Two Populations: Point and Interval Estimation

For all quizzes in part 3: Be sure you have **formula sheet** and **Table 1**.

Learning Objectives

To estimate **difference between two population Means** $(\mu_1 - \mu_2)$, know how to use the **formula**

$\mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \pm E$ and TI (**option 9**)

To estimate **difference between two population proportions** $(P_1 - P_2)$, know how to use the **formula**

$P_1 - P_2 = (\hat{p}_1 - \hat{p}_2) \pm E$ and TI (**option B**)

Important Note 1: As you study each page of **topics Review,** do all the problems listed at the bottom of the page from practice problem before going to the next page.

Quizzes for Part 3

Be sure you have formula sheet and **Table 1** and **Table 2**.

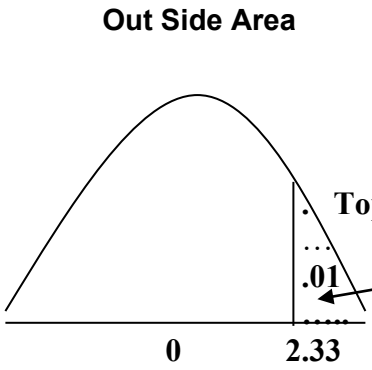
Quiz # 10: This quiz covers **all materials on quiz 8, quiz 9** plus the content of the following pages.

Example:

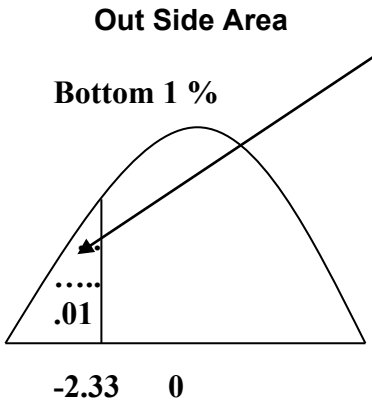
1. Estimate the **average difference** in battery life between Diehard and Everlast brand? $\mu_D - \mu_E = ?$
2. Estimate the **percentage difference** between female and male who pass stat class? $P_f - P_m = ?$

Estimating the ***difference*** between Two Populations ***Means*** or ***Proportions***	
Mean $\mu_1 - \mu_2$	Proportion $P_1 - P_2$
$\mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \pm E$	$P_1 - P_2 = (\hat{p}_1 - \hat{p}_2) \pm E$
Point **estimate** $= (\bar{x}_1 - \bar{x}_2)$	Point **estimate** $= (\hat{p}_1 - \hat{p}_2)$
$E = z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$E = z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$
TI-83/84 ***stat*** $\rightarrow$ ***test*** $\rightarrow$ ***Option 9***	TI-83/84 ***stat*** $\rightarrow$ ***test*** $\rightarrow$ ***B***

Based on Standard Normal Distribution $\mu = 0$ and $\sigma = 1$



OR

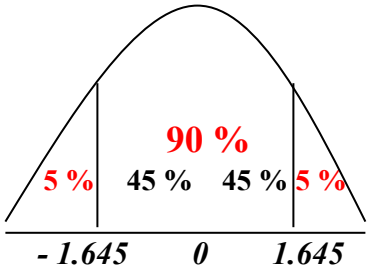


Confidence Level	Out Side Area On left or right Cut-off Point	Z - Value (±) Critical Value $= Z_{\alpha/2}$
99%	.005	± 2.5758
98%	.01	±2.3263
97%	.015	±2.1701
96%	.02	±2.0537
95%	.025	±1.9600
94%	.03	±1.8808
92%	.04	±1.7507
90%	.05	±1.6450
88%	.06	±1.5548
86%	.07	±1.4758
84%	.08	±1.4051
82%	.09	±1.3408
80%	.10	±1.2816
78%	.11	±1.2265
76%	.12	±1.1750
70%	.15	±1.0364
60%	.20	±0.8416
50%	.25	±0.6749
40%	.30	±0.5244

How to find the Z -value for confidence intervals.

