Hypothesis Testing

7 – Step Process

1. Starting Claim, Opposite Claim
2. Standard Set –up, H_0, H_1
3. Establishing Guideline
4. Collecting Sample (Test Statistics)
5. Drawing Conclusion
6. Comment
7. P-value

Learning Objectives

What do we hypothesize? **Population Parameter** such as **Mean** ($\mu = ?$) or **Proportion** ($P = ?$)
Why do we hypothesize? To investigate any claim about **Population Parameter**
Is **average** weight of cereal boxes 24 oz? Do **average** life of Die hard batteries exceed 60 months?
Is less than **10%** of drivers text while driving? Will more than **45%** of people vote in the next election?

7-Step Process (overview)

Very very important:
From topics review you **must** read and practice one **step at a time.** Read the first step from topics review and then go to pages 3 through 6 and see how that step is done and then continue doing that for all the 7 steps

Step 1: Finding what the starting claim is. Is that about the **average** (μ) or proportopn(P);
Write the starting claim as **SC** and try to oppose it as **OC** in statistical notation

Step 2: Rewriting ***SC*** and ***OC*** as H_0 and H_1

H_0 (**must** have one of the $=$ or $\leq$ or $\geq$ sign) and H_1 (**must** have one of the $\neq$ or $<$ or $>$ sign).
Draw the appropriate graph as ***Left tail***, ***two tails*** or ***right tail***.

Step 3: Finding critical value or values by using the **t- table**,. Critical value depends on **three factors**
a) significance level (α)
b) being one-tailed or two-tailed.
c) sample size (Hint: if $n > 30$ use the bottom of the table otherwise use the top.)

Step 4: (called **Test Statistics**) is using the evidence from our sample and converting that to ***Z or t*** **score** that can be done by **formula or Ti**

Step 5: (called conclusion) is about **step 2** to see if to accept or reject H_0 .

Step 6: (called comment) is about **step 1** to see if to accept or reject **SC (Starting Claim)**.

Step 7: (**p-value**) to read the **p-value** from **TI** screen on step 4 and to find out if it is smaller or larger that significance level (α).

7-Steps of hypothesis testing (Detailed Outline)

1) From the problem write (**SC: Starting Claim**) and then write its (**OC: O**pposing **C**laim) in statistical notation.

		SC	OC
Examples:	**Average** life of "Diehard" batteries exceeds **60** months	$\mu > 60$	$\mu \leq 60$
	Average time to do a certain task is less than **25** minutes	$\mu < 25$	$\mu \geq 25$
	Average net weight of a certain cereal is **24** oz.	$\mu = 24$	$\mu \neq 24$

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2) The next step is rewriting SC, and OC in a new set up called
H_0 (Null Hypothesis), and **H_1** (Alternative Hypothesis):
As how to change **SC,** and **OC** to **H_0**, and **H_1**, you need to follow the next rule remembering that **H_0** (Null Hypothesis) **must** contain some form of equality, and **H_1** (Alternative Hypothesis) **must** contain **no** form of equality. The mathematical setup is explained right below,

H_0 (Null Hypothesis): (contains equal sign) $=$ or $\geq$ or $\leq$
H_1 (Alternative Hypothesis): (contains not equal sign) $\neq$ or $<$ or $>$

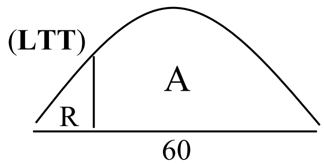
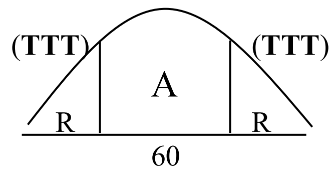
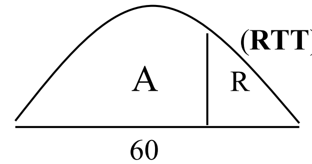
There are **three-possibilities** for setting up the hypothesis (a left-tailed test, two-tailed, right-tailed).

