

## Part III

## Point and Interval Estimation

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For all quizzes in part 3: Be sure you have formula sheet and Table 1 and Table 2.

## Learning Objectives

What do we estimate? **Population Mean** ( $\mu = ?$ ) or **Population Proportion** ( $P = ?$ )

Why do we estimate? Due to our limited resources (Time, Money, manpower, destruction of tested subjects, widely scattered data, hardly accessible subjects).

Know all the new **terminologies** and related **notations** (Point estimate  $\bar{x}$ ,  $\hat{p}$ , Margin of error)

Know all the new **formulas** on **formula sheet** and their related **TI commands**.

Know in estimating **population mean** ( $\mu = ?$ ) when to use **normal distribution** versus **t-distribution**.

Know how to use TI (**option 7 or 8**) or (**formula  $\mu = \bar{x} \pm E$  + table**) to estimate **population mean** ( $\mu = ?$ ).

Know how to use TI (**option B**) or (**formula + table**) to estimate **difference between two population proportions** ( $P_1 - P_2$ ).

Know how to use (**formula + table**) to **determine sample size** for **Population Mean** ( $\mu = ?$ ) or **Population Proportion** ( $P = ?$ )

**Important Note 2:** As you study each page of **topics Review**, do all the problems listed at the bottom of the page from practice problem before going to the next page.

**Important Note 3:** For all practice problems the answers and complete solutions are given on later pages.

**Important Note 4: Doing all related practice problems.**

# Overview

One major application of inferential statistics involves the use of sample data to estimate the value of a population parameter such as means ( $\mu$ ) and proportions ( $P$ ).

## Objectives:

- To introduce methods for estimating values of two important population parameters: mean ( $\mu$ ) and proportions ( $P$ ). This can be done by using a point estimate ( $\bar{X}$  or  $\hat{P}$ ) that is the value of a statistic to estimates the value of a parameter ( $\mu$  or  $P$ ).
- To present methods for determining sample sizes necessary to estimate those parameters.

## Example:

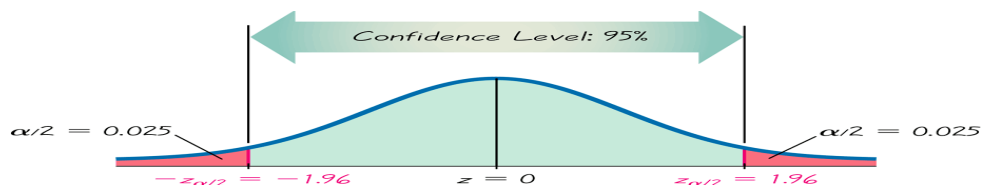
- Estimate the **average** life of Diehard batteries?  $\mu = ?$
- Estimate the **average** waiting time at a supermarket register?  $\mu = ?$
- Estimate the **average** clarity (in depth) of water at Lake Tahoe?  $\mu = ?$
- Estimate the **percentage** of residents in North America that only speak English at home?  $P = ?$
- Estimate the **percentage** of drivers text while driving?  $P = ?$
- Estimate the **percentage** of registered voters will vote for next democratic candidates?  $P = ?$
- Estimate the **average difference** in battery life between Diehard and Everlast brand?  $\mu_D - \mu_E = ?$
- Estimate the **percentage difference** between female and male who pass stat class?  $P_f - P_m = ?$

## Definitions:

**Point estimate:** Sample statistics such as ( $\bar{X}$  or  $\hat{P}$ )

**Confidence Interval:** A confidence interval (or interval estimate) is a range (or an interval) of values used to estimate the true value of a population parameter. A confidence interval is sometimes abbreviated as CI.

**A confidence level:** a confidence level is the probability ( $1 - \alpha$ ) (often expressed as the equivalent percentage value) usually 90%, 95%, or 99%. that is the proportion of times that the confidence interval actually does contain the population parameter, assuming that the estimation process is repeated a large number of times. Percentage outside confidence level is called **critical area** ( $\alpha$ ). So for example with  $95\% = (1 - \alpha)$  confidence level then the critical **area** will be ( $\alpha = 5\%$ ).



