Abe Mirza	Topics Review Part II (Section 5)	Statistic

Addition Rule (Keywords: or, at least, at most)

$$P(A\ or\ \ B) = P(A) + P(B) - p(A\ \ and\ \ B)$$

If there is no **overlapping** between event A and B then they are called mutually exclusive $P(A\ and\ \ B) = 0$

$$P(A\ or\ \ B) = P(A) + P(B)$$

A.1, If we draw a card from a deck of card what is the probability that it will be red **or** King?

$$P(\text{R}\ \ or\ K) = P(R) + P(K) - P(R\ \ and\ \ K) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13} = 53.85\%$$

A.2, If we draw a card from a deck of card what is the probability that it will be Queen **or** King?

$$P(Q\ \ or\ K) = P(Q) + P(K) - P(Q\ \ and\ \ K) = \frac{4}{52} + \frac{4}{52} - \frac{0}{52} = \frac{8}{52} = \frac{2}{13} = 15.38\%$$

A.3 If we roll a die what is the probability getting an even number **or** multiple of 3?

Solution: even $\{2,4,6\}$ and multiple of 3 $\{3,6\}$, even and multiple of 3 $\{6\}$

$$P(even\ \ or\ \ mult\ 3) = P(even) + P(mult\ 3) - P(even\ \ and\ \ mult\ 3) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3} = 33.33\%$$

A.4 If we roll a die what is the probability getting an even number **or** multiple of 5?

Solution: even $\{2,4,6\}$ and multiple of 5 $\{5\}$

$$P(even\ \ or\ \ mult\ 5) = P(even) + P(mult\ 5) - P(even\ \ and\ \ mult\ 5) = \frac{3}{6} + \frac{1}{6} - \frac{0}{6} = \frac{4}{6} = \frac{2}{3} = 33.33\%$$

A.5 Of the 60 people who answered "yes" to a question, 35 were male. Of the 40 people who answered "no" to the question, 10 were male.

	Yes	**No**	
Male	35	10	**?**
Female	?	?	**?**
	60	40	

Use the given information to complete the table.

	Yes	**No**	
Male	35	10	**45**
Female	25	30	**55**
	60	**40**	**100**

If **one** person is selected at random from the group, answers the following questions

Find the probability that the person answered "yes" **or** is male? $P(yes\ \ or\ \ male) = \frac{60}{100} + \frac{45}{100} - \frac{35}{100} = \frac{70}{100} = 70\%$

Find the probability that the person answered "no" **or** is female? $P(no\ \ or\ \ female) = \frac{40}{100} + \frac{55}{100} - \frac{30}{100} = \frac{65}{100} = 65\%$

B. The distribution of master degree in a college is listed as such

Major	Frequency
Math	**110**
Engineering	**250**
Business	**300**
Education	**100**
English	**240**
Total	**1000**

If one student is selected at random then what is the probability that he/she is majoring in Math or English?

$$P(M) + P(E) = \frac{110}{1000} + \frac{240}{1000} = \frac{350}{1000} = 35\%$$

If one student is selected at random then what ids the probability that he/she is majoring in Math or English or Business?

$$P(M) + P(E) + P(B) = \frac{110}{1000} + \frac{240}{1000} + \frac{300}{1000} = \frac{640}{1000} = 64\%$$

C. The table below shows a random sample of 500 students getting traffic tickets in terms of their **gender** and **living arrangements.**

	Home	**A**partment	**D**orm	
Male	102	72	39	***213***
Female	209	33	45	***287***
	311	***105***	***84***	***500***

If **one** student who got traffic ticket is randomly selected then find the following **probability** that

1. The student is **M**ale **or** lives at **H**ome $\quad P(M) + P(H) - P(M\ and\ H) = \frac{213}{500} + \frac{311}{500} - \frac{102}{500} = \frac{422}{500} = 84.4\%$

2. The student is **F**emale **or** lives at **D**orm $\quad P(F) + P(D) - P(F\ and\ D) = \frac{287}{500} + \frac{84}{500} - \frac{45}{500} = \frac{326}{500} = 65.2\%$

3. The student is Female or lives at Home $\quad P(F) + P(H) - P(F\ and\ H) = \frac{287}{500} + \frac{311}{500} - \frac{209}{500} = \frac{389}{500} = 77.8\%$

4. The student lives at Dorm or at Apt. $\quad P(D) + P(A) - P(D\ and\ A) = \frac{84}{500} + \frac{105}{500} - \frac{0}{500} = \frac{189}{500} = 37.8\%$

5. The student is Female or lives at Apt. **Ans:** $\frac{359}{500} = 71.8\%$

6. The student lives at Male or not living at Apt. **Ans:** $\frac{467}{500} = 93.4\%$

7. The student is **M**ale **or** lives at **A**pt **or D**orm: **Ans:** $\frac{291}{500} = 58.2\%$

Principles of Counting

Objective: To find the total possible number of arrangements (ways) an event may occur.