Hint: if $\mathbf{H_1}$: $\mu <$ it is a left-tailed test

if $\mathbf{H_1}$: $\mu \neq$ it is a two-tailed test

if $\mathbf{H_1}$: $\mu >$ it is a right-tailed test

Label the region, as **A** (Accepting H_0), or **R** (Rejecting H_0) Rejections or acceptances labels are based on $\mathbf{H_0}$.

three -possibilities	$\mathbf{H_0}$: $\mu \geq 60$ $\mathbf{H_1}$: $\mu < 60$	$\mathbf{H_0}$: $\mu = 60$ $\mathbf{H_1}$: $\mu \neq 60$	$\mathbf{H_0}$: $\mu \leq 60$ $\mathbf{H_1}$: $\mu > 60$
left-tailed (**LTT**) two-tailed, (**TTT**) right-tailed (**RTT**)			

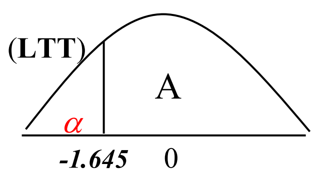
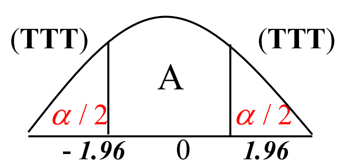
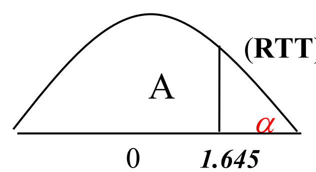
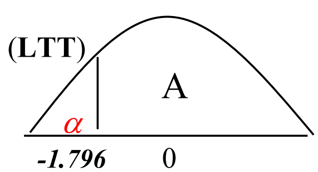
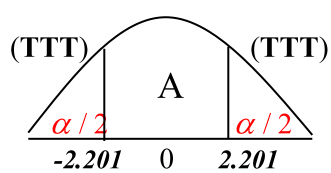
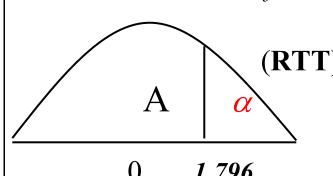
3) What is **Critical value(s)** and how to find it?
Critical value(s) is limit(s) or boundary(ies) that if it is exceeded (by our sample data) then $\mathbf{H_0}$ will be **rejected.**

How to find it? By looking up **t- table**, when we know the followings;

a) Significance level **= α** (Alpha Level) = Critical Region = Critical area = **type I** error
In other words the determining the probability of rejectioing $\mathbf{H_0}$, when $\mathbf{H_0}$ is true.
It is like finding some one to be quilty when he is innocent.
So not that to let that happen we choose **significance level** or **α value** to be small between 1% to 10%.
Hint: If significance level = α is not given assume α =.05 = 5%
Critical Region is also the area designated by Significance level and is shown by **α** or **R**
Also remember if our sample size is 30 or less, then on **Table 2** use **df** = degree of freedom = $n-1$

b) **One-tailed or two-tailed**, and

For **<u>sample sizes</u>** $n > 30$ then use **last row** of **Table 2** to find the critical value(s)..

Given α =.05 and $n > 30$			
For **<u>sample sizes</u>** $n \leq 30$ then use **Table 2**, to find critical value(s). Be sure you find df= degree of freedom = $n-1$			
Given α =.05 and $n \leq 30$ Need to find degree of freedom first!	α =.05 $n = 12$ $df = 11$ 	α =.05 $n = 12$ $df = 11$ 	α =.05 $n = 12$ $df = 11$ 

4. Compute **Test Statistics** (based on sample information) from the following formulas.

a. $z = \dfrac{\sqrt{n}(\overline{x} - \mu)}{s}$ To test the Mean (μ) for large sample sizes

TI-83/84 *stat* $\rightarrow$ *test* $\rightarrow$ *Option 1*

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b. $t = \dfrac{\sqrt{n}(\overline{x} - \mu)}{s}$ To test the Mean (μ) for $n \leq 30$ and, when σ is unknown