**Critical Value(s):** The  $z_{\alpha/2}$  value that can be found from the table based on different confidence level.

**Margin of error:** (also called error, error bound or maximum error) is the maximum likely difference observed between point estimates ( $\bar{X}$  or  $\hat{P}$ ) and population parameter ( $\mu$  or  $P$ ).

Be sure you *always have this page and Normal and T-Distribution as a reference for every estimation problem*

Estimating One Population **Mean**  $\mu = \bar{x} \pm E$

**Important: If confidence level is not given use 95% as a default.**

$\bar{X}$ = Point estimate (Sample Mean)		E = Margin of error(error bound)	
Decision making process based on $\sigma$ ( population standard deviation)			
Margin of Error	$E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$	$\sigma$ (known or given)	(For $z_{\alpha/2}$ , use Table page 3)
	$E = z_{\alpha/2} \frac{s}{\sqrt{n}}$	$\sigma$ (unknown or not given) and $n > 30$	(For $z_{\alpha/2}$ , use Table page 3)
	$E = t_{\alpha/2} \frac{s}{\sqrt{n}}$	$\sigma$ (unknown or not given) and $n \leq 30$	(For $t_{\alpha/2}$ , use Table page 4)
Population is normally distributed			
Interval Estimate	$\mu = \bar{x} \pm E$		$\mu = \bar{x} \pm E$
TI-83/84	stat $\rightarrow$ tests $\rightarrow$ <b>Option 7(Z-interval)</b>		stat $\rightarrow$ tests $\rightarrow$ <b>Option 8(t-interval)</b>
<p><b>Width</b> (difference between upper and lower bounds) = <math>2E = UB - LB</math> so <math>E = (UB - LB) / 2</math></p> <p><b>Point Estimate</b> (middle of upper and lower bounds) = <math>\bar{x} = (UB + LB) / 2</math></p>			

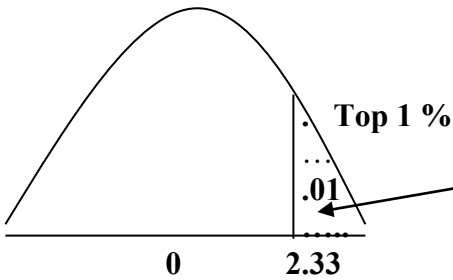
Estimating Population <b>Proportion</b> $P = \hat{p} \pm E$	
$\hat{p} = \frac{x}{n}$ (Called p-hat is sample proportion and point estimate for population proportion)	E = Margin of error $E = z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
<p><b>Width</b> (difference between upper and lower bounds) = <math>2E = UB - LB</math> so <math>E = (UB - LB) / 2</math></p> <p><b>Point Estimate</b> (middle of upper and lower bounds) = <math>\hat{p} = (UB + LB) / 2</math></p>	
TI-83 stat $\rightarrow$ test $\rightarrow$ <b>Option A</b>	

Estimating the <i>difference</i> between <b>Two</b> Populations <b>Means</b> or <b>Proportions</b>	
<b>Mean</b> $\mu_1 - \mu_2$	<b>Proportion</b> $P_1 - P_2$
$\mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \pm E$	$P_1 - P_2 = (\hat{p}_1 - \hat{p}_2) \pm E$
Point estimate = $(\bar{x}_1 - \bar{x}_2)$	Point estimate = $(\hat{p}_1 - \hat{p}_2)$
$E = z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$E = z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$
TI-83/84 stat $\rightarrow$ test $\rightarrow$ <b>Option 9</b>	TI-83/84 stat $\rightarrow$ test $\rightarrow$ <b>B</b>

<b>Sample Size Determination for the Estimation of Population</b>	
<b>Mean</b> = $\mu$	<b>Proportion</b> = $P$
$n = (Z_{\alpha/2} S / E)^2$	$n = (Z_{\alpha/2} / E)^2 \hat{p}(1-\hat{p})$
If $S$ is unknown then estimate it by $S = \text{Range} / 4$	If $\hat{p}$ is unknown then estimate it by $\hat{p} = 0.5$

