

## Part Two (Section 6)

## Binomial Probability

### Assumptions;

- Each trial must have only **two outcomes**. Pass/Fail, Boy/Girl, Agree/Disagree, Tail/Head
- The probability must remain **constant** for each trial. Example: always 50% chance to be tail when you flip a coin
- The trials must be **independent**. Each flip of coin results in an independent outcome.
- The experiment should have a **fixed number of trials**. Example: number of times flipping a coin.

$$P(x) = {}^nC_x p^x (1-p)^{n-x} \quad p = \text{probability of Success} \quad n = \text{Total number of trials}$$

$$x = \text{Number of success outcomes} \quad {}^nC_x = \frac{n!}{x!(n-x)!} \quad \text{Combination Rule}$$

$$\text{Mean or Expected values} = \mu = np \quad \text{St. Dev.} = \sigma = \sqrt{np(1-p)}$$

**Hint: To find binomial probability, for each case**

- Draw a triangle, put **n** at the top, label each branch,
- put **x** on the left branch and **n - x** on the right branch
- Put **p** (probability of success) at the **bottom of left branch** and **1 - p** (probability of failure) at the **bottom of right branch**.
- Use the formula  $P(x) = {}^nC_x p^x (1-p)^{n-x}$ .

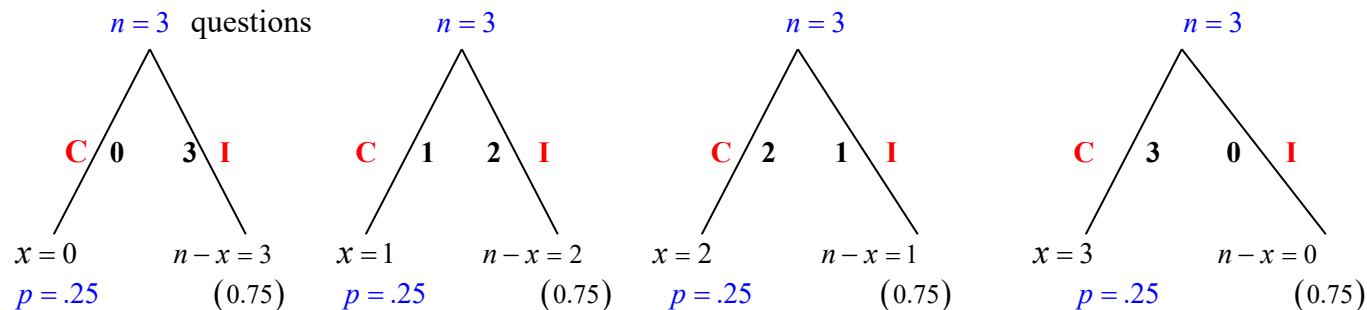
**Example 1.** John wants to guess 3 multiple choice questions (each question has 4 choices).

So the **random variable** = **X** = **number of correct guesses that he can make and they can be 0,1,2,3**

$n = 3$  is number of questions to be guessed and  $p = 1/4 = 0.25$  is the chance to guess each question correctly and  $1 - p = 3/4 = 0.75$  is the chances to guess incorrectly. So To find  $P(0)$ , probability that **no question** to be

guessed correctly can be found by setting the triangle, label all necessary information and then use the formula  $P(x) = {}^nC_x p^x (1-p)^{n-x}$   ${}^3C_0 (0.25)^0 (1-0.25)^{3-0} = {}^3C_0 (0.25)^0 (0.75)^3 = 1(1)(.4219) = 0.4219$  or **TI 83/84**

For each triangle  $n = 3$ ,  $p = 1/4 = .25$  and **X** will be changed to = 0, 1, 2, 3 Also **C:Correct** **I:Incorrect**



The probability that **no one correct** =  ${}^3C_0 (0.25)^0 (1-0.25)^{3-0} = {}^3C_0 (0.25)^0 (0.75)^3 = 1(1)(.4219) = 0.4219$