TI-83/84 *stat* $\rightarrow$ *test* $\rightarrow$ *Option 2*

`**5) Conclusion**: The decision is made by comparing **Test Statistics** with **Critical value**, and find where the test statistics falls (inside the **CR**: **C**ritical **R**egion or not);
If **Test Statistics** falls inside the **CR**: **C**ritical **R**egion the decision is to **Reject** H_0 or saying that there is sufficient evidence to Reject H_0. If it falls outside the **CR**: **C**ritical **R**egion the decision is to **Fail to Reject H_0** or **Accept H_0** that there is not sufficient evidence to Reject H_0. **When the result of a hypothesis test are determined to be significant then we reject the null hypotheses.**

6) Comment: Decision as to accept or reject **SC(** the stated claim)? **Two possibilities:**

1) If **SC** and H_0 are the same then any decision you make for H_0 will be the same for **SC** and you write that as your comment.

2) If **SC** and H_0 are different then whatever decision you make for H_0, you should make the opposite decision of that for **SC** and you write that as your comment.

7) P-value: It is the **area corresponding to the test statistics** and is always shown on the display of **TI-8 3/84** **as** $P = ($when you compute the test statistics). Basically it is the minimum α - value that is needed to reject the Null hypothesis H_0. **As a rule** you **reject** reject the Null hypothesis **when P-value** is smaller than α **- value**

Type I and Tpe II errors

Remember that we do not know for certain that if H_0 is true or false but after the test is set up, data collected, then we either Accept H_0: or Reject H_0:

The table below summarizes all possible scenarios that might happen when testing procedure is completed.

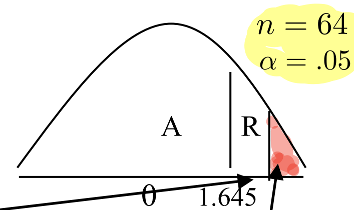
	H_0: True	H_0: False
Accept H_0:	Correct Decision	Type **II** error or called **Beta**(β)
Reject H_0:	Type *I* error or called **Alpha**(α)	Correct Decision = **Power** of a test $1-\beta$

Large Samples about Mean

Example 1. **Average** life of "Die Long" batteries **exceeds** 60 months. A sample of 64 batteries had an average life of 63 months and st. dev. of 10 months. Let **α = .05**

SC: $\mu > 60$ **H₀** : $\mu \le 60$ **Hint**: Use **H₁** to determine if it is LTT ,TTT or RTT test

OC: $\mu \le 60$ **H₁** : $\mu > 60$ **Note:** μ **in H₁ is more than,** then it is a **RTT**



When **α = .05 ,** n > 30 and one –tailed test then by using bottom row of ***Table 2.***

Critical value = CV= Z = 1.645

Test Statistics = $z = \dfrac{\sqrt{n}(\bar{x}-\mu)}{s} = \dfrac{\sqrt{64}(63-60)}{10} = 2.4$ ***Falls inside CR*** **p-value(area from test statistics)**

Conclusion: Accept or reject **H₀**? Inside ***CR*** then reject **Ho**

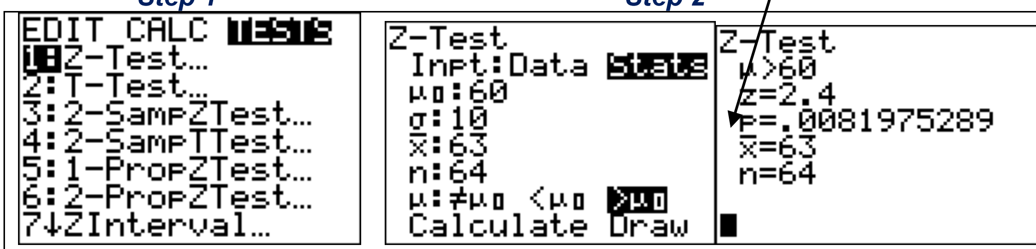
Comment: Accept or reject **SC**? **Accept** that the **average** life of batteries **exceeds 60** months.