Based on Standard Normal Distribution  $\mu=0$  and  $\sigma=1$

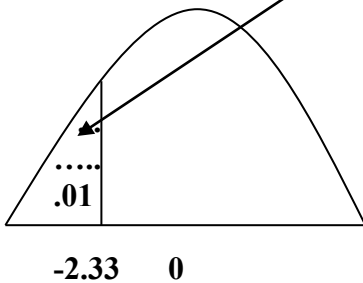
Out Side Area



OR

Out Side Area

Bottom 1 %

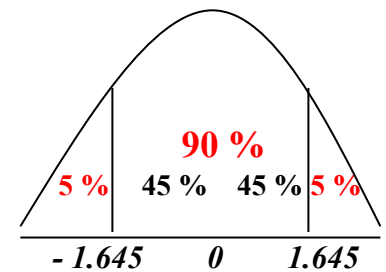


Confidence Level	Out Side Area On left or right Cut-off Point	Z - Value ( ± ) Critical Value = $Z_{\alpha/2}$
99%	.005	± 2.5758
98%	.01	± 2.3263
97%	.015	± 2.1701
96%	.02	± 2.0537
95%	.025	± 1.9600
94%	.03	± 1.8808
92%	.04	± 1.7507
90%	.05	± 1.6450
88%	.06	± 1.5548
86%	.07	± 1.4758
84%	.08	± 1.4051
82%	.09	± 1.3408
80%	.10	± 1.2816
78%	.11	± 1.2265
76%	.12	± 1.1750
70%	.15	± 1.0364
60%	.20	± 0.8416
50%	.25	± 0.6749
40%	.30	± 0.5244

How to find the Z -value for confidence intervals.

**Example: Find the Z - value for 90% confidence interval**

1. Divide 90% = 0.90 by 2,  $\Rightarrow .90 / 2 = 0.45$
2. Subtract 0.45 from 0.5  $\Rightarrow .5 - 0.45 = .05$
3. Look for area close to 0.05 from **inside** the table (page1).
4. Find its corresponding Z-value (- 1.645)

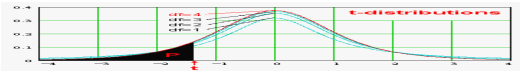


TI-83/84 2nd  $\rightarrow$  Distr  $\rightarrow$  Option 3 input (% , 0 , 1)

Example: 2nd  $\rightarrow$  Distr  $\rightarrow$  Option 3 input (.05 , 0 , 1) enter , then the answer will be - 1.645

Example: 2nd  $\rightarrow$  Distr  $\rightarrow$  Option 3 input (.95 , 0 , 1) enter , then the answer will be 1.645

Hint for TI % is the area to the left of the cut off point.