X	P(X)	$p(x)$ by TI 83/84	$x p(x)$
0	${}^3C_0 (0.25)^0 (1-0.25)^{3-0} = {}^3C_0 (0.25)^0 (0.75)^3 = 1(1)(.4219) = 0.4219$	<b>binompdf(3,0.25,0) = 0.4219</b>	0
1	${}^3C_1 (0.25)^1 (1-0.25)^{3-1} = {}^3C_1 (0.25)^1 (0.75)^2 = 3(.25)(.5625) = 0.4219$	<b>binompdf(3,0.25,1) = 0.4219</b>	.4219
2	${}^3C_2 (0.25)^2 (1-0.25)^{3-2} = {}^3C_2 (0.25)^2 (0.75)^1 = 3(.625)(.75) = 0.1406$	<b>binompdf(3,0.25,2) = 0.1406</b>	.2812
3	${}^3C_3 (0.25)^3 (1-0.25)^{3-3} = {}^3C_3 (0.25)^3 (0.75)^0 = 1(.512)(1) = 0.0156$	<b>binompdf(3,0.25,3) = 0.0156</b>	0.4688

$$\text{Mean} = \sum x p(x) = .75 \quad \text{or short-cut } \mu = np = 3(.25) = 0.75$$

Based on above table, find the probability that

- All three will be correct.  **$P(X=3) = 0.0156$**
- None will be correct.  **$P(X=0) = 0.4219$**
- At least 2 will be correct.  $0.1406 + 0.0156 = 0.1562$
- At most 1 will be correct.  $.4219 + .4219 = 0.8438$
- Expected number of correct answers.  $\mu = np = 3(.25) = 0.75$
- Standard deviation of correct answers.  $\sigma = \sqrt{np(1-p)} = \sqrt{3(.25)(1-.25)} = .75$

## TI-83/84

Using TI 83/84 to do the problem from last page.

For example to find the probability for  $x = 1$ , we can use TI 83/84 as such

2<sup>nd</sup> Distribution, select binompdf type 3,1/4,1

press enter

```

0:5:1:3 DRAW
0:1:Fcdf(
1:1:binompdf(
2:1:binomcdf(
3:1:Poissonpdf(
4:1:Poissoncdf(
5:1:geometpdf(
6:1:geometcdf(
    
```

```

binompdf(3,1/4,1
    
```

```

binompdf(3,1/4,1
.421875
    
```

And you can continue, the same way for getting the probability for  $x = 1, 2, 3$

If you want TI to calculate all the probabilities for  $x=0,1,2,3$  then the following short cut will be very helpful

Enter 0,1,2,3 in L1

go to the very top of L2

2<sup>nd</sup> Distribution, select binompdf

```

L1
0
1
2
3
-----
L1(5)=
    
```

```

L1  L2
0  -----
1  -----
2  -----
3  -----
-----
L2 =
    
```

```

0:5:1:3 DRAW
0:1:Fcdf(
1:1:binompdf(
2:1:binomcdf(
3:1:Poissonpdf(
4:1:Poissoncdf(
5:1:geometpdf(
6:1:geometcdf(
    
```

Enter 3,1÷4,L1 and then enter

Answers will now be listed on L2

```

L1  L2  L3  2
0  -----  -----
1  -----  -----
2  -----  -----
3  -----  -----
-----
L2 =...df(3, 1/4, L1
    
```

```

L1  L2  L3  2
0  .421875  -----
1  .421875  -----
2  .140625  -----
3  .015625  -----
-----
L2(1)=.421875
    
```

Find the probability that out of 6 multiple questions at most 4 are guessed correctly.

The short cut is

```

0:5:1:3 DRAW
1:1:normalpdf(
2:1:normalcdf(
3:1:invNorm(
4:1:invT(
5:1:tpdf(
6:1:tcdf(
7:1:x²pdf(
    
```

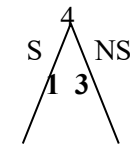
```

binomcdf(6,1/4,4
    
```

```

binomcdf(6,1/4,4
.9953613281
    
```

**L.** The past study suggests that 40 % of adult with health insurance are satisfied (**S**)with their coverage. If we have a random sample of 4 adults who have health insurance, discuss **why** we can use a **binomial** probability distribution and **what is the random variable** in this problem, then compute the corresponding probabilities



$$4C_1 (0.4)^1 (1-0.4)^{4-1} = 4C_1 (0.4)^1 (0.6)^3 = 4(.4)(.216) = 0.3456$$

$$4C_0 (0.4)^0 (1-0.4)^{4-0} = 4C_0 (0.4)^0 (0.6)^4 = 1(1)(.1296) = 0.1296$$

$x$	$p(x)$	$p(x)$ by TI 83/84	$x p(x)$
0	$4C_0 (0.4)^0 (1-0.4)^{4-0} = 4C_0 (0.4)^0 (0.6)^4 = 1(1)(.1296) = 0.1296$	$\text{binompdf}(4,0.4,0) = 0.1296$	0
1	$4C_1 (0.4)^1 (1-0.4)^{4-1} = 4C_1 (0.4)^1 (0.6)^3 = 4(.4)(.216) = .3456$	$\text{binompdf}(4,0.4,1) = 0.1296$	.3456
2	.3456	$\text{binompdf}(4,0.4,2) = 0.1296$	.6912
3	.1536	$\text{binompdf}(4,0.4,3) = 0.1296$	.4608
4	.0256	$\text{binompdf}(4,0.4,4) = 0.1296$	.1024