P-value: 0 .008 less than **α = .05** reject **Ho** (remember when **p-value is less than α, we reject Ho**)

P-value can be found by TI calculator

TI-83/84 *stat* ⟶ *test* ⟶ *Option 1*

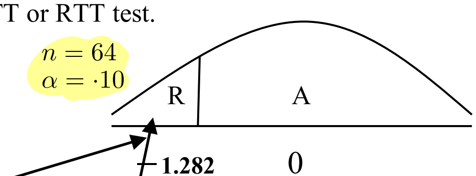
Step 1 ***Step 2*** ***Step 3***



Example 2. **Average** life of "Die Long" batteries is **less than** 60 months. A sample of 64 batteries had an average life of 58 months and st. dev. of 10 months. Let **α =0 .10**

SC: $\mu < 60$ **H₀**: $\mu \ge 60$ **Hint**: Use **H₁** to determine if it is LTT ,TTT or RTT test.

OC: $\mu \ge 60$ **H₁**: $\mu < 60$ **Note:** μ **in H₁ is less than,** then it is a **LTT**



When **α = .10 ,** n > 30 and one –tailed test then by using bottom row of **Table 2.**

Critical value = CV=Z = − 1.282

Test Statistics = $z = \dfrac{\sqrt{n}(\bar{x}-\mu)}{s} = \dfrac{\sqrt{64}(58-60)}{10} = -1.6$ ***Falls inside CR*** **p-value(area from test statistics)**

Conclusion: Accept or reject **H₀**? Inside ***CR*** then reject **Ho**

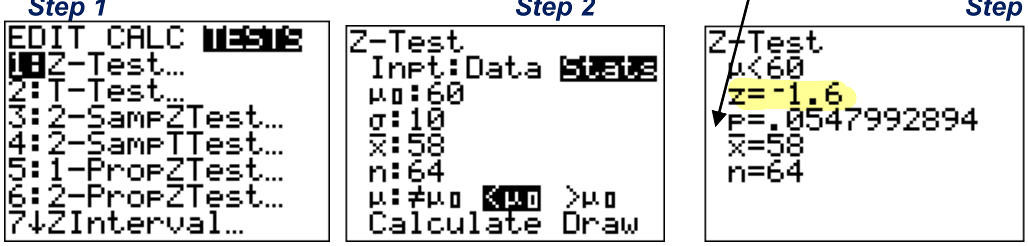
Comment: Accept or reject **SC**? **Accept** that the **average** life of batteries is **less than 60** months

P-value: **0 .0548** less than **α = 0.10** reject **Ho** (remember when **p-value is less than α, we reject Ho**)

P-value can be found by TI calculator

TI-83/84 *stat* ⟶ *tes* ⟶ *Option 1*

Step 1 ***Step 2*** ***Step 3***



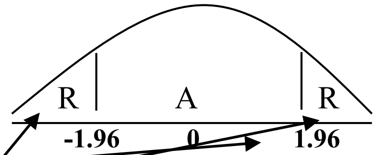
Example 3. **Average** life of "Die Long" batteries **is different** than 60 months. A sample of 64 batteries had an average life of 62 months and st. dev. of 10 months. Let **α = .05**

SC: $\mu \neq 60$ **H_0 :** $\mu = 60$ **Hint**: Use **H_1** to determine if it is LTT ,TTT or RTT test.

OC: $\mu = 60$ **H_1 :** $\mu \neq 60$ **Note:** μ **in H_1 is not equal,** then it is a **TTT**

$n = 64$
$\alpha = .05$

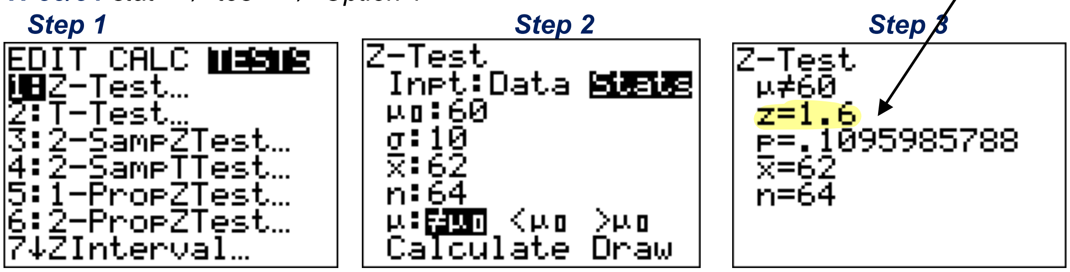
When **α = .05** , n > 30 and two –tailed test then by using bottom row of ***Table 2.***



