Abe Mirza	Topics Review	Statistic

Part II

Probability

Topics	Page

Quizzes for Part 2

Quiz # 5: This quiz covers Addition Rule, Counting Principles, Setting Probability Distribution Table and computing expected value

Quiz # 6: This quiz covers definition of binomial probability distribution and its corresponding assumptions. Knowing how to find mean and standard deviation for binomial probability distribution. Solving various problems related to binomial probability distribution.
Knowing how to use TI calculator to do binomial probability problems.

Quiz # 7: This quiz covers Normal Probability Distribution and its corresponding applications.
Knowing how to use TI calculator to do normal probability problems

Learning Objectives

Addition Rules and its Applications. **Watch PowerPoint 3C**

$$P(A \text{ or } B) = P(A) + P(B) - p(A \text{ and } B)$$

Counting Principles

Basic Counting and their **applications. Watch PowerPoint 3D**

Knowing when to use **Factorial, Combination, Permutation** and their **applications. Watch PowerPoint 3D**

Definition of **Random Variables**. **Watch PowerPoint 4A**

Difference between **Discrete** and **Continuous** Random Variables. **Watch PowerPoint 4A**

Definition of **Probability Distribution** and its properties**. Watch PowerPoint 4A**

Setting up **Probability Distribution Table** for various types of problems. **Watch PowerPoint 4A**

Using **Probability Distribution Table** to find **Expected Value (mean) by formula** $= \mu = \sum x\, p(x)$
Using **Probability**

Binomial Probability

Binomial Probability and its **four** important assumptions. **Watch PowerPoint 4B**

Drawing the **triangle** and **put all information around it**

Know the formula **Binomial Probability.**

Setting up the table for **Binomial Probability.**

Using **TI83/84** to find **Probabilities** for one value**. (See YouTube link # 2 for binomial)**

TI-83/84 *2nd* → ***DISTR*** → *Option 0* ***then*** ***input*** *(n,p,x)* → *enter*

Using **TI83/84** to find **Probabilities** for Binomial Table **(See YouTube link # 2 for binomial)**

How to use the formula $\mu = n\,p$ to find **Expected Value (mean)** for the Binomial Probability**.**

Various Applications for Binomial Probability

Normal Probability

Properties of **Normal Probability Distribution. Watch PowerPoint 5**

Difference between **Standard** and **non-standard Normal Probability Distribution**

To know **Z value** in correspondence with **Standard** Normal Probability Distribution.

To be able to **graph** a normal carve and **draw the boundary** or boundaries.

How to create a missing boundary either lower or upper use

Formulas to create **missing** Upper Boundary $UB = \mu + 5\sigma$
Formulas to create **missing** Lower Boundary $LB = \mu - 5\sigma$

How to use **TI83/84** to find **probability (percentage)** between boundaries.

TI-83/84 *2nd* $\longrightarrow$ ***DISTR*** $\longrightarrow$ *Option 2* ***then*** *input (LB,UB,* μ , σ *)* $\longrightarrow$ *enter*

How to use **TI83/84** to find a **Cut-off points** by **a given percentage**.

TI-83/84 *2nd* $\longrightarrow$ ***DISTR*** $\longrightarrow$ *Option 3* *input (%,* μ , σ *)* $\longrightarrow$ *enter*

How to find a **Cut-off points** by **a given percentage** by **table**.

$$x = \mu + \sigma z$$

Various Applications for Normal Probability

Addition Rule (Keywords: or, at least, at most)

$$P(A \; or \; B) = P(A) + P(B) - p(A \; and \; B)$$

If there is no **overlapping** between event A and B then they are called mutually exclusive $P(A \; and \; B) = 0$

$$P(A \; or \; B) = P(A) + P(B)$$

A.1 A spinner has regions numbered 1 through 10. What is the probability that the spinner will stop on an odd number **or** a multiple of 5?

$$P(odd \; or \; mult\ 5) = P(odd) + P(mult\ 5) - P(odd \; and \; mult\ 5) = \frac{5}{10} + \frac{2}{10} - \frac{1}{10} = \frac{6}{10} = 60\%$$

A.2 A spinner has regions numbered 1 through 12. What is the probability that the spinner will stop on an even number **or** a multiple of 3?

$$P(even \; or \; mult\ 3) = P(even) + P(mult\ 3) - P(even \; and \; mult\ 3) = \frac{6}{12} + \frac{4}{12} - \frac{2}{12} = \frac{8}{12} = 66.67\%$$

$$P(even \; or \; odd) = P(even) + P(odd) - P(even \; and \; odd) = \frac{6}{12} + \frac{6}{12} = \frac{12}{12} = 100\%$$

A.2 Of the 60 people who answered ″yes″ to a question, 35 were male. Of the 40 people who answered ″no″ to the question, 10 were male.

	Yes	No	
Male	35	10	**?**
Female	?	?	**?**
	60	40	

Use the given information to complete the table.

	Yes	No	
Male	35	10	**45**
Female	25	30	**55**
	60	**40**	**100**

If **one** person is selected at random from the group, answers the following questions

Find the probability that the person answered ″yes″ or is male? $P(yes \; or \; male) = \frac{60}{100} + \frac{45}{100} - \frac{35}{100} = \frac{70}{100} = 70\%$

Find the probability that the person answered ″no″ or is female? $P(no \; or \; female) = \frac{40}{100} + \frac{55}{100} - \frac{30}{100} = \frac{65}{100} = 65\%$

Facts: In a deck of 52 cards there are 26 reds, 13 spades (♠),13 hearts (♥), 13 diamonds (♦) and 13 clubs (♣), 12 faces, 4 aces, 3 cards are diamond and faces, 6 cards are red and faces, and I card is diamond and ace.

A.3. If we draw one card at random, then what is the probability that it is **D**iamond or **A**ce? Knowing that in a deck of card there are 13 **D**iamonds, 4 **A**ces and one card that is **D**iamond **A**ce

The card is **D**iamond or **A**ce $P(D \; or \; A) = P(D) + P(A) - P(D \; and \; A) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = 30.77\%$

B. The table below shows a random sample of 500 students in terms of their **gender** and **living arrangements.**

	Home	**A**partment	**D**orm	
Male	102	72	39	**213**
Female	209	33	45	**287**
	311	**105**	**84**	**500**

If **one** student is randomly selected then find the following probability that

1. The student is **M**ale or lives at **H**ome $P(M)+P(H)-P(M\ and\ H)=\frac{213}{500}+\frac{311}{500}-\frac{102}{500}=\frac{422}{500}=84.4\%$

2. The student is **F**emale or lives at **D**orm $P(F)+P(D)-P(F\ and\ D)=\frac{287}{500}+\frac{84}{500}-\frac{45}{500}=\frac{326}{500}=65.2\%$

3. The student is Male or lives at Dorm.

4. The student is Female or lives at Home

5. The student lives at Dorm or at Apt.

6. The student is Female or lives at Apt.

If **two** students are selected at random find the following **probability** that

7. Both students live at Dorm. $\frac{84}{500}\cdot\frac{83}{499}=2.79\ \%$

8. Both students live at Home.

9. Both are not living at home.

10. Both are female.

(**Answers/P.26**)

C. The table below shows 250 shirts in terms of colors and size. (**Answers/P.26**)

	Blue	**R**ed	**W**hite
Large	55	65	25
Small	45	25	35

If **one** shirt is randomly selected then find the following probability that

1. It is red or small

2. It is blue or large

3. It is blue or white

4. It is large or white

5. Red or white or small

If two shirts are selected without replacement, then find the following probability that

6. **Both** are red.

7. Both are small

8. Both are blue

***Extra Practice*: Problem A from practice problem part II on page 1.**

Principles of Counting

Objective: To find the total possible number of arrangements (ways) an event may occur.

a) Identify **the number of parts** (Area Codes, Zip Codes, License Plates, Password, Short Melodies)
b) Start with **the most restricted part** and write the number of possible choices.
c) Write the **number of choices** for other parts
d) Multiply all the numbers.

1) How many different zip codes are possible? D D D D D = $10 \times 10 \times 10 \times 10 \times 10 = 100,000$

2) How many different zip codes are possible with no zero at the beginning?
D D D D D = $9 \times 10 \times 10 \times 10 \times 10 = 90,000$

3) How many different 7- part license plates are possible with one digit first, 3 letters after followed by another 3 digits?
D L L L D D D = $10 \times 26 \times 26 \times 26 \times 10 \times 10 \times 10 = 175,760,000$

4) How many different 7- part license plates are possible if each part can use letter or digit?

D L L L D D D = $36 \times 36 \times 36 \times 36 \times 36 \times 36 \times 36 = 78,364,164,096$

5) How many different 6-part password can be written (case sensitive with 10 digits, 52 letters and 8 symbols)

$70 \times 70 \times 70 \times 70 \times 70 \times 70 = 117,649,000,000$

6) How many different 12-note melodies can be made by a 44-key keyboard?
$44^{12} = 52,654,090,776,777,588,736$

7) How many different 4- digit even numbers can we write with (0,5,6,3,8,7)?
D D D D $5 \times 6 \times 6 \times 3 = 540$

Hint: To be 4- digit **zero can not be used** as the first digit, and to be an even number the **last numbe**r can be 0,6,8, that give us 3 choices.

Extra Practice*: *Problems on page 2 from practice problem part II.