t -Distribution for small sample  $n < 30$  and  $\sigma$  Unknown

df = n-1	←----- alpha $\alpha$ -----→							
2-Tailed	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.005
1-Tailed	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.0025
Conf. Lev.	60%	70%	80%	90%	95%	98%	99%	99.5%
1	1.376	1.963	3.078	6.314	12.706	31.821	63.656	127.321
2	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.089
3	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453
4	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598
5	<b>0.920</b>	<b>1.156</b>	<b>1.476</b>	<b>2.015</b>	<b>2.571</b>	<b>3.365</b>	<b>4.032</b>	<b>4.773</b>
6	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317
7	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029
8	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833
9	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690
10	<b>0.879</b>	<b>1.093</b>	<b>1.372</b>	<b>1.812</b>	<b>2.228</b>	<b>2.764</b>	<b>3.169</b>	<b>3.581</b>
11	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497
12	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428
13	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372
14	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326
15	<b>0.866</b>	<b>1.074</b>	<b>1.341</b>	<b>1.753</b>	<b>2.131</b>	<b>2.602</b>	<b>2.947</b>	<b>3.286</b>
16	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252
17	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222
18	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197
19	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174
20	<b>0.860</b>	<b>1.064</b>	<b>1.325</b>	<b>1.725</b>	<b>2.086</b>	<b>2.528</b>	<b>2.845</b>	<b>3.153</b>
21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135
22	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119
23	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104
24	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091
25	<b>0.856</b>	<b>1.058</b>	<b>1.316</b>	<b>1.708</b>	<b>2.060</b>	<b>2.485</b>	<b>2.787</b>	<b>3.078</b>
26	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067
27	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057
28	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047
29	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038
30	<b>0.854</b>	<b>1.055</b>	<b>1.310</b>	<b>1.697</b>	<b>2.042</b>	<b>2.457</b>	<b>2.750</b>	<b>3.030</b>
n>30 ⇒ Z	<b>0.842</b>	<b>1.036</b>	<b>1.282</b>	<b>1.645</b>	<b>1.96</b>	<b>2.326</b>	<b>2.576</b>	<b>2.807</b>
2-T	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.005
1-T	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.0025
Conf. Lev.	60%	70%	80%	90%	95%	98%	99%	99.5%

for  
 $n > 30$   
Use  
Bottom  
row

# T-Distribution vs. the Normal Distribution for Confidence Interval for Means

## Main Point to Remember:

You must use the t-distribution table when working problems when the population standard deviation ( $\sigma$ ) is not known and the sample size is small ( $n < 30$ ).

## General Correct Rule:

If  $\sigma$  is not known, then using t-distribution is correct. If  $\sigma$  is known, then using the normal distribution is correct.

## What is Most Common Practice:

Since people often prefer to use the normal, and since the t-distribution becomes equivalent to the normal when the number of cases becomes large, common practice often is:

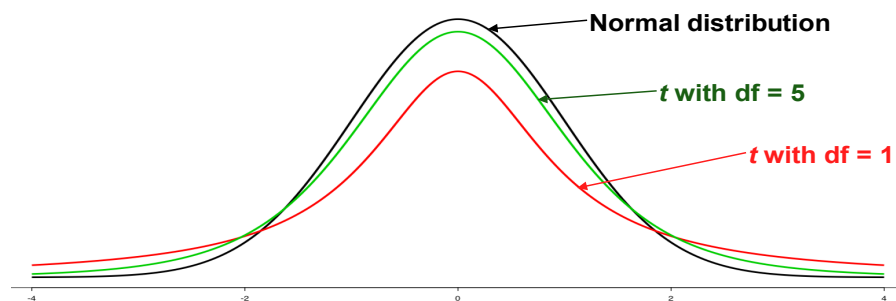
- If  $\sigma$  known, then use normal.
- If  $\sigma$  not known:
  - If  $n$  is large, then use normal.
  - If  $n$  is small, then use t-distribution.

## What is Another Common Way Textbooks Teach This:

Textbooks often simplify this to “large-sample” vs. “small-sample” methods; use normal distribution with large samples and t-distribution with small samples. This is right almost all the time, because in real sampling problems we seldom have a basis for knowing  $\sigma$ . However, there can be some situations when we do have a basis for assuming a value for  $\sigma$ , such as using a  $\sigma$  based on past data, and in those situations even if sample size is small the correct procedure would be to use the normal distribution, so the simplified “large-sample” vs. “small sample” approach would lead to an error.

## **t distribution**

- **t** distribution looks like a normal distribution, but has “thicker” tails. The tail thickness is controlled by the **degrees of freedom**



- The smaller the degrees of freedom, the thicker the tails of the **t** distribution
- If the degrees of freedom is large (if we have a large sample size), then the t distribution is pretty much identical to the normal distribution