Example: Find the Z - value for 90% confidence interval

1. Divide 90% = 0.90 by 2, $\Rightarrow$ $.90 / 2 = 0.45$
2. Subtract 0.45 from 0.5 $\Rightarrow$ $.5 - 0.45 = .05$
3. Look for area close to 0.05from **inside** the table (page1).
4. ***Find its corresponding Z-value (- 1.645)***



TI-83/84 *2nd* $\rightarrow$ *Distr* $\rightarrow$ *Option 3* *input* *(%, 0, 1)*

Example: *2nd* $\rightarrow$ *Distr* $\rightarrow$ *Option 3* *input* *(.05, 0, 1) enter, then the answer will be - 1.645*

Example: *2nd* $\rightarrow$ *Distr* $\rightarrow$ *Option 3* *input* *(.95, 0, 1) enter, then the answer will be 1.645*

Hint for TI *% is the area to the left of the cut off point.* $n > 30$

Estimating Difference between Two Populations Means $(\mu_1 - \mu_2)$

$\mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \pm E$ $\quad E = z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ **(for z- value use table)**

a) What is the point estimate for the **difference** between two population **means** $\mu_1 - \mu_2$?
b) What is the error formula for the **difference** between two population **means**?
c) How to use TI to estimate for the **difference** between two population **means**?
d) What conclusion can we draw if the lower bound of the estimate happened to be zero or negative?
e) What conclusion can we draw if the both bounds of the estimate happened to be negative?

3 different ways the **final** answer may look like.

1) Both end signs **positive** $+ < \mu_1 - \mu_2 < +$ Indicating **group 1** has higher average than **group 2.**
2) **Different** end signs $- < \mu_1 - \mu_2 < +$ I**nconclusive** as which group has higher average.
3) Both end signs **negative** $- < \mu_1 - \mu_2 < -$ Indicating **group 2** has higher average than **group 1.**

1) Use **99%** confidence level to estimate the difference in average life (in months) of "Diehard" and "Everlast" batteries

Diehard	Everlast
$n_1 = 44$	$n_2 = 36$
$\bar{x}_1 = 51.8$	$\bar{x}_2 = 47.4$
$s_1 = 2.5$	$s_2 = 3.7$

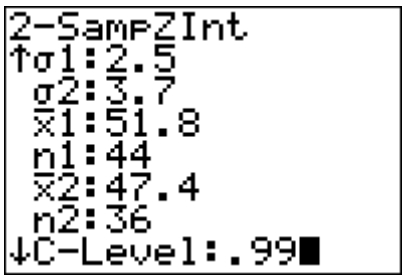
$\mu_1 - \mu_2 = (51.8 - 47.4) \pm E$ $\quad E = 2.5758\sqrt{\frac{2.5^2}{44} + \frac{3.7^2}{36}} = 1.86$

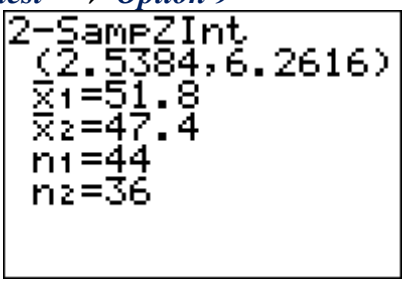
$\mu_1 - \mu_2 = 4.4 \pm 1.86$ $\quad 2.54 < \mu_1 - \mu_2 < 6.26$

Conclusion: Both **signs are positive** indicating **group** 1 **(**Diehard**)** has higher average than **group** 2 **(**Everlast**).** By 99% confidence, Diehard batteries on the average last between 2.54 to 6.26 months longer than Everlast.