Counting

Factorial	**Combination**	**Permutation**
Number of ways **n** objects can be arranged or selected	Number of ways **x** objects **out of n** objects can be arranged or selected	Number of ways **x** objects **out of n** objects can be arranged or selected
n objects or subjects	**n objects or subjects**	**n objects or subjects**
Using all	Using **x out of n** subjects or objects.	Using **x out of n** subjects or objects.
The order of arrangement **is relevant.** **(picture line up, book arrangements)**	**The order** of arrangement **is irrelevant.** ***(committee, field trip, party)***	**The order** of arrangement **is relevant.** ***(different positions, prizes, routes)***
$n!$	$_nC_x = \frac{n!}{x!(n-x)!}$	$_nP_x == \frac{n!}{(n-x)!}$
$0! = 1,$ $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$	$_5C_3 = \frac{5!}{3!2!} = 10$ $_5C_3 = \frac{5!}{3!2!} = 10$	$_3P_2 = \frac{3!}{(3-2)!} = 6$ $_5P_2 = \frac{5!}{(5-2)!} = 20$

Learn how to use you **calculator** to do **Factorial, Combination, and Permutation!!!!**

Factorial: Number of ways **n** objects or subjects can be arranged**.**

In how many ways 3 people can line up for a picture? $3! = 3 \cdot 2 \cdot 1 = 6$

ABC, ACB, BAC, BCA, CAB, CBA

In how many ways five people can line up for a picture? $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

In how many ways can we arrange **3** books in a bookshelf? $3! = 3 \cdot 2 \cdot 1 = 6$

Combination: Number of ways **x** objects **out of n** objects can be arranged

TI-83/84 *n* → *math* → *PRB* → *Option 3* → *x*

In how many ways can we select **two** out of **five** letters? ${}_5C_2 = \frac{5!}{2!3!} = 10$ **ways**

AB, AC, AD, AE, BC, BD, BE, CD, CE, DE

${}_6C_1 = \frac{6!}{1!5!} = 6$ ${}_5C_4 = 5$ ${}_8C_4 = \frac{8!}{4!4!} = 70$ ${}_4C_2 = \frac{4!}{2!2!} = 6$ ${}_5C_0 = \frac{5!}{0!5!} = 1$ ${}_5C_5 = \frac{5!}{5!0!} = 1$

- In how many ways a teacher can select 5 of his 23 students for a fieldtrip? ${}_{23}C_5$ **Ans:** 33,649
- In how many ways can we select 3- member committee from a group of 8 people? ${}_8C_3$ **Ans:** 56

Permutation: Number of ways **x** objects **out of n** objects can be arranged

TI-83/84 *n* → *math* → *PRB* → *Option 2* → *x*

In how many ways can we select **two** out of **three** people for 1st and 2nd Prize? ${}_3P_2 = \frac{3!}{(3-2)!} = 6$ **ways**

AB, BA, AC, CA, BC, CB

${}_5P_2 = 20$ ${}_8P_5 = 6720$ ${}_8P_7 = 40320$ ${}_7P_6 = 5040$

1- In how many ways a teacher can give different prizes to 5 of his 18 students? ${}_{18}P_5$ **Ans**: *1,028,160*

2 - How many ways can a president and a treasurer be selected in a club of 11 members? ${}_{11}P_2$ **Ans:** *110*

3 - How many ways can a president, vice-president, and a treasurer be selected in a club with 10 members? ${}_{10}P_3$ **Ans:** *720*

4 - How many different signals can be made by 5 flags from 8-flags of different colors? ${}_8P_5$ **Ans:** *6720*

Probability Distribution

<table>
<tr><td colspan="2">X= Random Variable
A variable that has a single numerical value, determined by chance, for each outcome of a procedure.</td></tr>
<tr><td>Discrete (countable)</td><td>Continuous (measurable)</td></tr>
<tr><td>Examples

- Number of applicants passing DMV test each day

- Number of traffic violation on campus.

- Number of emergency visits each day at Hospital.</td><td>Examples

- Average rainfall each year in Sacramento

- Length of new born babies

- Height of Redwood tree.</td></tr>
<tr><td>Probability distribution used in the text,

- General discrete type
Expected Value = Mean $= \mu = \sum(x\, p(x))$
Standard deviation $= \sigma = \sqrt{\sum x^2 p(x) - \mu^2}$
- Binomial
Expected Value = Mean $= \mu = np$
Standard deviation $= \sigma = \sqrt{np(1-p)}$</td><td>Probability distribution used in the text,

- Uniform distribution
- Normal probability distribution</td></tr>
</table>

Example 1. Let **Random Variable** = **X** to be the number of **absent employees** in an office in a given day.

X	f (days)
2	10
3	20
4	15
5	5 +

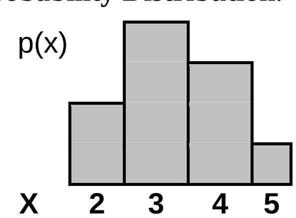
To find **probability values** $p(x)$ **in the 3rd column** divide each frequency by their sum in this case 50

To draw **probability distribution** use **x values** as **x- axis** and $p(x)$ values as **y-axis**.

To find the mean (**expected value**) create last column $x\, p(x)$ **by multiplying** x **and** $p(x)$ in each row. The mean (**expected value**) is the summation of $x\, p(x)$ column.

X	f (days)	P(X) $= f \div n$	$x\, p(x)$
2	10	$10/50 =$ 0.20	0.40
3	20	0.40	1.20
4	15	0.30	1.20
5	5 +	0.10 +	0.50 +
	$n =$ 50	**1.0 ?**	***3.3***
			Mean $= \mu = \sum(x\, p(x))$ **= 3.3**

Probability Distribution.

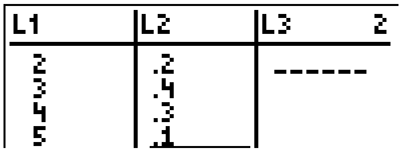


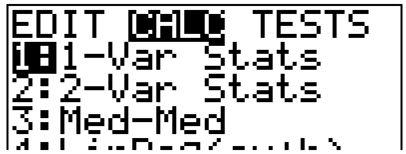
absent employees

It is **most likely** that 3 employees will be absent/day
It is **least likely** that 5 employees will be absent/day.

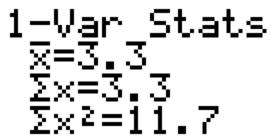
1. Find the probability that at **least** there will be 4 **absent** in a given day. 0.30 + 0.10 =.40
2. Find the probability that at **most** there will be 4 **absent** in a given day. 0.30 + 0.40 + 0.20 = .90
3. Find the **expected** number of number of absentees in a given day. **Mean** $= \mu = \sum xp(x) = 3.3$

TI-83/84 , to find expected values: enter x values in L1 and P(x) values into L2 then stat, calc, option 1, L1, L2,enter



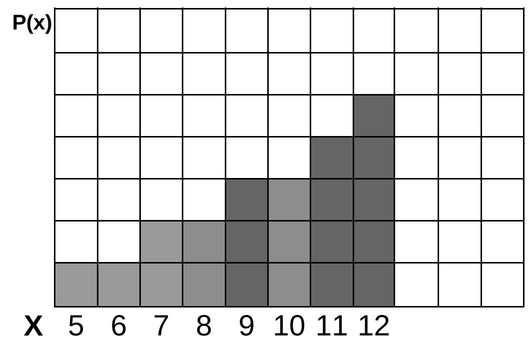


Answer is ***3.3***



E. Let **Random Variable = X** = the number of **reported car accidents** at Sun City in a given day.

x	f	$p(x)$ %	$x\,p(x)$
5	2	.02=2%	0.10
6	3		
7	8		
8	9	.09=9	0.72
9	15		
10	18	.18=18	1.8
11	20		
12	25	.25 +	3 +
	100	1.0 ?	***Mean = ?***

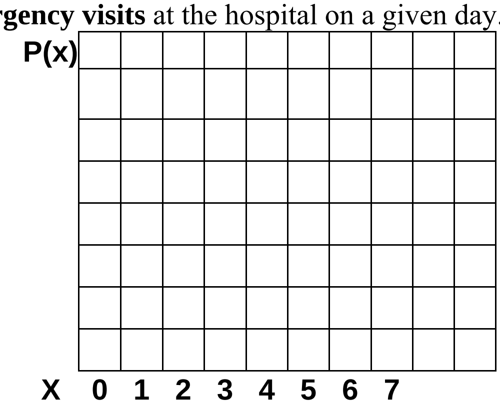


- Complete the table and draw probability distribution **(Answers/P.26)** and find the probability that.

1. At **least** there will be 10 returned accidents in a given day. **Ans: 63 %**
2. At **most** there will be 7 returned accidents in a given day. **Ans: 13 %**
3. Find the **expected number** of accidents in a given day. ***Mean =9.91***

F. Let **Random Variable** = **X** = the number of **emergency visits** at the hospital on a given day.

F			
x	f	$p(x)$ %	$x\,p(x)$
0	2		
1	17		
2	10		
3	11		
4	10		
5	4		
6	8		
7	2		
		?	***Mean = ?***



- Complete the table, draw probability distribution **(Answers/P.26)** and find the probability that,

1. At **least** there will be 5 emergency visits in a given day. **Ans: 22 %**
2. At **most** there will be 3 emergency visits in a given day. **Ans: 63 %**
3. Find the **expected number** of emergency visits in a given day. ***Mean = 3.00***

Extra Practice: Problem B from practice problem part II on page 1.