**Estimating one population Mean**  $\mu = \bar{x} \pm E$

- a) What do we estimate? Population mean ( $\mu$ ) or sample mean ( $\bar{x}$ ) or both?
- b) Why do we need to estimate? Cite some reasons?
- c) What is the point estimate?
- d) What is the confidence level?
- e) What is the criteria of t-distribution?
- f) Under what condition we use t-distribution?
- g) What is the formula for degree of freedom  $df = ?$
- h) What is the margin of error and what are three different possible formulas for it?
- i) Where you can find the  $z$  table and under what condition you will be using this table?
- j) Where you can find the  $t$  table and under what condition you will be using this table?
- k) What is the width of a confidence interval?
- l) How we can use the upper and lower boundaries of a confidence interval to find point estimate?
- m) How we can use the width of a confidence interval to find margin of error?
- n) How to use **TI calculator** to find the boundaries of a confidence interval when we use **normal distribution**?
- o) How to use **TI calculator** to find the boundaries of a confidence interval when we use **t-distribution**?

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**A)** For the following problems decide to  $z$  or  $t$  value or neither?

- 1) Sample size  $n = 20$ ,  $s = 4$  and the population is normally distributed?  $z$  or  $t$  value :  $t$  value
- 2) Sample size  $n = 19$ ,  $\sigma = 4$  and the population is normally distributed?  $z$  or  $t$  value:  $z$  value
- 3) Sample size  $n = 18$ ,  $s = 4$  and the population is normally distributed?  $z$  or  $t$  value :  $t$  value
- 4) Sample size  $n = 10$ ,  $\sigma = 4$  and the population is normally distributed?  $z$  or  $t$  value:  $z$  value
- 5) Sample size  $n = 100$ ,  $s = 4$  and the population is normally distributed?  $z$  or  $t$  value:  $z$  value

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**Find the margin of error for the following problems by using the z-table(page 4) or t-table(page 5)?**  
**Be sure you if you use t-table, you subtract 1 from n and then use (n-1) row to find the t-value**

**B)**  $E = z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$

**B)**  $E = z_{\alpha/2} \left( \frac{s}{\sqrt{n}} \right) \quad n > 30$

**C)**  $E = t_{\alpha/2, df} \left( \frac{s}{\sqrt{n}} \right) \quad n \leq 30$

- 1) Sample size  $n = 36$ ,  $\sigma = 4$  and 95% confidence level?
- 2) Sample size  $n = 16$ ,  $s = 8$  and 99% confidence level?
- 3) Sample size  $n = 18$ ,  $\sigma = 30$  and 90% confidence level?
- 4) Sample size  $n = 100$ ,  $s = 4$  and 97% confidence level?
- 5) Sample size  $n = 14$ ,  $s = 10$  and the 95% confidence level?

**Answer: 1.31**

**Answer: 5.894**

**Answer: 11.6**

**Answer: 0.868**

**Answer: 5.773**

*Solution on the next page!*

- 1) Sample size  $n = 36$ ,  $\sigma = 4$ , 95% and confidence level? **Z-value** when  $\sigma$  is given  $E = 1.96 \frac{4}{\sqrt{36}} = 1.31$
- 2) Sample size  $n = 16$ ,  $s = 8$ , and 99% confidence level? **t-value** when  $\sigma$  unknown,  $n < 30$   $E = 2.947 \frac{8}{\sqrt{16}} = 5.894$
- 3) Sample size  $n = 18$ ,  $\sigma = 30$ , and 90% confidence level? **Z-value** when  $\sigma$  is given  $E = 1.645 \frac{30}{\sqrt{18}} = 11.6$
- 4) Sample size  $n = 100$ ,  $s = 4$ , and 97% confidence level? **Z-value** when  $n > 30$   $E = 2.17 \frac{4}{\sqrt{100}} = 0.868$
- 5) Sample size  $n = 14$ ,  $s = 10$ , and 95% confidence level? **t-value** when  $\sigma$  unknown,  $n < 30$   $E = 2.16 \frac{10}{\sqrt{14}} = 5.77$