Solution by TI 83/84 Calculator

TI-83/84 ***stat*** $\rightarrow$ ***test*** $\rightarrow$ ***Option 9***





$E = (UB - LB) / 2 = (6.26 - 2.54) / 2 = 1.86$

2) Use **90%** confidence level to estimate the difference in average life (in months) of "Diehard" and "Everlast" batteries

	Diehard μ_1	**Everlast** μ_2
n	$n_1 = 40$	$n_2 = 49$
$\overline{x}$	$\overline{x}_1 = 52$	$\overline{x}_2 = 50.5$
S	$s_1 = 5.5$	$s_2 = 4.5$

$\mu_1 - \mu_2 = (\overline{x}_1 - \overline{x}_2) \pm E$ $\quad E = z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

$\mu_1 - \mu_2 = (?-?) \pm E$ $\quad E = ?\sqrt{\frac{?^2}{?} + \frac{?^2}{?}} = ?$

$? < \mu_1 - \mu_2 < ?$

Answer: $-0.279 < \mu_1 - \mu_2 < 3.279$

Conclusion: Because **one side is negative** then we can conclude **that** One brand **does not** have longer average life than the other.

3) Use 99% confidence level to estimate the difference in weights sugar between Regular Coke and Regular Pepsi.

	Regular Coke μ_1	**Regular Pepsi** μ_2
n	$n_1 = 36$	$n_2 = 36$
$\overline{x}$	$\overline{x}_1 = 0.82410$	$\overline{x}_2 = 0.81682$
S	$s_1 = 0.007507$	$s_2 = 0.005701$

$\mu_1 - \mu_2 = (?-?) \pm E$ $\quad E = ?\sqrt{\frac{?^2}{?} + \frac{?^2}{?}} = ?$

$? < \mu_1 - \mu_2 < ?$

Answer: $0.00324 < \mu_1 - \mu_2 < 0.01133$

Conclusion:

3) According to AMA, the average annual earnings of radiologists in the US are \$250,000 and those of surgeons are \$240,000. Suppose that these means are based on random samples of 400 radiologists and 500 surgeons and that the population st. dev. of the annual earnings of radiologists and surgeons are \$30,000 and \$35, 000. Construct a 97 % confidence interval for the difference between the annual mean earnings of radiologists and surgeons

$\mu_R - \mu_S = (?-?) \pm E$ $\quad E = ?\sqrt{\frac{?^2}{?} + \frac{?^2}{?}} = ?$

$? < \mu_R - \mu_S < ?$

Answer $5295.56 < \mu_R - \mu_S < 14704.44$

Estimating Difference between two Populations Proportions ($P_1 - P_2$)

$$P_1 - P_2 = (\hat{p}_1 - \hat{p}_2) \pm E \qquad E = z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$ **(for z- value use table 1)**

a) What is the point estimate for the **difference** between two population **proportions** $P_1 - P_2$?
b) What is the error formula for the **difference** between two population **proportions**?
c) How to use TI to estimate for the **difference** between two population **proportions**?
d) What conclusion can we draw if the lower bound of the estimate happened to be zero or negative?
e) What conclusion can we draw if the both bounds of the estimate happened to be negative?

3 different ways the **final** answer may look like.

1) Both end signs **positive** $+ < P_1 - P_2 < +$ Indicating **group 1** has higher percentage than **group 2**
2) **Different** end signs $- < P_1 - P_2 < +$ **Inconclusive** as which group has higher percentage.
3) Both end signs **negative** $- < P_1 - P_2 < -$ Indicating **group 2** has higher percentage than **group 1.**

YouTube TI Calculator: https://www.youtube.com/watch?v=-YO0_3VqZ1g **Difference of two Proportions**

1) Use **95%** confidence level to estimate the difference in percentage between Female and Male that are passing stat class.