$$\text{Mean} = \sum xp(x) = 1.6 \text{ or short-cut } np = 4(.4) = 1.6$$

Based on above table, find the probability that

- All are satisfied with their coverage. **.0256**
- None is satisfied with their coverage. **.1296**
- At least 2 are satisfied with their coverage. **.3456 + .1536 + .0256 = .5248**
- At most 2 are satisfied with their coverage. **.1296 + .3456 + .3456 = .8208**
- Expected number of adults who are satisfied with their coverage.  $\sum xp(x) = 1.6$  or short cut  $np = 4(.4) = 1.6$
- Standard deviation of number of who are satisfied with their coverage.  $\sigma = \sqrt{np(1-p)} = \sqrt{4(.4)(1-.4)} = .98$

**M.** According to Abe, **55%** of his students pass his stat class, if **5** of his students are randomly selected and **random variable = X = number of his students that will pass his stat class**, then complete the probability distribution table,

$X$	$P(X)$	$p(x)$ by TI 83/84
0	.0185	$\text{binompdf}(5,0.55,0) = 0.0185$
1	.1128	$\text{binompdf}(5,0.55,1) = 0.11285$
2	$5C_2 (0.55)^2 (1-0.55)^{5-2} = 10(.3025)(.0911) = .276$	$\text{binompdf}(5,0.55,2) = 0.2760$
3	.3369	$\text{binompdf}(5,0.55,3) = 0.3369$
4	$5C_4 (0.55)^4 (1-0.55)^{5-4} = 5(.0915)(.45) = .2059$	$\text{binompdf}(5,0.55,4) = 0.2059$
5	.0503	$\text{binompdf}(5,0.55,5) = 0.0503$

Based on above table, find the probability that

- All lucky five will pass. **.0503**
- None will pass. **.0185**
- At least 3 will pass. **.3369 + .2059 + .0503 = .5931**
- At most 3 will pass. **.0185 + .1128 + .276 + .3369 = .7431**
- Expected number of students that will pass.  $\mu = np = 5(.55) = 2.75$
- Standard deviation of number of students that will pass.  $\sigma = \sqrt{np(1-p)} = \sqrt{5(.55)(1-.55)} = 1.11$

**Extra Practice: Problems D, E from practice problem part II on page 3.**

## More Practices for Binomial Probability

**Hint: It helps that you draw a table for each problem**

For each problem define the random variable X.

1. A die is tossed 3 times. What is the probability of

(a) 1 five? Random Variable = X = ? = number of five

$$\text{binompdf}(3, 1/6, 1) = 0.3472$$

(b) 3 fives? Random Variable = X = ? = number of five

$$\text{binompdf}(3, 1/6, 3) = 0.00463$$

(c) No fives turning up? Random Variable = X = ? = number of five

$$\text{binompdf}(3, 1/6, 0) = 0.5787$$

Random Variable = X = Number of times getting 5 by tossing a die 3 times (0, 1, 2, 3)

L1
0
1
2
3
-----
L1(5)=

0: fcdf(
1: binompdf(
2: binomcdf(
3: poissonpdf(
4: poissoncdf(
5: geometpdf(
6: geometcdf(

L1	L3
0	-----
1	-----
2	-----
3	-----
-----	-----
L2 = ...df(3, 1/6, L1	

L1	L2	L3
0	0.5787	-----
1	0.3472	-----
2	0.0694	-----
3	0.00463	-----
-----	-----	-----
L2(1) = .5787037037...		

2. Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover? Random Variable = X = ? = number of patients recovered

$$\text{binompdf}(6, 0.25, 4) = 0.03296$$

3. In the old days, there was a probability of 0.8 of success in any attempt to make a telephone call.

Calculate the probability of having 7 successes in 10 attempts. Random Variable = X = ? number of success

$$\text{binompdf}(10, 0.8, 7) = 0.20133$$

4. A (blindfolded) marksman finds that on the average he hits the target 4 times out of 5. If he fires 4 shots, what is Random Variable = X = ? = number of hits and what is the probability of

(a) more than 2 hits?  $\text{binompdf}(4, 4/5, 3) + \text{binompdf}(4, 4/5, 4) = 0.8192$

(b) at least 3 misses?  $\text{binompdf}(4, 1/5, 3) + \text{binompdf}(4, 1/5, 4) = 0.0272$

5) A quiz consists of 10 true false questions. To pass the quiz a student must get 70% or better on the quiz. If a student randomly guesses, what is the probability that the student will pass the quiz?