C) Important properties about the **relationship** of sample size and **confidence level** and **increase, decrease**, of

$$E = z \frac{\sigma}{\sqrt{n}}$$

- a) As the **sample size (n) decreases**, the **margin of error (E) increases**
- b) As the **confidence level (C) decreases**, the **margin of error (E) decreases**

i)  $\bar{x} \pm z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$       ii)  $\bar{x} \pm z_{\alpha/2} \left( \frac{s}{\sqrt{n}} \right)$   $n > 30$       iii)  $\bar{x} \pm t_{\alpha/2, df} \left( \frac{s}{\sqrt{n}} \right)$   $n \leq 30$

- 1) A random sample of 36 life insurance policy holders showed that the average premiums paid on their life insurance policies was \$340 per year with a standard deviation of \$24. Construct a 95% confidence interval for the population mean.  $n = 36$      $\bar{x} = 340$      $\sigma =$     **or**     $s = 24$

**Because sample size is more than 30, we use normal distribution**

$$E = z \left( s / \sqrt{n} \right) = 1.96 \frac{24}{\sqrt{36}} = 7.84 \qquad \mu = 340 \pm 7.84 \qquad \$332.16 < \mu < \$347.84$$

- 2) A random sample of 9 life insurance policy holders showed that the average premiums paid on their life insurance policies was \$340 per year with a standard deviation of \$24. Construct a 95% confidence interval for the population mean.  $n = 9$      $\bar{x} = 340$      $\sigma =$     **or**     $s = 24$

**Because sample size is less than 30, we use t distribution**

$$E = t \left( s / \sqrt{n} \right) = 2.306 \frac{24}{\sqrt{9}} = 18.45 \qquad \mu = 340 \pm 18.45 \qquad \$321.55 < \mu < \$358.45$$

- 3) A random sample of 9 life insurance policy holders showed that the average premiums paid on their life insurance policies was \$340 per year and population standard deviation of \$24. Construct a 95% confidence interval for the population mean.  $n = 9$      $\bar{x} = 340$      $\sigma = 24$     **or**     $s =$

**Because sample size is less than 30, but population standard deviation is given then we use normal distribution**

$$E = z \left( \sigma / \sqrt{n} \right) = 1.96 \frac{24}{\sqrt{9}} = 15.68 \qquad \mu = 340 \pm 15.68 \qquad \$324.32 < \mu < \$355.68$$



- 4) A company that produces white bread is concerned about the distribution of the amount of sodium in its bread. The company takes a simple random sample of 25 slices of bread and computes the sample mean to be 100 milligrams of sodium per slice. Construct a 90% confidence interval for the unknown mean sodium level assuming that the sample standard deviation is 10 milligrams.

$$n = 25 \quad \bar{x} = 100 \quad \sigma = 10 \quad \text{or} \quad s = 10 \quad \text{Because sample size is less than 30, we use t distribution}$$

$$E = t\left(s / \sqrt{n}\right) = 1.711 \frac{10}{\sqrt{25}} = 3.42 \quad \mu = 100 \pm 3.42 \quad 96.58 < \mu < 103.42$$

- 5) The football coach randomly selected eight players and timed how long it took to perform a certain drill. The times in minutes were: 12, 9, 13, 7, 8, 13, 16, 10. Assuming that the times follow a normal distribution, find a 90% confidence interval for the population mean.  $n = 8 \quad \bar{x} = 11 \quad \sigma =$  or  $s = 3.02$

**Because sample size is less than 30, we use t distribution**

$$E = t\left(s / \sqrt{n}\right) = 1.895 \frac{3.02}{\sqrt{8}} = 2.02 \quad \mu = 11 \pm 2.02 \quad 8.98 < \mu < 13.02$$

Estimating the  $\mu$  = average life of Diehard batteries by using **95%** confidence Level. A sample of **64** batteries has  $\bar{x} = 50$  months. From prior study, we know  $\sigma = 10$  months

**Solution by Formula**

$$\sigma \text{ known, } \rightarrow E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 1.96 \frac{10}{\sqrt{64}} \quad E = 2.45 \quad \mu = 50 \pm 2.45 \quad 47.55 < \mu < 52.45$$

By 95% confidence, the average life of Diehard batteries is between 47.55 to 52.45 months