Expected Value Problems **Hint: To find the expected value use the formula** $\sum(x \times p(x))$

G. A $1 slot machine in a casino has a winning prize of $6 for each play with winning probability $15/100$. What are the expected results for the player and the house each time the game is played.

Outcome	x	$p(x)$	$x\,p(x)$
Win	6-1	$15/100$	$5 \times .15 = .75$
Lose	-1	$85/100$	$-1 \times .85 = -.85$
		$\sum p(x) = 1$	$\sum xp(x) = -0.10$

- Each time the game is played, player has an expected loss of $.10 and the house an expected gain of $.10

- If a slot machine is played 1000 times a day and 360 days a year then each machine is expected to generate revenue of $1000 \times 360 \times .10 = \$36,000$ per year. If a typical casino has 100 slot machines then the total revenue will be $\$36,000 \times 100 = \$3,600,000$!!!!

H. A $1 slot machine in a casino has a winning prize of $6 for each play with winning probability $10/100$. What are the expected results for the player and the house each time the game is played.

How much will be the expected to generate revenue if a typical casino has 100 slot machines and each slot machine is played 1000 times a day and 360 days a year. **Ans: $14,400,000 per year**. *Solution: page 26*

I) In a game, you have a 4 probability of winning $110 and a 46 probability of losing $10. What is your expected value?

Outcome	x	$p(x)$	$x\,p(x)$
Win	110-10	4/50	$100 \times .8 = 8$
Lose	-10	$46/50$	?
		$\sum p(x) = 1$	$\sum xp(x) = -1.2$

J) A contractor is considering a sale that promises a profit of $20,000 with a probability of 0.60 or a loss (due to bad weather, strikes, and such) of $10,000 with a probability of 0.4. What is the expected profit?

Outcome	x	$p(x)$	$x\,p(x)$
profit	?	?	?
loss	?	?	?
		$\sum p(x) = 1$	$\sum xp(x)$ =8,000

K) Suppose you pay $3.00 to roll a fair die with the understanding that you will get back $5.00 for rolling a 5 or a 4, nothing otherwise. What is your expected value of your gain or loss? **3)** _____

Outcome	x	$p(x)$	$x\,p(x)$
Win	?	?	?
Lose	?	?	?
		$\sum p(x) = 1$	$\sum xp(x) =$

Solution: page 26

L) In a game, you have a 1 probability of winning $116 and a 44 probability of losing $7. What is your expected value? **1)** _____

A) $-\$4.27$ **B)** $2.58 **C)** $-\$6.84$ **D)** $9.42

M) A contractor is considering a sale that promises a profit of $38,000 with a probability of 0.7 or a loss (due to bad weather, strikes, and such) of $18,000 with a probability of 0.3. What is the expected profit? **2)** _____

A) $21,200 **B)** $20,000 **C)** $26,600 **D)** $39,200

N) Suppose you pay $3.00 to roll a fair die with the understanding that you will get back $5.00 for rolling a 5 or a 4, nothing otherwise. What is your expected value of your gain or loss? **3)** _____

A) −$3.00 **B)** $5.00 **C)** $3.00 **D)** −$1.33

O) Suppose you buy 1 ticket for $1 out of a lottery of 1000 tickets where the prize for the one winning ticket is to be $5000. What is your expected value? **4)** _____

A) $40.00 **B)** $4.00 **C)** $0.40 **D)** −$0.40

P) A 28-year-old man pays $159 for a one-year life insurance policy with coverage of $140,000. If the probability that he will live through the year is 0.9994, what is the expected value for the insurance policy? **5)** ____

A) -$158.90 **B)** $139,916.00 **C)** −$75.00 **D)** $84.00

Q) The prizes that can be won in a sweepstakes are listed below together with the chances of winning each one:$3500 (1 chance in 8100); $1900 (1 chance in 5400); $700 (1 chance in 3400); $400 (1 chance in 2500). Find the expected value of the amount won for one entry if the cost of entering is 66 cents. **6)** ____

A) −$0.49 **B)** $0.49 **C)** 4.9 **D)** −$4.9

R) On a multiple-choice test, a student is given five possible answers for each question. The student receives 1 point for a correct answer and loses ¼ point for an incorrect answer. If the student has no idea of the correct answer for a particular question and merely guesses, what is the student's expected gain or loss on the question? **7)** _____

A) 0 **B)** 0.25 **C)** 0.133 **D)** −0.33

S) Suppose also that on one of the questions you can eliminate two of the five answers as being wrong. If you guess at one of the remaining three answers, what is your expected gain or loss on the question? **8)** ____

A) 0 **B)** 0.167 **C)** 0.133 **D)** 0.63

T) A dairy farmer estimates for the next year the farm's cows will produce about 25,000 gallons of milk. Because of variation in the market price of milk and cost of feeding the cows, the profit per gallon may vary with the probabilities given in the table below. Estimate the profit on the 25,000 gallons. **9)** ___

Gain per gallon	$1.10	$0.90	$0.70	$0.40	$0.00	-$0.10
Probability	0.30	0.38	0.20	0.06	0.04	0.02

A) $21,850 **B)** $20,508 **C)** $20,580 **D)** $20,850

U) At many airports, a person can pay only $1.00 for a $100,000 life insurance policy covering the duration of the flight. In other words, the insurance company pays $100,000 if the insured person dies from a possible flight crash; otherwise the company gains $1.00 (before expenses). Suppose that past records indicate 0.45 deaths per million passengers. How much can the company expect to gain on one policy? **10)** ___

A) $0.895 **B)** $0.955 **C)** $0.95 **D)** $0.855

On 100,000 policies?

A) $89,500 **B)** $95,500 **C)** $95,000 **D)** $85,500

Solutions to Expected values Problems

L-Game

x	p(x)	x . P(x)
116	0.022	2.578
-7	0.978	-6.844
	1	-4.267

M- Contractor

x	p(x)	x . P(x)
38000	0.7	26600
-18000	0.3	-5400
		21200

N-Fair Die

x	p(x)	x . P(x)
2	0.333	0.667
-3	0.667	-2.000
	1.000	-1.333

O-Lottery

x	p(x)	x . P(x)
4999	0.001	4.999
-1	0.999	-0.999
	1	4

P- Life Insurance

	x	p(x)	x . P(x)
Die	140000	0.0006	84
Survive	-159	0.9994	-158.9046
		1	-74.9046

Q- Sweepstakes

x	p(x)	x . P(x)
3499.34	0.00012	0.43202
1899.34	0.00019	0.35173
699.34	0.00029	0.20569
399.34	0.0004	0.15974
-0.66	0.999	-0.6593
	1	0.48983

R- Multiple choice

	X	P(x)	X*P(X)
Correctly	1	0.2	0.2
Incorrectly	-0.25	0.8	-0.2
		1	0

S- Multiple choice

	X	P(x)	X*P(X)
Correctly	1.000	0.333	0.333
Incorrectly	-0.250	0.667	-0.167
		1	0.167

T- Gallon of Milk

X	P(x)	X*P(X)
1.1	0.3	0.33
0.9	0.38	0.342
0.7	0.2	0.14
0.4	0.06	0.024
0	0.04	0
-0.1	0.02	-0.002
	1	0.834

U- Plane Crash

	X	P(x)	X*P(X)
No Crash	-1	0.9999996	-1
Crash	99999	0.00000045	0.045
		1	-0.955

100000*.955=95,500

Binomial Probability

Assumptions;

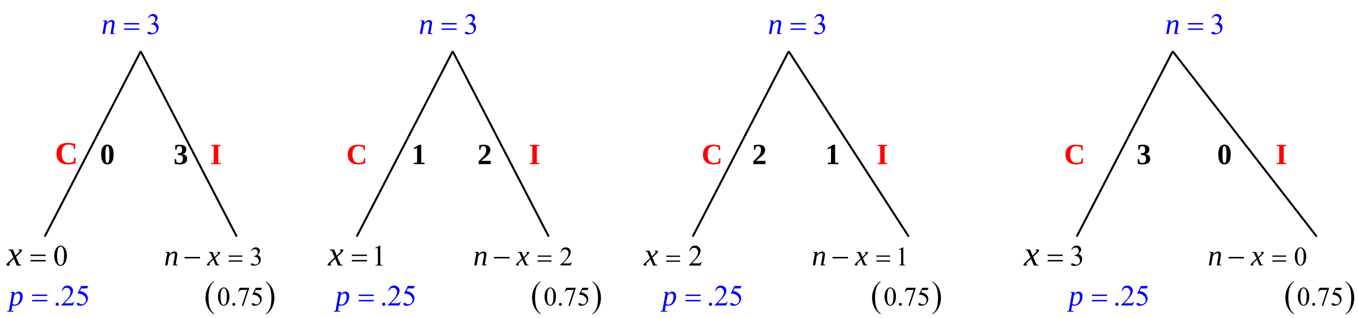
1. Each trial must have only **two outcomes.** **P**ass/**F**ail, **B**oy/**G**irl, **A**gree/**D**isagree, **T**rue/**F**alse
2. The probability must remain **constant** for each trial.
3. The trials must be **independent**.
4. The experiment should have a **fixed number of trials**.

