by selecting a bigger sample.
 - g) For a fixed margin of error, smaller samples will mean lower confidence.
 - h) For a given confidence level, a sample 9 times as large will make a margin of error one third as big.