**Solution by TI 83/84 Calculator**

$\sigma$  known  $\rightarrow$  TI-83/84 stat  $\rightarrow$  tests  $\rightarrow$  Option 7

```
ZInterval
Inpt:Data
σ:10
x̄:50
n:64
C-Level:.95
Calculate
```

```
ZInterval
(47.55,52.45)
x̄=50
n=64
```

$$E = (UB - LB) / 2 = (52.45 - 47.55) / 2 = 2.45$$

Estimating the  $\mu$  = average life of Diehard batteries by using **95%** confidence Level when a sample of **6** batteries provides these data 48,54,57,45, 56,52

**Solution by Formula**

**Hint:** to use the formula, you need to calculate  $\bar{x} = ? \cdot s = ? \cdot \bar{x} = 52$  months  $\cdot s = 4.69$  months

$$(\sigma \text{ unknown, and } n \leq 30) \text{ (for t-value use table page 4) } df = 6 - 1 \quad t_{\alpha/2} = 2.571 \quad E = 2.571 \frac{4.69}{\sqrt{6}}$$

$$E = 4.92 \quad \mu = 52 \pm 4.92 \quad 47.08 < \mu < 56.92$$

**Solution by TI 83/84 Calculator**

**input data in L1 then, ( $\sigma$  unknown, and  $n \leq 30$ )  $\rightarrow$  TI-83/84 stat  $\rightarrow$  tests  $\rightarrow$  Option 8**