$P(x) = nCx\ p^{x}(1-p)^{n-x}$ Mean $= \mu = n\,p$ St. Dev. $= \sigma = \sqrt{n\,p(1-p)}$

$p = probability\ of\ Success$ $n = Total\ number\ of\ trials$ $x = Number\ of\ success\ outcomes$
$nCx = Combination\ Rule$

Example.
1. John wants to guess the last 3 multiple choice question on the test (each question has 4 choices for the correct answers). So $n = 3$ and $p = 1/4 = 0.25$, The **random variable** = **X = number of correct answer(0,1,2,3),** then complete the probability distribution table, X = can be , 0, 1, 2, 3 $p = 1/4 = .25$

C:Correct I:Incorrect



The probability that **no one correct** $= 3C_0\ (0.25)^{0}(1-0.25)^{3-0} = 3C0\ (0.25)^{0}(0.75)^{3} = 1(1)(.4219) =$ **0.4219**

X	P(X)	$x\,p(x)$
0	$= 3C_0\ (0.25)^{0}(1-0.25)^{3-0} = 3C0\ (0.25)^{0}(0.75)^{3} = 1(1)(.4219) =$ **0.4219**	**0**
1	$= 3C_1\ (0.25)^{3}(1-0.25)^{3-1} = 3C1\ (0.25)^{1}(0.75)^{2} = 3(.25)(.5625) =$ **0.4219**	**.4219**
2	$= 3C_2\ (0.25)^{2}(1-0.25)^{3-2} = 3C2\ (0.25)^{2}(0.75)^{1} = 3(.625)(.75) =$ **0.1406**	**.2812**
3	$= 3C_3(0.25)^{3}(1-0.25)^{3-3} = 3C3\ (0.25)^{3}(0.75)^{0} = 1(.512)(1) =$ **0.0156**	**0.4688**

$\sum xp(x) = .75$

$\mu = np = 3(.25) = 0.75$

Based on above table, find the probability that

1. All three will be correct. **P(X = 3) = 0.0156**
2. None will be correct. **P(X =0) = 0.4219**
3. At least 2 will be correct. $0.1406 + 0.0156 = 0.1562$
4. At most 1 will be correct. $.4219 + .4219 = 0.8438$

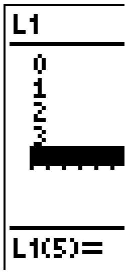
5. Expected number of correct answers. $\mu = np = 3(.25) = 0.75$

6. Standard deviation of correct answers. $\sigma = \sqrt{np(1-p)} = \sqrt{3(.25)(1-.25)} = .75$

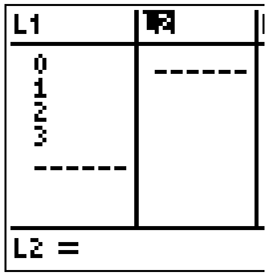
TI-83/84

To find P(x) values:

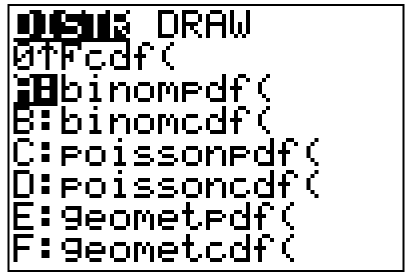
Enter 0,1,2,3 in L1



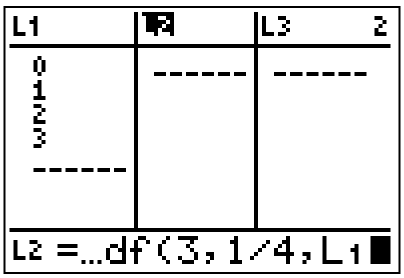
go to the very top of L2



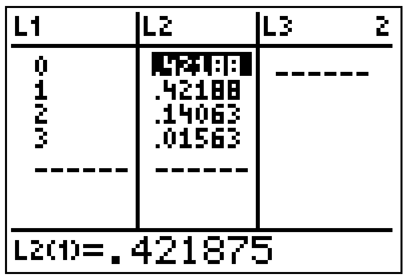
2nd Distribution, select binompdf



3,1÷4 ,L1 and then enter

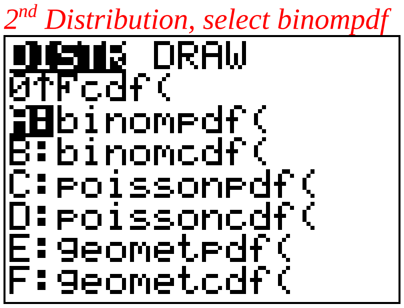


Answers now are in L2

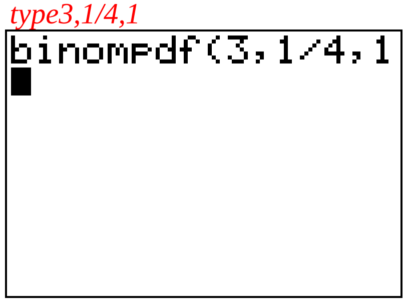


If you need to find the probability of a specific value let's say x=1, you do not need to create a table, the short cut is

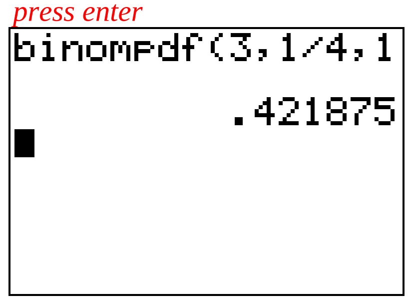
2nd Distribution, select binompdf



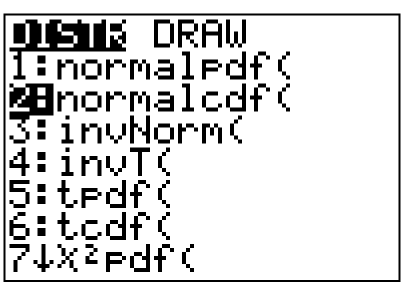
type3,1/4,1

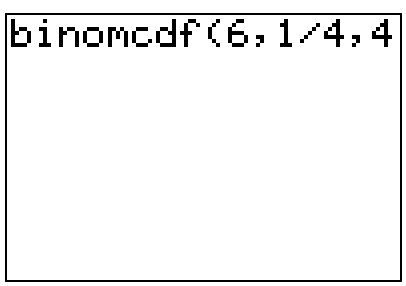


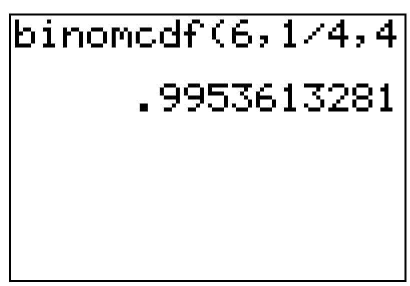
press enter



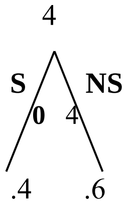
Find the probability that out of 6 multiple questions at most 4 are guessed correctly.
The short cut is

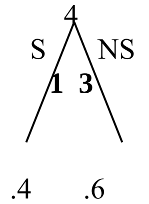






L. The past study suggests that 40 % of adult with health insurance are satisfied with their coverage. If we have a random sample of 4 adults who have health insurance, discuss **why** we can use a **binomial** probability distribution and **what is the random variable** in this problem, then compute the corresponding probabilities





$_4C_1\ (0.4)^1(1-0.4)^{4-1} = {_4C_1}\ (0.4)^1(0.6)^3 = 4(.4)(.216) = \mathbf{0.3456}$

$_4C_0\ (0.4)^0(1-0.4)^{4-0} = {_4C_0}\ (0.4)^0(0.6)^4 = 1(1)(.1296) = \mathbf{0.1296}$

X	**P(X)**	$x\,p(x)$
0	$_4C_0\ (0.4)^0(1-0.4)^{4-0} = {_4C_0}\ (0.4)^0(0.6)^4 = 1(1)(.1296) = \mathbf{0.1296}$	**0**
1	$_4C_1\ (0.4)^1(1-0.4)^{4-1} = {_4C_1}\ (0.4)^1(0.6)^3 = 4(.4)(.216) = \mathbf{.3456}$	***.3456***
2	***.3456***	
3	***.1536***	
4	***.0256***	

$\sum xp(x) = 1.6$

Based on above table, find the probability that

1. All are satisfied with their coverage.

2. None is satisfied with their coverage.

3. At least 2 are satisfied with their coverage.

4. At most 2 are satisfied with their coverage.

5. Expected number of adults who are satisfied with their coverage. $\mu = np =$

6. Standard deviation of number of who are satisfied with their coverage. $\sigma = \sqrt{np(1-p)} =$

Solution: page 26

M. According to Abe, **55%** of his students pass his stat class, if **5** of his students are randomly selected and **random variable** = **X = number of his students that will pass his stat class,** then complete the probability distribution table,

X	**P(X)**
0	**.0185**
1	**.1128**
2	$_5C_2\ (0.55)^2(1-0.55)^{5-2} = {_5C_2}\ (0.55)^2(0.45)^3 = 10(.3025)(.0911) = \mathbf{.276}$
3	**.3369**
4	$_5C_4\ (0.55)^4(1-0.55)^{5-4} = {_5C_4}\ (0.55)^4(0.45)^1 = 5(.0915)(.45) = \mathbf{.2059}$
5	**.0503**