Critical value = CV= **Z =** $\pm$**1.960**

Test Statistics = $z = \dfrac{\sqrt{n}(\bar{x}-\mu)}{s} = \dfrac{\sqrt{64}(62-60)}{10} = 1.6$ ***Falls Outside CR*** **p-value(area from test statistics)**

TI-83/84 *stat* → *tes* → *Option 1*



Conclusion: Accept or reject **H_0**? ***Outside*** CR then ***Fail to Reject H_0*** *or* ***Accept H_0***

Comment: Accept or reject **SC**? **Reject** that the **average** life of batteries **is different than 60** months

***P-value*:** **0 .1096** more than **α = 0.05** accept **Ho** (remember when **p-value is larger than α, we accept Ho**)

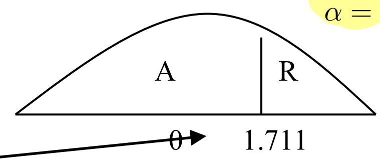
Small Samples about Mean $n \leq 30$

Example 4. **Average** life of "Die Long" batteries **exceeds** 60 months. A sample of 25 batteries had an average life of 63 months and st. dev. of 10 months. Let **α = .05**

SC: $\mu > 60$ **H_0 :** $\mu \leq 60$ **Hint**: Use **H_1** to determine if it is LTT ,TTT or RTT test

OC: $\mu \leq 60$ **H_1 :** $\mu > 60$ **Note:** μ **in H_1 is more than,** then it is a **RTT**

$n = 25$
$\alpha = .05$



When **α = .05** , n < 30 and one –tailed test then by using **24th** row of **Table 2.**

Critical value = CV= **t = 1.711**

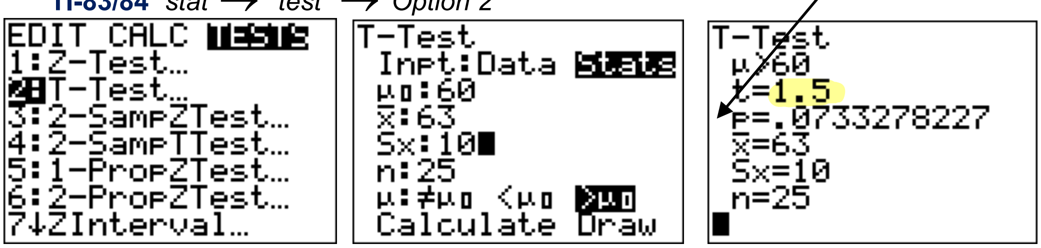
Test Statistics = $z = \dfrac{\sqrt{n}(\bar{x}-\mu)}{s} = \dfrac{\sqrt{25}(63-60)}{10} = 1.5$ ***Falls Outside CR*** **p-value(area from test statistics)**

Conclusion: Accept or reject **H_0**? ***Outside*** CR then **Fail to Reject H_0** or **Accept H_0**

Comment: Accept or reject **SC**? **Reject** that the **average** life of "Die Easy" batteries **exceeds 60** months

P-value: **0 .0733** more than **α = 0.05** accept **Ho** (remember when **p-value is larger than α, we accept Ho**)

TI-83/84 *stat* → *test* → *Option 2*

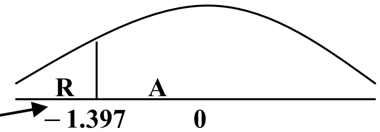


Example 5. **Average** life of "Die Long" batteries is **less than** 60 months. A sample of 9 batteries had an average life of 54 months and st. dev. of 10 months. Let **α = .10**