1) Identify **the purpose** (Area Codes, Zip Codes, zip codes, pin numbers, License Plates, Password, Melodies)
2) Number of parts for that purpose **(area code has 3 parts, zip code has 5 parts, pin number has 4 parts)**
3) What will go into each part (letter, digit, symbols, specific value or character)
4) How many choices are available **for each part?**
5) Finally multiply the number of choices!

1) How many different zip codes are possible? D D D D D $= 10\times10\times10\times10\times10 = 100,000$

2) How many different zip codes are possible with no zero at the beginning?
D D D D D $= 9\times10\times10\times10\times10 = 90,000$

3) How many different 7- part license plates are possible with one digit first, 3 letters after followed by another 3 digits?
D L L L D D D $= 10\times26\times26\times26\times10\times10\times10 = 175,760,000$

4) How many different 7- part license plates are possible if each part can use letter or digit?

D L L L D D D $= 36\times36\times36\times36\times36\times36\times36 = 78,364,164,096$

5) How many different 6-part password can be written (case sensitive with 10 digits, 52 letters and 8 symbols)
$70\times70\times70\times70\times70\times70 = 117,649,000,000$

6) How many different 12-note melodies can be made by a 44-key keyboard?
$44^{12} = 52,654,090,776,777,588,736$

7) How many different 4- digit even numbers can we write with (0,5,6,3,8,7)?
D D D D $5\times6\times6\times3 = 540$

Hint: As 4- digit number **zero can not be used** as the first digit, and for an even number 3 choices 0,6,8 at the end.

8) How many different combinations can we set for this lock? **Ans:60,466,176**

Factorial **Permutation** **Combination**

Learn how to use you **calculator** to do **Factorial, Permutation, and Combination!!!!**

Factorial: Number of ways **n different** objects or subjects can be arranged. $n!$
$5! = 5\cdot4\cdot3\cdot2\cdot1 = 120$ $3! = 3\cdot2\cdot1 = 6$ $0! = 1$

In how many ways **3 different prizes** can be given to **three different people**? $3! = 3\cdot2\cdot1 = 6$

In how many ways 3 people can **lineup** for a picture? $3! = 3\cdot2\cdot1 = 6$

ABC, ACB, BAC, BCA, CAB, CBA

In how many ways five people can **line up** for a picture? $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

In how many ways can we arrange **3** books in a bookshelf? $3! = 3 \cdot 2 \cdot 1 = 6$

Permutation: Number of ways **x** objects **out of n** objects can be arranged. **<u>Order</u> in selection <u>does matter</u>!**

TI-83/84 *n → math → PRB → Option 2 → x*

Try these permutations $_5P_2 = 20$ $_8P_5 = 6720$ $_8P_7 = 40320$ $_7P_6 = 5040$

In how many ways **2 different prizes (W**atch, **T**ablet**)** can be given to **three different people (A,B,C)** ?

$_3P_2 = \frac{3!}{(3-2)!} = 6$ **ways** **AW, AT, BW, BT, CW, CT**

In how many ways can we select **two** out of **three different people (A,B,C)** for 1st and 2nd Prize?