Based on above table, find the probability that

1. All lucky five will pass.

2. None will pass.

3. At least 3 will pass.

4. At most 3 will pass.

5. Expected number of students that will pass.

6. Standard deviation of number of students that will pass.

Solution: page 26

***Extra Practice**: Problems D, E from practice problem part II on page 3.*

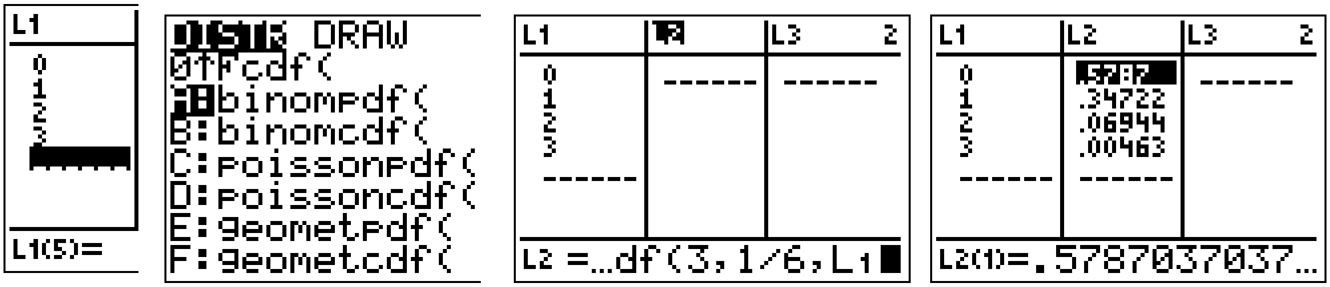
More Prtactices for Binomial   Probability

For each problem define the random variable X.

1. A die is tossed 3 times. What is the probability of

(a) 1 five? ***Ans:* 0.3472**   (b) 3 fives? ***Ans: 0.00463***   (c) No fives turning up? ***Ans: 0.5787***

Random Variable =X= Number of times getting 5 by tossing a die 3 times (0, 12,2,3)



---

2. Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover? ***Ans: 0.03296***

Random Variable =X= ?

---

3. In the old days, there was a probability of 0.8 of success in any attempt to make a telephone call.

Calculate the probability of having 7 successes in 10 attempts. ***Ans: 0.20133***

Random Variable =X= ?

---

4. A (blindfolded) marksman finds that on the average he hits the target 4 times out of 5. If he fires 4 shots, what is the probability of

(a) more than 2 hits? ***Ans: 0.8192***   b) at least 3 misses? ***Ans: 0.0272***

Random Variable =X= ?

---

5. A multiple choice test contains 20 questions. Each question has five choices for the correct answer. Only one of the choices is correct. What is the probability of making an 80 with random guessing? ***Ans: 0.000000013***

Random Variable =X= ?

---

6) A study indicates that 4% of American teenagers have tattoos.   You randomly sample 30 teenagers.   What is the likelihood that exactly 3 will have a tattoo? ***Ans: 0.0863***

Random Variable =X= ?

---

7. A manufacturer of metal pistons finds that on the average, 12% of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain

a) no more than 2 rejects? ***Ans: 0.8913***   b) at least 2 rejects? ***Ans: 0.34173***

Random Variable =X= ?

8. Suppose a die is tossed 5 times. What is the probability of getting exactly 2 fours? ***Ans: 0.161***

Random Variable =X= ?

---

9. Find the mean for the number of sixes that appear when rolling 30 dice. ***Ans: 5***

Random Variable =X= ?

---

10. Knowing that about 12% of people are left handed, ***Ans: 0.1025*** **Random Variable =X= ?**

a) find the probability of having five left-handed students in a class of twenty five. ***Ans:* 0.103**

b) How many are expected to be left handed? ***Ans: 3***

---

11. Find the mean for the number of corrected answers on a 20 multiple choice questions (5 choices), if all answers were guessed. ***Ans: 4*** **Random Variable =X= ?**

---

12) A company owns 400 laptops. Each laptop has an 8% probability of not working. You randomly select 20 laptops for your salespeople. **Random Variable =X= ?**

(a) What is the likelihood that 5 will be broken? ***Ans: 0.0145***

(b) What is the likelihood that they will all work? ***Ans: 0.1887***

(c) What is the likelihood that they will all be broken? ***Ans:*** **0.0000000000000000000001**

---

13) An XYZ cell phone is made from 55 components. Each component has a .002 probability of being defective. What is the probability that an XYZ cell phone will not work perfectly? ***Ans: 0.104***

Random Variable =X= ?

---

14) The ABC Company manufactures toy robots. About 1 toy robot per 100 does not work. You purchase 35 ABC toy robots. What is the probability that exactly 4 do not work? ***Ans: 0.00038***

Random Variable =X= ?

---

15) The LMB Company manufactures tires. They claim that only .007 of LMB tires are defective. What is the probability of finding 2 defective tires in a random sample of 50 LMB tires? ***Ans: 0.428***

Random Variable =X= ?

---

16) An HDTV is made from 100 components. Each component has a .005 probability of being defective. What is the probability that an HDTV will work perfectly? ***Ans: 0.606*** Random Variable =X= ?

---

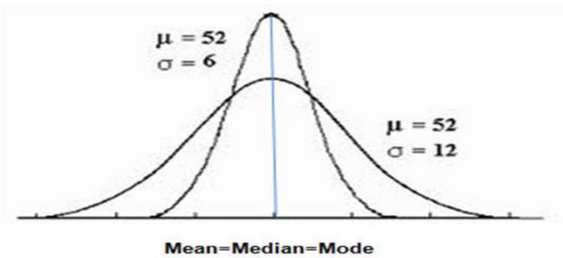
17. The ratio of boys to girls at birth in Singapore is quite high at 1.09:1. What proportion of Singapore families with exactly 6 children will have at least 3 boys? (Ignore the probability of multiple births.) ***Ans: 0.69565***

Random Variable =X= ?

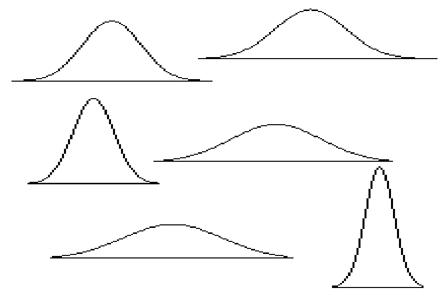
[**Interesting and disturbing trivia:** In most countries the ratio of boys to girls is about 1.04:1, but in China it is 1.15:1.

Normal Probability Distribution

$$y = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(\bar{x}-\mu)^2/2\sigma^2}$$



Smaller standard deviation will results in narrower normal width.



Normal distributions are a family of distributions that have the same general shape. They are symmetric with scores more concentrated in the middle than in the tails. Normal distributions are sometimes described as bell shaped. Examples of normal distributions are shown above on the left. Notice that they differ in how spread out they are. The area under each curve is the same. The height of a normal distribution can be specified mathematically in terms of two parameters: the mean (μ) and the standard deviation (σ).

Properties

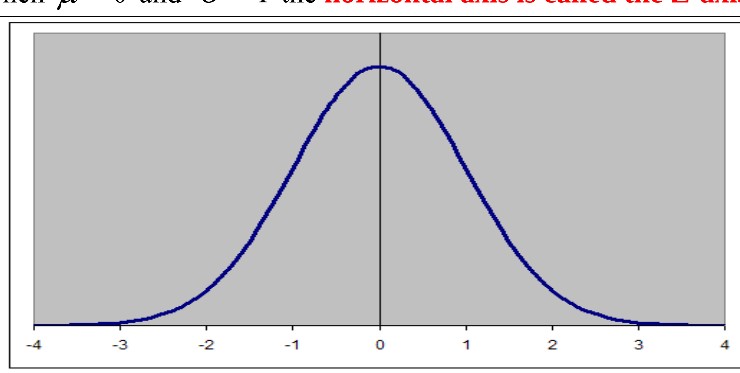
1. Normal Probability Distribution deals with continuous random variables.
   (age, speed, temp, weight, length, time, …)
2. The ***entire area*** under the curve is *100% = 1*, 50% of area to the left and 50 % to the right.
3. The **larger** the **standard deviation** the ***wider the distribution*** will be.
4. The ***area*** under the curve represents the ***probability***.
5. The ***graph of the standard normal curve approaches zero*** as z increases in positive direction or decreases in negative direction.
6. The ***area or percentage under the curve*** (area between two boundaries) can be about ***an individual*** or the ***entire population***.

Standard Normal Probability Distribution (SNPD)

It is a special case of normal distribution when $\mu = 0$ and $\sigma = 1$ the **horizontal axis is called the Z-axis**.

The graph of the standard normal curve approaches zero as z increases in positive direction or decreases in negative direction

$\mu = 0$ and $\sigma = 1$



Finding area (percentage) under Standard Normal Probability distribution by using TI 83/84

Note 1: When using TI 83/84,

You need a Lower Boundary *LB* *or, an* Upper Boundary ***UB* *and*** $\mu = 0$ and $\sigma = 1$

Note 3: Sketch a normal curve, draw both boundaries and shade the area in between the boundaries.