SC: $\mu < 60$ **H₀**: $\mu \geq 60$ **Hint**: Use **H_1** to determine if it is LTT ,TTT or RTT test

OC: $\mu \geq 60$ **H₁**: $\mu < 60$ **Note:** μ **in H_1 is less than,** then it is a **LTT**

$n = 9$
$\alpha = 10$

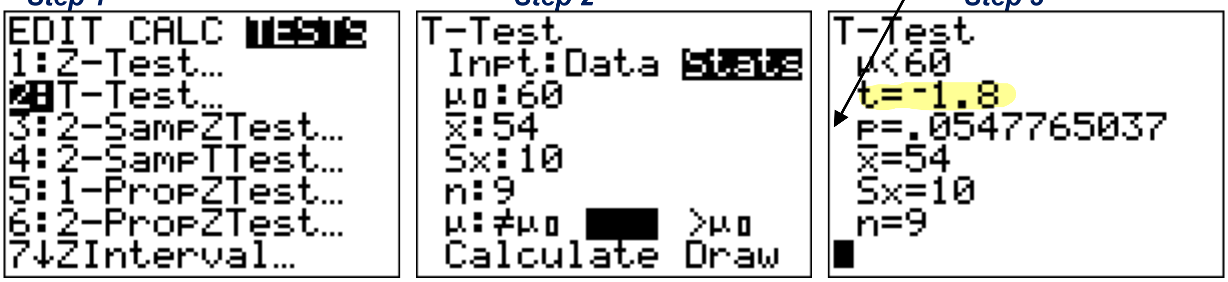


When **α = .10** , n < 30 and one –tailed test then by using **8[th]** row of **Table 2.**
Critical value = CV= **t** = **−1.397**

Test Statistics = $z = \frac{\sqrt{n}(\bar{x}-\mu)}{s} = \frac{\sqrt{9}(54-60)}{10} = -1.8$ ***Falls inside CR*** p-value(area from test statistics)

TI-83/84 *stat* ⟶ *test* ⟶ *Option 2*

Step 1 ***Step 2*** ***Step 3***



Conclusion: Accept or reject **H₀**? Inside ***CR*** then reject **Ho**
Comment: Accept or reject **SC**? **Accept** that the **average** life of "Die Easy" batteries is **less than 60** months
P-value: 0 .05478 less than **α = 0.10** reject **Ho**

Example 6. **Average** life of "Die Long" batteries **is different** than 60 months. A sample of **16** batteries had an average life of 66 months and st. dev. of 10 months. Let **α = .02**

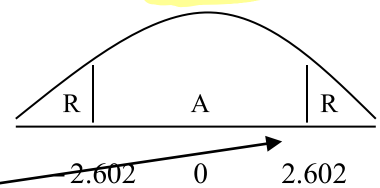
$n = 16$
$\alpha = 0.02$

SC: $\mu \neq 60$ **H₀** : $\mu = 60$ **Hint**: Use **H_1** to determine if it is LTT ,TTT or RTT test

OC: $\mu = 60$ **H₁** : $\mu \neq 60$ **Note:** μ **in H_1 is not equal,** then it is a **TTT**

When **α = .02** , n < 30 and two –tailed test then by using **15[th]** row of **Table 2.**

Critical value =*C V*= **t** = ± ***2.602***



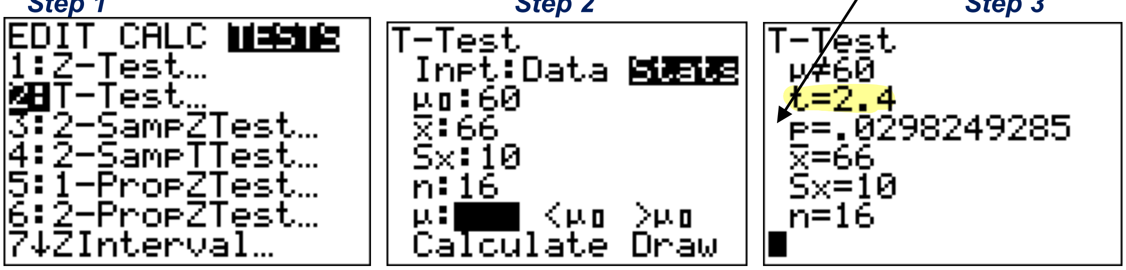
Test Statistics = $t = \frac{\sqrt{n}(\bar{x}-\mu)}{s} = \frac{\sqrt{16}(66-60)}{10} = 2.4$ ***Falls Outside CR*** p-value(area from test statistics)