$_3P_2 = \frac{3!}{(3-2)!} = 6$ **ways** **A1, A2, B1, B2, C1, C2**

1- In how many ways a teacher can give **different** prizes to 5 of his 18 students? $_{18}P_5$ **Ans**: *1,028,160*

2 - How many ways can a **president** and a **treasurer** be selected in a club of 11 members? $_{11}P_2$ **Ans:** *110*

3 - How many ways can a **president**, **vice-presiden**t, and a **treasurer** be selected in a club with 10 members? $_{10}P_3$ **Ans:** *720*

Combination: Number of ways **x** objects **out of n** objects can be arranged. **<u>Order</u> in selection <u>does not matter</u>!**

TI-83/84 *n → math → PRB → Option 3 → x*

Try these

$_6C_1 = \frac{6!}{1!5!} = 6$ $_5C_4 = 5$ $_8C_4 = \frac{8!}{4!4!} = 70$ $_4C_2 = \frac{4!}{2!2!} = 6$ $_5C_0 = \frac{5!}{0!5!} = 1$ $_5C_5 = \frac{5!}{5!0!} = 1$

In how many ways **2 same prizes (W**atch**)** can be given to **three different people (A,B,C)** ?

Solution: $_3C_2 = \frac{3!}{2!(3-2)!} = \frac{3!}{2!1} = 3$ **ways** **AW, BW** **or** **AW, CW** **or** **BW, CW**

In how many ways can we select **two** out of **five** letters (A, B, C, D, E)? $_5C_2 = \frac{5!}{2!3!} = 10$ **ways**

AB, AC, AD, AE, BC, BD, BE, CD, CE, DE

- In how many ways a teacher can select 5 of his 23 students for a fieldtrip? $_{23}C_5$ **Ans:** 33,649
- In how many ways can we select 3- member committee from a group of 8 people? $_8C_3$ **Ans:** 56

Probability Distribution

<table>
<tr><td colspan="2">X= Random Variable
A variable that has a single numerical value, determined by chance, for each outcome of a procedure.</td></tr>
<tr><td>Discrete (countable)</td><td>Continuous (measurable)</td></tr>
<tr><td>Examples
- Number of applicants passing DMV test each day

- Number of traffic violation on campus.

- Number of emergency visits each day at Hospital.</td><td>Examples
- Average rainfall each year in Sacramento

- Length of new born babies

- Height of Redwood tree.</td></tr>
<tr><td>Probability distribution used in the text,

- General discrete type

Expected Value = Mean $= \mu = \sum(x\,p(x))$

Standard deviation $= \sigma = \sqrt{\sum x^2 p(x) - \mu^2}$
- Binomial
Expected Value = Mean $= \mu = np$
Standard deviation $= \sigma = \sqrt{np(1-p)}$</td><td>Probability distribution used in the text,

- Uniform distribution

- Normal probability distribution</td></tr>
</table>

Probability distribution is information about **all the outcomes** of an experiment and their **corresponding probabilities**.

For any probability distribution the **summation** of **all the probabilities must add up to 1** or $\sum p(x) = 1$

Example 1: Create a probability distribution table for rolling a die also graph probability distribution. **x** = **number on the die**

Outcome x	1	2	3	4	5	6	
P(X)	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$\sum p(x) = 1$

Example 2: In a bag, we have of 5 reds 6 blues and 4 yellow marbles. The bottom table is the probability of getting a certain color marble form a bag. Is the table represent a probability distribution?

x Outcome		P(X)
Red		$5/15$
Blue		$6/15$
Yellow		$4/15$
		$\sum p(x) = 1$

No, because in a probability distribution the outcomes must be explained numerically not just by label.

Example 3. Let **X** to be the number of **absent employees** in class on any given day.

X	f (days)
2	10
3	20
4	15
5	5 +

To find **probability values** $p(x)$ **in the 3rd column** divide each frequency by their sum in this case 50

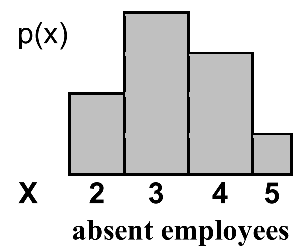
To draw **probability distribution** use **x values** as **x- axis** and $p(x)$ values as **y-axis**.

To find the mean (**expected value**) create last column $x\,p(x)$ **by multiplying** x **and** $p(x)$ in each row. The mean (**expected value**) is the summation of $x\,p(x)$ column.

X	f (days)	P(X) $= f \div n$	$x\,p(x)$
2	10	$10/50 =$ 0.20	0.40
3	20	0.40	1.20
4	15	0.30	1.20
5	5 +	0.10 +	0.50 +
	$n =$ 50	**1.0**	***3.3***

Mean $= \mu = \sum(x\,p(x))$ **= *3.3***

Probability Distribution.