To pass student must answer 7 or 8 or 9 or 10 questions correctly.

$$\text{binompdf}(10, 1/2, 7) + \text{binompdf}(10, 1/2, 8) + \text{binompdf}(10, 1/2, 9) + \text{binompdf}(10, 1/2, 10) = 0.1719$$

6) A quiz consists of 10 multiple choice questions, each with five possible answers, one of which is correct. To pass the quiz a student must get 70% or better on the quiz. If a student randomly guesses, what is the probability that the student will pass the quiz? To pass student must answer 7 or 8 or 9 or questions correctly.

$$\text{binompdf}(10, 1/5, 7) + \text{binompdf}(10, 1/5, 8) + \text{binompdf}(10, 1/5, 9) + \text{binompdf}(10, 1/5, 10) = 0.0009$$

7. A multiple choice test contains 20 questions. Each question has five choices for the correct answer. Only one of the choices is correct. What is the probability of making an 80 with random guessing? Ans: 0.000000013

Random Variable = X = ? number of correct guesses

To get 80% means to get 16 out of 20 correct  $\text{binompdf}(20, 1/5, 16) = 0.000000013$

8) A study indicates that 4% of American teenagers have tattoos. You randomly sample 30 teenagers. What is the likelihood that exactly 3 will have a tattoo? **Ans: 0.0863**

**Random Variable =X= ? = number teenagers with tattoos      $\text{binompdf}(30,0.04,3) = 0.0863$**

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9. A manufacturer of metal pistons finds that on the average, 12% of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain

**Random Variable =X= ? = number of rejected pistons**

a) no more than 2 rejects?  **$\text{binompdf}(10,0.12,0) + \text{binompdf}(10,0.12,1) + \text{binompdf}(10,0.12,2) = 0.8913$**

b) at least 2 rejects?  **$1 - (\text{binompdf}(10,0.12,0) + \text{binompdf}(10,0.12,1)) = (1 - 0.65827) = 0.34173$**

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10) Suppose a die is tossed 5 times. What is the probability of getting exactly 2 fours?

**Random Variable =X= number of times getting 4 when toss a die 5 times     Ans:  $\text{binompdf}(5,1/6,2) = 0.161$**

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11) Find the **mean** for the number of sixes that appear when rolling 30 dice.

**Random Variable =X= number of times getting 6 when toss 30 dice     Ans:  $\mu = np = 30(1/6) = 5$**

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12) Knowing that about 12% of people are left handed,

a) Find the probability of that out of 25 students 5 are left handed. **Ans:  $\text{binompdf}(25,0.12,5) = 0.103$**

b) How many are expected to be left handed? **Ans:  $\mu = np = 25(0.12) = 3$**

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13) Find the mean for the number of corrected answers on a 20 multiple choice questions (5 choices), if all answers were guessed. **Ans: Random Variable =X= ?      $\mu = np = 20(1/5) = 4$**

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14) A company owns 400 laptops. Each laptop has an 8% probability of not working. You randomly select 20 laptops for your salespeople. **Random Variable =X= ?**

(a) What is the likelihood that 5 will be broken?

**$\text{binompdf}(20,0.08,5) = 0.0145$**

(b) What is the likelihood that they will all work?

**$\text{binompdf}(20,0.08,0) = 0.1887$**

(c) What is the likelihood that they will all be broken?  **$\text{binompdf}(20,0.08,20) = 0.00000000000000000001$**

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15) An XYZ cell phone is made from 55 components. Each component has a .002 probability of being defective. What is the probability that an XYZ cell phone will not work perfectly?

**Random Variable =X= number of defective components?  $1 - \text{binompdf}(55,0.002,0) = 1 - 0.8957 = 0.1043$**

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16) The ABC Company manufactures toy robots. About 1 toy robot per 100 does not work. You purchase 35 ABC toy robots. What is the probability that exactly 4 do not work?

**Random Variable =X= number of robots do not work?  $\text{binompdf}(35,0.012,4) = 0.00038$**