$$E = (UB - LB) / 2 = (56.92 - 47.08) / 2 = 4.92$$

## Summary in deciding **Normal** or **t-Distribution** in estimating One Population

**Mean**  $\mu = \bar{x} \pm E$

$\sigma$ ( <i>population st. dev.</i> ) is known	$n > 30$ $\sigma$ is unknown	$\sigma$ is unknown, and $n < 30$	<b>Raw Data Only</b> $n < 30$
<p style="text-align: center;"><math>\sigma = 10</math> <b>Sample</b> <math>n = 9</math> <math>\bar{x} = 50</math></p> <p style="text-align: center;"><b>Use 95 % Confidence Level</b></p> <p style="text-align: center;"><math>\mu = \bar{x} \pm E = 50 \pm E</math></p> <p><math>\sigma</math> is known, use <b>Normal distribution</b></p> <p style="text-align: center;"><math>E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}</math></p> <p>(for <math>z_{\alpha/2}</math> - <b>value</b> use table on <b>page 4</b>)</p> <p>For 95% confidence</p> <p style="text-align: center;"><math>z_{\alpha/2} = 1.96</math></p> <p style="text-align: center;"><math>E = 1.96 \frac{10}{\sqrt{9}} = 6.53</math></p> <p style="text-align: center;"><math>\mu = 50 \pm 6.53</math></p> <p style="text-align: center;"><math>43.47 &lt; \mu &lt; 56.53</math></p> <p style="text-align: center;"><b>Or</b></p> <p style="text-align: center;">(43.47 , 56.53)</p> <p style="text-align: center;"><b>By 95% confidence, the population average is between 43.47 and 56.53</b></p>	<p style="text-align: center;"><b>Sample</b> <math>n = 64</math> <math>\bar{x} = 50</math> <math>s = 10</math></p> <p style="text-align: center;"><b>Use 95 % Confidence Level</b></p> <p style="text-align: center;"><math>\mu = \bar{x} \pm E = 50 \pm E</math></p> <p><math>n &gt; 30</math>, use <b>Normal distribution</b></p> <p style="text-align: center;"><math>E = z_{\alpha/2} \frac{s}{\sqrt{n}}</math></p> <p>(for <math>z_{\alpha/2}</math> - <b>value</b> use table on <b>page 4</b>)</p> <p>For 95% confidence</p> <p style="text-align: center;"><math>z_{\alpha/2} = 1.96</math></p> <p style="text-align: center;"><math>E = 1.96 \frac{10}{\sqrt{64}} = 2.45</math></p> <p style="text-align: center;"><math>\mu = 50 \pm 2.45</math></p> <p style="text-align: center;"><math>47.55 &lt; \mu &lt; 52.45</math></p> <p style="text-align: center;"><b>Or</b></p> <p style="text-align: center;">(47.55 , 52.45)</p> <p style="text-align: center;"><b>By 95% confidence, the population average is between 47.55 and 52.45</b></p>	<p style="text-align: center;"><b>Sample</b> <math>n = 9</math> <math>\bar{x} = 50</math> <math>s = 10</math></p> <p style="text-align: center;"><b>Use 95 % Confidence Level</b></p> <p style="text-align: center;"><math>\mu = \bar{x} \pm E = 50 \pm E</math></p> <p><math>\sigma</math> is unknown, and <math>n &lt; 30</math> use <b>t- distribution</b></p> <p style="text-align: center;"><math>E = t_{\alpha/2} \frac{s}{\sqrt{n}}</math></p> <p>(for <math>t_{\alpha/2}</math> - <b>value</b> use table on <b>page 5</b>)</p> <p>Use 95% column with df=9-1=8</p> <p style="text-align: center;"><math>t_{\alpha/2} = 2.306</math></p> <p style="text-align: center;"><math>E = 2.306 \frac{10}{\sqrt{9}} = 7.69</math></p> <p style="text-align: center;"><math>\mu = 50 \pm 7.69</math></p> <p style="text-align: center;"><math>42.31 &lt; \mu &lt; 57.69</math></p> <p style="text-align: center;"><b>Or</b></p> <p style="text-align: center;">(42.31 , 57.69)</p> <p style="text-align: center;"><b>By 95% confidence, the population average is between 42.31 and 57.69</b></p>	<p style="text-align: center;"><b>Sample</b> 42,48,52,58 <math>n = 4</math> <math>\bar{x} = 50</math> <math>s = 6.73</math></p> <p style="text-align: center;"><b>Use 95 % Confidence Level</b></p> <p style="text-align: center;"><math>\mu = \bar{x} \pm E = 50 \pm E</math></p> <p><math>\sigma</math> is unknown, and <math>n &lt; 30</math> use <b>t- distribution</b></p> <p style="text-align: center;"><math>E = t_{\alpha/2} \frac{s}{\sqrt{n}}</math></p> <p>(for <math>t_{\alpha/2}</math> - <b>value</b> use table on <b>page 5</b>)</p> <p>Use 95% column with df=4-1=3</p> <p style="text-align: center;"><math>t_{\alpha/2} = 3.182</math></p> <p style="text-align: center;"><math>E = 3.182 \frac{6.73}{\sqrt{4}} = 10.71</math></p> <p style="text-align: center;"><math>\mu = 50 \pm 10.71</math></p> <p style="text-align: center;"><math>39.29 &lt; \mu &lt; 60.71</math></p> <p style="text-align: center;"><b>Or</b></p> <p style="text-align: center;">(39.29 , 60.71)</p> <p style="text-align: center;"><b>By 95% confidence, the population average is between 39.29 and 60.71</b></p>
<b>TI</b> Instruction  <i>stat</i> → <i>tests</i> → <b>Option 7</b>	<b>TI</b> Instruction  <i>stat</i> → <i>tests</i> → <b>Option 7</b>	<b>TI</b> Instruction  <i>stat</i> → <i>tests</i> → <b>Option 8</b>	<b>TI</b> Instruction  <i>stat</i> → <i>tests</i> → <b>Option 8</b>

- 6) The actual time it takes to cook a ten-pound turkey is a normally distributed. Suppose that a random sample of 9 ten pound turkeys is taken. Given that an average of 2.9 hours and a standard deviation of .24 hours was found for a sample of 9 turkeys, calculate a 95% confidence interval for the average cooking time of a ten-pound turkey.  $n =$                        $\bar{x} =$                        $\sigma =$                       **or**                       $s =$

$E =$

$\mu =$

$$2.72 < \mu < 3.08$$