Note 4: If one boundary is missing either Lower or Upper, then use the following rule to create one.

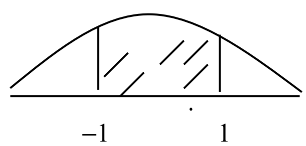
Formulas to create **missing** Lower Boundary $LB = \mu - 5\sigma$

Formulas to create **missing** Upper Boundary $UB = \mu + 5\sigma$

Steps to use TI-83/84

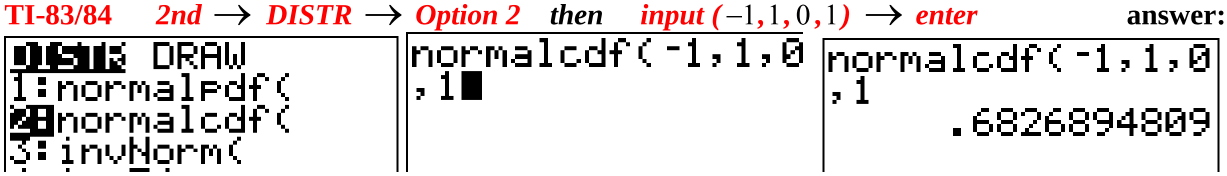
2nd $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 2*** ***then*** ***input (LB,UB,0,1)*** $\longrightarrow$ ***enter***

Example 1 Find the area (percentage) between $z = -1$ and $z = 1$ $P(-1 < Z < 1) = ?$ (68% empirical rule)

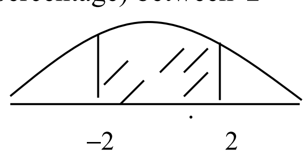


TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 2*** ***then*** ***input (LB,UB,0,1)*** $\longrightarrow$ ***enter***

TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 2*** ***then*** ***input*** $(-1, 1, 0, 1)$ $\longrightarrow$ ***enter*** **answer: 68.27%**



Example 2 Find the area (percentage) between $z = -2$ and $z = 2$ $P(-1 < Z < 2) = ?$ (**95%** empirical rule)



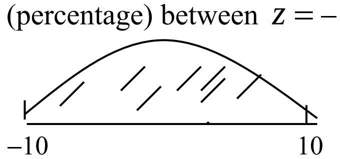
TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 2*** ***then*** ***input*** $(-2, 2, 0, 1)$ $\longrightarrow$ ***enter*** **answer: 95.45%**

Example 3 Find the area (percentage) between $z = -3$ and $z = 3$ (basically applying **99.7%** empirical rule)

TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 2*** ***then*** ***input*** $(-3, 3, 0, 1)$ $\longrightarrow$ ***enter*** **answer: 99.73%**

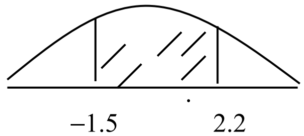
Example 4 Find the area (percentage) between $z = -10$ and $z = 10$ (between **10** standard deviation)

Important



TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 2*** ***then*** ***input*** $(-10, 10, 0, 1)$ $\longrightarrow$ ***enter*** **answer: 99.99%**

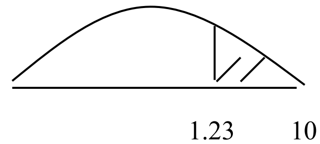
Example 5 Find the area (percentage) between $z = -1.5$ and $z = 2.2$ $P(-1.5 < Z < 2.2) = ?$



TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***input*** $(-1.5, 2.2, 0, 1)$ $\rightarrow$ ***enter*** **answer: 91.92%**

Example 6 **Find the area (percentage) greater than** $z = 1.23$

$P(1.23 < Z) = ?$

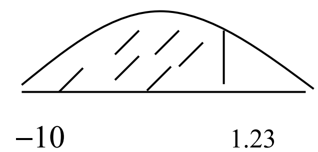


Upper boundary is missing: create an upper boundary $UB = \mu + 5\sigma$ **in this case** $UB = 0 + 5(1) = 5$

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***input*** $(1.23, 5, 0, 1)$ $\rightarrow$ ***enter*** **answer: 10.93%**

Example 7 **Find the area (percentage) less than** $z = 1.23$

$P(Z < 1.23) = ?$

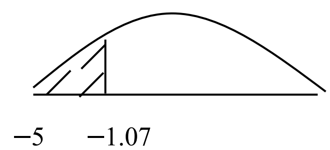


Lower boundary is missing: create a lower boundary $LB = \mu - 5\sigma$ **in this case** $LB = 0 - 5(1) = -5$

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***input*** $(-5, 1.23, 0, 1)$ $\rightarrow$ ***enter*** **answer: 89.065%**

Example 8 **Find the area (percentage) less than** $z = -1.07$

$P(Z < -1.07) = ?$

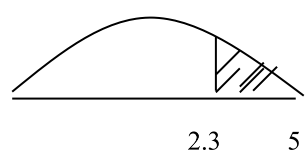


Lower boundary is missing: create a lower boundary $LB = \mu - 5\sigma$ **in this case** $LB = 0 - 5(1) = -5$

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***input*** $(-5, -1.07, 0, 1)$ $\rightarrow$ ***enter*** **answer: 14.23%**

Example 9 **Find the area (percentage) greater than** $z = 2.35$

$P(2.35 < Z) = ?$



Upper boundary is missing: create an upper boundary $UB = \mu + 5\sigma$ **in this case** $UB = 0 + 5(1) = 5$

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***input*** $(2.35, 5, 0, 1)$ $\rightarrow$ ***enter*** **answer: 0.94%**

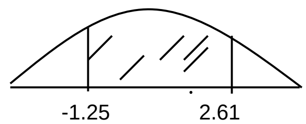
More Practice on S N P D when $\mu = 0$ and $\sigma = 1$ the **horizontal axis is Z-axis**.

TI-83/84 2nd → DISTR → Option 2 then input (LB,UB,0,1) → enter

Formulas to create **missing** Upper Boundary $UB = \mu + 5\sigma$ $UB = 0 + 5(1) = 5$

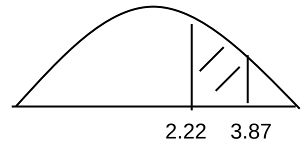
missing Lower Boundary $LB = \mu - 5\sigma$ $LB = 0 - 5(1) = -5$

1) P(-1.25 < Z < 2.61) =



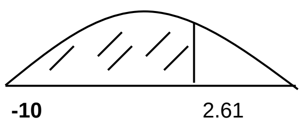
Answer =0 *.8899*

2) P(2.22 < Z < 3.87) =



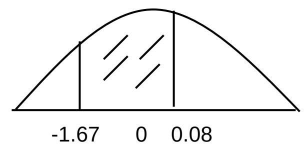
Answer = *0.0131*

3) P(Z < 2.61) =



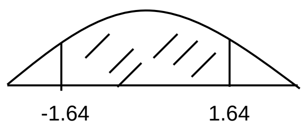
Answer = *.9955*

4) P(-1.67 < Z < 0.08) =



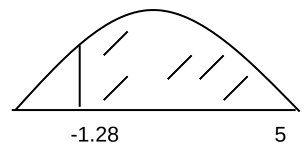
Answer = *0.4844*

5) P(-1.64 < Z < 1.64) =



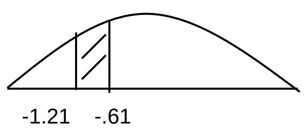
Answer = *.8990*

6) P(-1.28 < Z) =



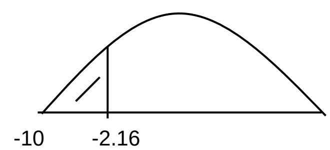
Answer = *.8997*

7) P(-1.21 < Z < -0.61) =



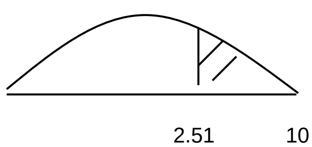
Answer = *.1578*

8) P(Z < -2.16) =



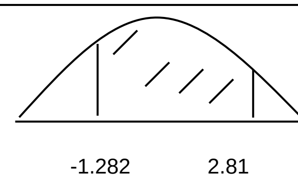
Answer = *0.0154*

9) P(2.51 < Z) =



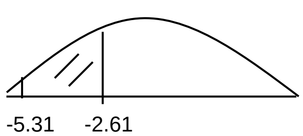
Answer = *0.0060*

10) P(-1.82 < Z < 2.81) =



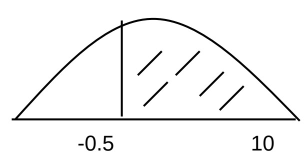
Answer = *0.9631*

11) P(-5.34 < Z < -2.61) =



Answer = *0.0044*

12) P(-0.5 < Z) =



Answer = *0.6915*

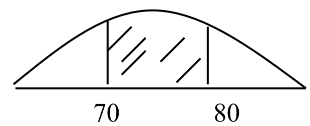
Extra Practice*: *Problem 1-12 on top of page 4 from practice problem part II.