Conclusion: Accept or reject **H₀**? **Outside** CR then **Fail to Reject H₀** or **Accept H₀**
Comment: Accept or reject **SC**? **Reject** that the **average** life of "Die Easy" batteries **is different than 60** months.

P-value: 0 .0298 more than **α = 0.02** accept **Ho**

TI-83/84 *stat* ⟶ *test* ⟶ *Option 2*

Step 1 ***Step 2*** ***Step 3***



Example 7) Leno Co. claims that the mean life of their batteries is 60 months. Test this claim with $\alpha = 0.02$ if a sample of 6 batteries has a life of 62, 58, 59, 64, 63, 61, months.

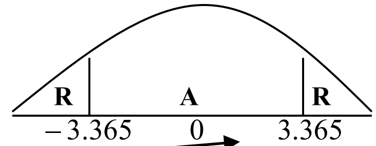
$n = 6$
$\alpha = 0.02$

SC: $\mu = 60$ **H₀**: $\mu = 60$ **Hint**: Use **H₁** to determine if it is LTT ,TTT or RTT test

OC: $\mu \neq 60$ **H₁**: $\mu \neq 60$ **Note:** μ **in H₁ is not equal,** then it is a **TTT**

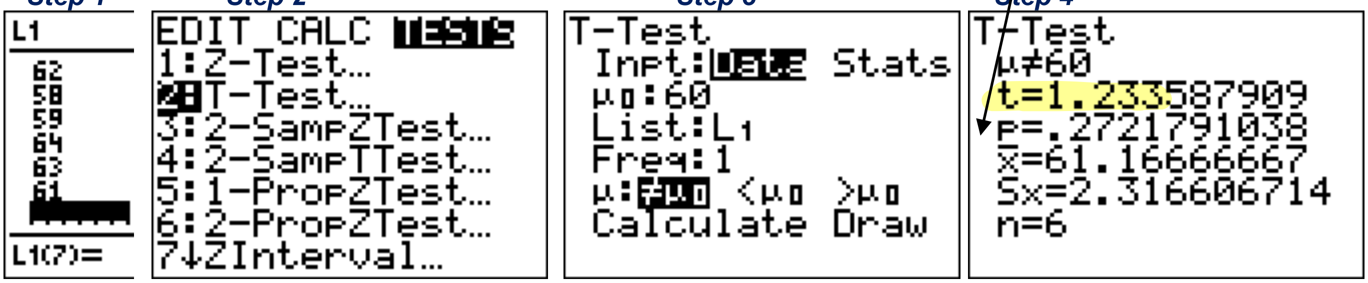
When **α = .02** , n < 30 and two –tailed test then by using **5th** row of **Table 2.**

Critical value =*C V*= **t** = ± ***3.365***



Test Statistics = $t = \dfrac{\sqrt{n}(\bar{x}-\mu)}{s} = \dfrac{\sqrt{6}(61.17-60)}{2.317} = 1.23$ ***Falls Outside CR*** **p-value(area from test statistics)**

Step 1 ***Step 2*** ***Step 3*** ***Step 4***



Conclusion: Accept or reject **H₀**? **Outside** CR then **Fail to Reject H₀** or **Accept H₀**

Comment: Accept or reject **SC**? **Fail to Reject** or **Accept** that the **average** life of "Die Easy" batteries **exceeds 60** months

P-value: 0 .0272 more than **α = 0.02** accept **Ho**