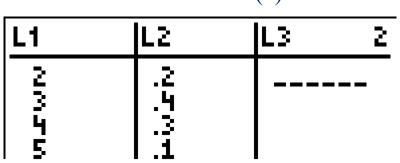


It is **most likely** that 3 employees will be absent/day
It is **least likely** that 5 employees will be absent/day.

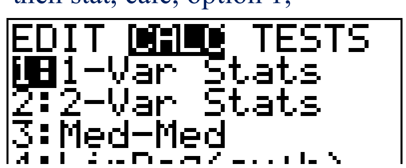
1. Find the probability **at least** 4 will be **absentees** on any given day. 0.30 + 0.10 =.40
2. Find the probability **at most** 4 will be **absentees** on any given day. 0.30 + 0.40 + 0.20 = .90
3. Find the **expected** or the **mean** number of **absentees** on any given day. **Mean** $= \mu = \sum xp(x) = 3.3$

TI-83/84 , to find expected values:

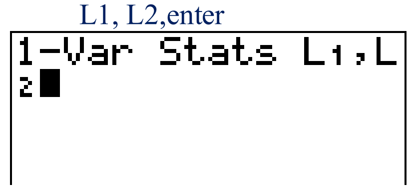
enter x values in L1 and P(x) values into L2



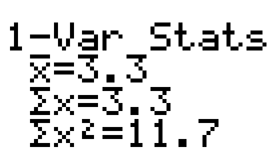
then stat, calc, option 1,



L1, L2,enter

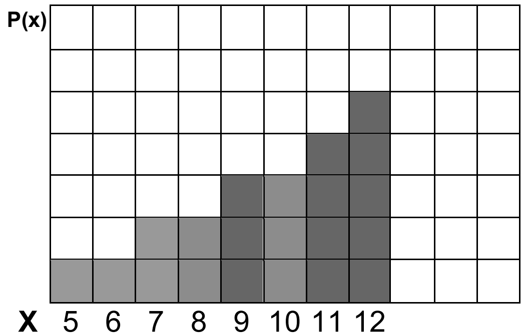


Answer is ***3.3***



E. Let **X** = the number of **car accidents** at Sun City on any given day.

x	f	$p(x)$ %	$x\,p(x)$
5	2	.02	0.10
6	3		
7	8		
8	9	.09	0.72
9	15		
10	18	.18	1.8
11	20		
12	25	.25 +	3 +
	100	1.0	***Mean = ?***

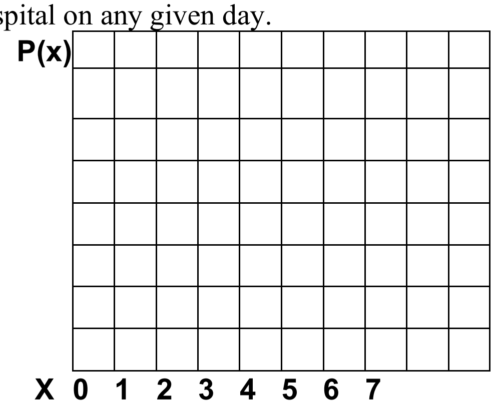


- Complete the table and draw probability distribution, locate approximately mode, median and the mean and,

1. Find the probability that there will be **at least** 10 accidents on any given day. **Ans: .18+.20+.25 = 63 %**
2. Find the probability that there will be **at most** 7 accidents on any given day. **Ans: .08+.03+.02 = 13 %**
3. Find the **expected number** or the **mean** of accidents on any given day. ***Mean =9.91***

F. Let **X** = the number of **emergency visits** at the hospital on any given day.

F			
x	f	$p(x)$ %	$x\,p(x)$
2	4	0.08	0.16
3	6		
4	8		
5	12	0.24	1.2
6	10		
7	6		
8	4	+	
		1	***Mean = ?***



- Complete the table, draw probability distribution, locate approximately mode, median and the mean and,

1. Find the probability that there will be **at least** 5 emergency visits on any given day. **Ans: 64 %**
2. Find the probability that there will be **at most** 3 emergency visits on any given day. **Ans: 20 %**
3. Find the **expected number** or the **mean** of emergency visits on any given day. ***Mean = 5.***

Expected Value Problems

Hint: To find the expected value use the formula $\sum(x \times p(x))$

A. A \$1 slot machine in a casino has a winning prize of \$6 for each play with winning probability $15/100$. What are the **expected results** for the player each time the game is played.