Non-Standard Normal Probability Distribution

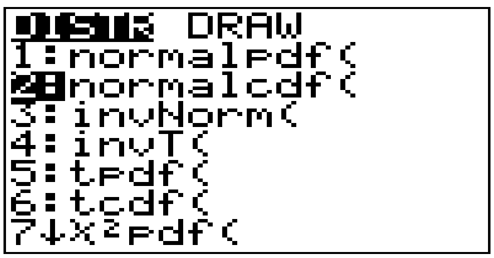
TI-83/84 *2nd* $\longrightarrow$ *DISTR* $\longrightarrow$ *Option 2* ***then*** *input (LB,UB,* μ,σ *)* $\longrightarrow$ *enter*

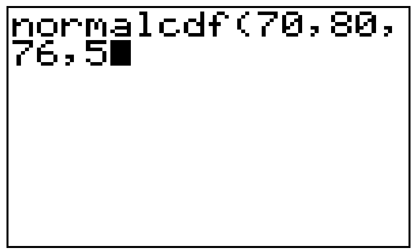
The average score for final stat exam was 76 with a standard deviation 5. If scores are normally distributed answer the following questions: **A normal distribution that** $\mu = 76$, $\sigma = 5$ **and the horizontal axis is called the X-axis.**

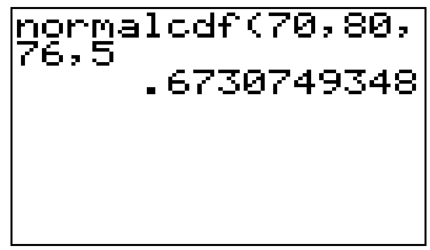
1. What percentage of students got scores between **70** and **80**?



TI-83/84 *2nd* $\longrightarrow$ *DISTR* $\longrightarrow$ *Option 2* ***then*** *input (* $70, 80, 76, 5$ *)* $\longrightarrow$ *enter* **answer: 67.31%**







---

2. What percentage of students got scores between **80** and **90**?

TI-83/84 *2nd* $\longrightarrow$ *DISTR* $\longrightarrow$ *Option 2* ***then*** *input (* $80, 90, 76, 5$ *)* $\longrightarrow$ *enter* **answer: 20.93%**

---

3. What percentage of students got scores less than **70**? Lower boundary is missing

In this case, the logical choice for Lower boundary is $LB = 0$

TI-83/84 *2nd* $\longrightarrow$ *DISTR* $\longrightarrow$ *Option 2* ***then*** *input (* $0, 70, 76, 5$ *)* $\longrightarrow$ *enter* **answer: 11.51%**

---

4. What percentage of students got scores more than **90**? Upper boundary is missing

In this case, the logical choice for upper boundary is $UB = 100$

TI-83/84 *2nd* $\longrightarrow$ *DISTR* $\longrightarrow$ *Option 2* ***then*** *input (* $90, 100, 76, 5$ *)* $\longrightarrow$ *enter* **answer: 0.255%**

---

5. What percentage of students got scores **within** one standard deviation of the mean?
   For this problem

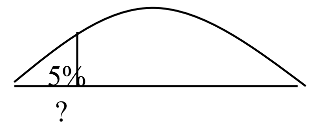
Upper boundary: $UB = \mu + 1\sigma = 76 + 5 = 81$

Lower boundary: $LB = \mu - 1\sigma = 76 - 5 = 71$

TI-83/84 *2nd* $\longrightarrow$ *DISTR* $\longrightarrow$ *Option 2* ***then*** *input (* $81, 91, 76, 5$ *)* $\longrightarrow$ *enter* **answer: 68.27%**

Finding the *cut-of point* with a given %

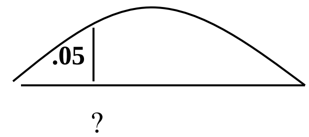
Ex:1
According to grading policy, the **bottom 5% of the class get a grade of F**
Find the cutting score for F



TI-83/84 *2nd* $\rightarrow$ *DISTR* $\rightarrow$ *Option 3* *then* *input* (0.05 , 76 , 5) $\rightarrow$ *enter* **answer:** $x = 67.778$
To use the formula with the help of **table (given on the last page)** $x = \mu + \sigma\, z = 76 + 5(-1.645) = 67.78$

Ex: 2 According to grading policy, the **top 5% of the class get a grade of A**

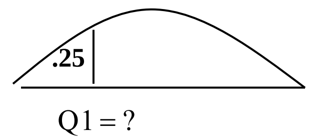
In using TI, area on the top must be subtract area from 1(in this case $1 - 0.05 = .95$)



TI-83/84 *2nd* $\rightarrow$ *DISTR* $\rightarrow$ *Option 3* *then* *input* (0.95 , 76 , 5) $\rightarrow$ *enter* **answer:** $x = 84.22$

To use the formula with the help of **table (given on the last page)** $x = \mu + \sigma\, z = 76 + 5(1.645) = 84.22$

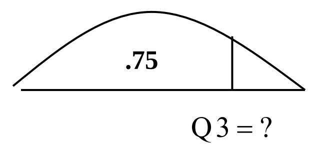
Ex: 3 Find the score that corresponds to the Q1



TI-83/84 *2nd* $\rightarrow$ *DISTR* $\rightarrow$ *Option 3* *then* *input* (0.25 , 76 , 5) $\rightarrow$ *enter* **answer:** 72.63
To use the formula with the help of **table (given on the last page)**
Base on the table for 25% or 0.2 area the z – value, will be $-.6749$ $x = \mu + \sigma\, z = 76 + 5(-0.6749) = 72.63$

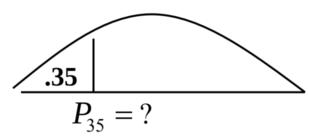
Ex: 4 Find the score that corresponds to the Q3

In using TI, area on the top must be subtract area from 1(in this case $1 - 0.05 = .95$)



TI-83/84 *2nd* $\rightarrow$ *DISTR* $\rightarrow$ *Option 3* *then* *input* (0.75 , 76 , 5) $\rightarrow$ *enter* **answer:** $x = 79.37$
To use the formula with the help of **table (given on the last page)**
Base on the table for 25% or 0.2 area the z – value, will be $.6749$ $x = \mu + \sigma\, z = 76 + 5(0.6749) = 72.63$

Ex:5 Find the score that corresponds to the 35TH Percentile $= P_{35}$

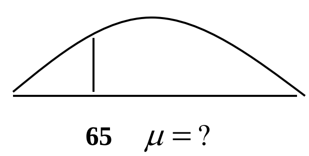


TI-83/84 *2nd* $\rightarrow$ *DISTR* $\rightarrow$ *Option 3* *then* *input* (0.75 , 76 , 5) $\rightarrow$ *enter* **answer:** $x = 79.37$
To use the formula with the help of **table (given on the last page)** $x = \mu + \sigma\, z = 76 + 5(0.6749) = 72.63$

Finding the *mean* from *cut-of point* with a given %

Hint: HW problems 97, 99 will be done using the method discussed in the following example.

Ex: 1 In a different test 20% of the class were below 65 points.
Given that the standard deviation was 6, what was class average?



Only cut off point formula works for these types of problems.
To find z – value, use the **table (given on the last page)** $x = \mu + \sigma\, z$
Base on the table for 20% or 0.2 area the z – value, will be $-.8416$
$65 = \mu + 6(-.8416)$ $65 = \mu - 5.05$ $65 + 5.05 = \mu$ $\mu = 70.05$

Application of Normal Probability Distribution

1) On a given test the average test scores was 68 with standard deviation of 8. If the scores are normally distributed, then find the probability as what percentage of students got scores

a) Between 60 and 70? ***Answer*: *44.05%***

b) Between 70 and 80? ***Answer*: *33.45%***

c) Between 80 and 90? ***Answer*: *6.38%***

d) Less than 60? ***Answer*: *15.86%***

e) More than 90? ***Answer*: *0.29%***

f) Find the cut-off point for F if the bottom 1% will be getting "F". ***Answer*: *49.39***

g) Find the cut-off point for "A" if the top 2% will be getting "A" ***Answer*: *84.43***

h) Find the score for Q1 ***Answer*: *62.60***

i) Find the P_{30} ***Answer*:*63.80***

j) Find the P_{70} ***Answer*: *72.18***

k) Find the P_{50} ***Answer*: *68***

---

2) The average time for workers to finish a specific task is 38 minutes with a standard deviation 8 minutes. If that data are normally distributed then what percentage of workers finishes the task;

a) Between 30 and 36 minutes ***Answer*: *24.26%***

b) Less than 42 minutes ***Answer*: *69.15%***

c) More than 40 minutes ***Answer*: *40.13%***

d) Within 4 minutes of the mean ***Answer*: *38.3%***

e). Find the time that separates the **fastest 10%** of workers finishing this task.
Note: this is a **cut-off** point and fastest means the bottom 10%

TI-83/84 *2nd* $\rightarrow$ *DISTR* $\rightarrow$ *Option 3* ***then*** *input* (0.10 , 38 , 8) $\rightarrow$ *enter* **answer: X = 27.74**

Note:
Also rather using TI-83/84 to find cut-off point, we can use formula $x = \mu + \sigma z$ *and z value* $= -1.28$ *form page 3 of the table for* **bottom 10%** $x = 38 + 8(-1.28) = 27.76$

f). Find the time that separates the **slowest 15%** of workers finishing this task.
Note: this is a **cut-off** point and slowest means the top **15%**