	Female	**Male**
Number of students passed	$x_1 = \mathbf{73}$	$x_2 = \mathbf{87}$
Number of students took stat	$n_1 = \mathbf{100}$	$n_2 = \mathbf{150}$
	$\hat{p}_1 = 73/100 = .73$	$\hat{p}_2 = 87/150 = .58$

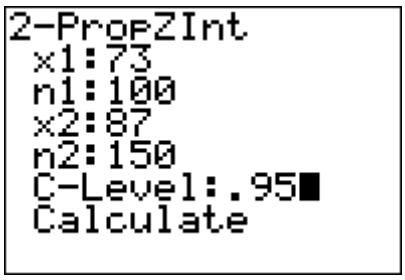
Point Estimate $= \hat{p}_1 - \hat{p}_2 = 0.73 - 0.58 = 0.15$

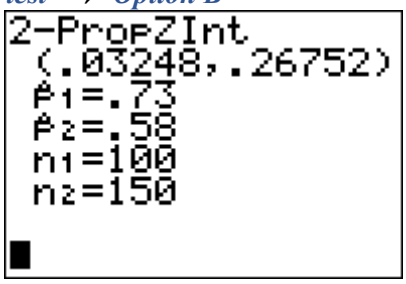
$$E = 1.96\sqrt{\frac{0.73(1-0.73)}{100} + \frac{0.58(1-0.58)}{150}} = 0.118 = 11.8\% \quad P_1 - P_2 = 0.15 \pm 0.118 = 15\% \pm 11.8\%$$

Answer $3.2\% < P_1 - P_2 < 26.8\%$

Conclusion: Both **signs are positive** indicating **group** 1 **(female)** has higher percentage in passing stat class than **group** 2**(male).**

TI-83/84 *stat* → *test* → *Option B*





$0.032 < P_1 - P_2 < 0.268$ $3.2\% < P_1 - P_2 < 26.8\%$

Both **signs are positive** indicating **group** 1 **(female)** has higher percentage in passing stat class than **group** 2**(male).** $E = (UB - LB)/2 = (0.268 - 0.032)/2 = 0.118$

2) Use **90%** confidence level to estimate the difference in percentage between Female and Male that are passing DMV test.

	Female	Male
Number of students passed	$x_1 = \mathbf{30}$	$x_2 = \mathbf{65}$
Number of students took driving test	$n_1 = \mathbf{78}$	$n_2 = \mathbf{80}$
	$\hat{p}_1 =$	$\hat{p}_2 =$

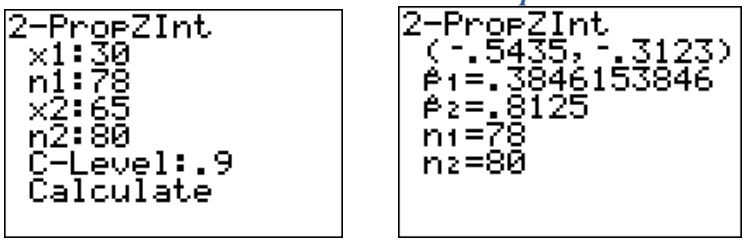
Answer $-54.4\% < P_1 - P_2 < -31.2\%$

$31.2\% < P_2 - P_1 < 54.4\%$

Conclusion:

Comparing Success rate in passing DMV driving test between Female and Male Applicants

TI-83/84 *stat* → *test* → *Option B*



$-0.544 < P_1 - P_2 < -0.312$ $3.123\% < P_2 - P_1 < 54.35\%$

Both signs are negative indicating **group 2 (male)** has higher percentage in passing DMV test than **group 1(female).**

$$E = (UB - LB) / 2 = (-0.3123 - (-0.5435)) / 2 = 0.115$$

$$E = (UB - LB) / 2 = (0.5435 - 0.3123) / 2 = 0.115$$

Conclusion:

3) 300 men and 400 women we asked how they felt about taxing Internet sales. 75 of the men and 60 of the women agreed with having a tax. Find a 90% confidence interval for the difference in proportions of men and women. *(Write your answers in percentages with 2 decimal places)!*

Answer:

$P_m - P_w = (\hat{p}_m - \hat{p}_w) \pm E = 10\% \pm 5.05\%$ $4.95\% < P_m - P_w < 15.05\%$