Outcome	x	$p(x)$	$x\,p(x)$
Win	6-1	$15/100$	$5 \times .15 = .75$
Lose	-1	$85/100$	$-1 \times .85 = -.85$
		$\Sigma\, p(x) = 1$	$\Sigma\, xp(x) = -0.10$

Each time the game is played, player has an **expected loss** of \$0.10 and the house an **expected** gain of \$0.10

- If a slot machine is played 1000 times a day and 360 days a year then each machine is expected revenue?
$.10 \times 1000 \times 360 = \$36,000$ per year.

If a typical casino has 100 slot machines, then the total **expected revenue** will be $\$36,000 \times 100 = \$3,600,000$!!!!

B. A \$1 slot machine in a casino has a winning prize of \$6 for each play with winning probability $10/100$. What are the **expected results** for the player and the house each time the game is played?

Outcome	x	$p(x)$	$x\,p(x)$
Win			
Lose			
		$\Sigma\, p(x) = 1$	$\Sigma\, xp(x) = -0.40$

How much will be the **expected** revenue if each slot machine is played 1000 times a day and 360 days a year and a typical casino has 100 slot machines. **Ans: \$14,400,000 per year**. ***Solution is same as above problem***

C) In a game, you have a 4 probability of winning \$100 and a 46 probability of losing \$10. What is your expected value? **Hint**, you do not need to subtract 10, from winning because you only pay if you lose! **Ans:\$-1.2**

Outcome	x	$p(x)$	$x\,p(x)$
Win	100	4/50	$100 \times .8 = 8$
Lose	-10	$46/50$	?
		$\Sigma\, p(x) = 1$	$\Sigma\, xp(x) =$

D) A contractor is considering a sale that promises a profit of \$20,000 with a probability of 0.60 or a loss (due to bad weather, strikes, and such) of \$10,000 with a probability of 0.4. What is the **expected outcome**?

Outcome	x	$p(x)$	$x\,p(x)$
profit			
loss			
		$\Sigma\, p(x) = 1$	$\Sigma\, xp(x) =$

Ans:\$8,000

E) Suppose you buy 1 ticket for \$1 out of a lottery of 1000 tickets where the prize for the one winning ticket is to be \$400. What is your **expected value**? **Ans:\$-0.60** **E)** _____

F) A 28-year-old man pays \$159 for a one-year life insurance policy with coverage of \$140,000. If the probability that he will live through the year is 0.9994, what is the **expected value** for the insurance policy? **Ans:\$ -74.90** **F)** ____

G) On a multiple-choice test, a student is given five possible answers for each question. **G)** _____

The student receives 1 point for a correct answer and loses ¼ point for an incorrect answer.
If student has no idea for the correct answer for a particular question and merely guesses, then what is the student's **expected points** on each question? **Ans:0**

H) Suppose also that on one of the questions you can eliminate two of the five answers as being wrong. If you guess at one of the remaining three answers, what is your **expected points** on each question? **H)** ____

Ans:0.167

E			
x	f	P(x)%	x P(x)
5	2	0.02	0.10
6	3	0.03	0.18
7	8	0.08	0.56
8	9	0.09	0.72
9	15	0.15	1.35
10	18	0.18	1.80
11	20	0.20	2.20
12	25	0.25	3.00
	100	***1.00***	***9.91***
		Mean =9.91	

F			
x	f	P(x)%	x P(x)
2	4	0.08	0.16
3	6	0.12	0.36
4	8	0.16	0.64
5	12	0.24	1.2
6	10	0.20	1.2
7	6	0.12	0.84
8	4	0.08	0.64
	50	***1.00***	***5.04***
		Mean = 5	

E-Lottery		
x	p(x)	x . P(x)
399	0.001	0.399
-1	0.999	-0.999
	1	-0.6

F- Life Insurance			
	x	p(x)	x . P(x)
Die	140000	0.0006	84
Survive	-159	0.9994	-158.9046
		1	-74.9046

G- Multiple choice			
	X	**P(x)**	**X*P(X)**
Correctly	1	0.2	0.2
Incorrectly	-0.25	0.8	-0.2
		1	0

H- Multiple choice			
	X	**P(x)**	**X*P(X)**
Correctly	1.000	0.333	0.333
Incorrectly	-0.250	0.667	-0.167
		1	0.167