TI-83/84 *2nd* $\rightarrow$ *DISTR* $\rightarrow$ *Option 3* ***then*** *input* (0.85 , 38 , 8) $\rightarrow$ *enter* **answer: X = 46.29**

Note:
Also rather using TI-83/84 to find cut-off point, we can use formula $x = \mu + \sigma z$ *and z value* $= -1.28$ *form page 3 of the table for* **top 15%** $x = 38 + 8(1.0364) = 46.29$

. Find the time that separates the fastest 10% of workers finishing this task. ***Answer*: *27.76***
$x = \mu + \sigma z \Rightarrow x = 38 + 8(-1.28) = 27.76$

. Find the time that separates the slowest 15% of workers finishing this task. ***Answer*: *46.32***
$x = \mu + \sigma z \Rightarrow x = 38 + 8(1.04) = 46.32$

3) The cholesterol level for adult males of a specific racial group is approximately normally distributed with a mean of 4.8 mmol/L and a standard deviation of 0.6 mmol/L.

a) What is the probability that a person has moderate risk if his cholesterol level is more than 1 but less than 2 standard deviations above the mean**:** ***Answer*****: 13.59%**

b) A person has high risk if his cholesterol level is more than 2 standard deviations above the mean, i.e., greater than 6.0 mmol/L. What proportion of the population has high risk ***Answer*****: 2.28%**

c) A person within 1 standard deviation of the mean has normal cholesterol risk What proportion of the population has high risk ***Answer*****: 31.73%**

d) What is the 90th percentile of the distribution (the cholesterol level that exceeds 90% of the population)? ***Answer*****: 5.569**

e) What is the 70th percentile of the distribution, i.e., the cholesterol level that exceeds 70% of the population? ***Answer*****: 5.11:**

4). Given the average height of adult male in United States is 65 inches with standard deviation of 8 inches and if the minimum and maximum acceptable heights for being recruited by ARMY is between 55 and 85 inches, then find the percentage of adult male that may be rejected because of their heights? ***Answer*****:** ***11.19***

---

5) The average life of a certain type of motor is 10 years, with a standard deviation of 2 years. Assume that the lives of the motors follow a normal distribution

a) What percentage of motors last longer than 15 years? ***Answer*****: .0062 =.62%**

b) What percentage of motors last less than 7 years? ***Answer*****: 0.668 = 6.68 %**

c) If the manufacturer is willing to replace only 3% of the motors that fail, how long a guarantee should he offer? ***Answer*****: 6.24 years**

d) If the manufacturer is willing to replace only 5% of the motors that fail, how long a guarantee should he offer? ***Answer*****: ? 6.71 years**

---

6) A company pays its employees an average wage of $8.25 an hour with a standard deviation of 0.80 cents. If the wages are approximately normally distributed, determine

a. the proportion of the workers getting wages between $6.75 and $10.75 an hour; ***Answer*****: 96%**

b. the minimum wage of the highest 5%. ***Answer*****: $9.57**

c. the minimum wage of the lowest 10%: ***Answer*****: $7.23**

d. What is the 90th percentile of the distribution ***Answer*****: $9.27**

e. What is the 30th percentile of the distribution ***Answer*****: $7.83**

f. What is the 75th percentile of the distribution ***Answer*****: $8.79**

Extra Practice: **Problems F, G 1-10 from practice problem part II on pages 4, 5.**

Answers

A. **3.** $P(M \text{ or } D) = \dfrac{213+84-39}{500} = 51.6\%$

4. $P(F \text{ or } H) = \dfrac{287+311-209}{500} = 77.8\%$

5. $P(D \text{ or } A) = \dfrac{84+105-0}{500} = 37.8\%$.

6. $P(F \text{ or } A) = \dfrac{287+105-33}{500} = 71.8\%$

7. 2.79 %

8. $\dfrac{311}{500}\cdot\dfrac{310}{499} = 38.64\ \%$

9. $\dfrac{189}{500}\cdot\dfrac{188}{499} = 14.24\ \%$

10. $\dfrac{287}{500}\cdot\dfrac{286}{499} = 32.90\ \%$

B. **1.** $P(R \text{ or } S) = \dfrac{90+105-25}{250} = 68\%$

2. $P(B \text{ or } L) = \dfrac{100+145-55}{250} = 76\%$

3. $P(B \text{ or } W) = \dfrac{100+60-0}{250} = 64\%$

4. $P(L \text{ or } W) = \dfrac{145+60-25}{250} = 72\%$

5. $P(R \text{ or } W \text{ or } S) = \dfrac{90+60+105-25-35}{250} = 78\%$

6. $P(R\ R) = \dfrac{90}{250}\cdot\dfrac{89}{249} = 12.87\%$

7. $P(S\ S) = \dfrac{105}{250}\cdot\dfrac{104}{249} = 17.54\%$

8. $P(B\ B) = \dfrac{100}{250}\cdot\dfrac{99}{249} = 15.90\%$

E			
x	f	P(x)%	x P(x)
5	2	0.02	0.10
6	3	0.03	0.18
7	8	0.08	0.56
8	9	0.09	0.72
9	15	0.15	1.35
10	18	0.18	1.80
11	20	0.20	2.20
12	25	0.25	3.00
	100	1.00	***9.91***
		Mean =9.91	

F			
x	f	P(x)%	x P(x)
0	2	0.03	0.00
1	17	0.27	0.27
2	10	0.16	0.31
3	11	0.17	0.52
4	10	0.16	0.63
5	4	0.06	0.31
6	8	0.13	0.75
7	2	0.03	0.22
	64	1.00	***3.00***
		Mean = 3.00	

H.

Outcome	x	$p(x)$	$x\,p(x)$
Win	5	$10/100$	$5\times.10 = .50$
Lose	-1	$90/100$	$-1\times.90 = -.90$
		$\sum p(x) = 1$	$\sum xp(x) = -0.40$

Outcome	x	$p(x)$	$x\,p(x)$
Win	5-3	2/6	4/6
Lose	-3	4/6	$-12/6$
		$\sum p(x) = 1$	$\sum xp(x) = -8/6$

100*1000*360*.40 = **$14,400,000 per year.**

L	
X	**P(X)**
0	**.1296**
1	.3456
2	.3456
3	.1536
4	**.0256**

1. 2.56 % **2.** 12.96 %
3. **52.48 %** 4. **82.08%** 3.
$\mu = 1.6$ $\sigma = 0.98$

X	**P(X)**
0	**.0185**
1	.1128
2	.2757
3	.3370
4	.2059
5	**.0503**

1. 5.03 % **2.** 1.85 %
59.32% 4. **74.39%**
$\mu = 2.75$

Part Two Formula Sheet

You are allowed to use this on the quizzes.

Addition Rule: $P(A \text{ or } B) = P(A) + P(B) - p(A \text{ and } B)$

Discrete Probability Distribution

X	f (days)	$f \div n =$ $p(x)$ %	$x\,p(x)$

Expected Value = Mean $= \mu = \sum x\,p(x)$ +

TI-83/84 *Inputting **x-values** in **L1** and **probabilities** in **L2***
then *stat → calc → Option 1 → enter → **L1 , L2** → → enter*

Counting

Factorial: Number of ways **n** objects or subjects can be arranged $= n!$

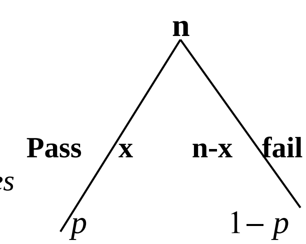
Combination: Number of ways that **x objects** or subjects can be selected from **n** objects or subjects

The **order** in selection is **not relevant**. $nCx = \dfrac{n!}{x!(n-x)!}$ **TI-83/84** *n → math → PRB → Option 3 → x*

Permutation: Number of ways that **x** objects or subjects can be selected from **n** objects or subjects

The **order** in selection is **relevant**. $nPx = \dfrac{n!}{(n-x)!}$ **TI-83/84** *n → math → PRB → Option 2 → x*

Binomial Probability



$P(x) = nCx\; p^x (1-p)^{n-x}$ Mean $= \mu = n\,p$ St. Dev. $= \sigma = \sqrt{n\,p(1-p)}$

p = Desired probability *n = Total number of trials* *x = Number of desired outcomes*

nCx = Combination Rule

TI-83/84 *2nd → **DISTR** → Option 0* ***then*** ***input** (n,p,x) → enter*

$P(x) = nCx\; p^x (1-p)^{n-x}$

Non - Standard Normal Probability (NSNPD)

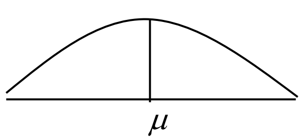
TI-83/84 *2nd → **DISTR** → Option 2* ***then*** *input (LB,UB, μ , σ) → enter*

To create Lower Boundary $LB = \mu - 5\sigma$ To create Upper Boundary $UB = \mu + 5\sigma$

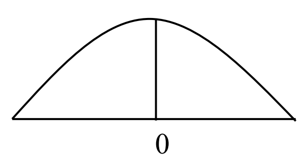
Cut-off point formula $x = \bar{x} + s\,z$ **or** $x = \mu + \sigma\,z$ **TI-83/84** *2nd → **DISTR** → Option 3 input (%, μ , σ)*

For finding **Z** , you need to look it up on **page 3** of the table **Hint for TI** *% is the area to the left of the cut off point.*

Converting a **non - standard** value to **standard value** by using

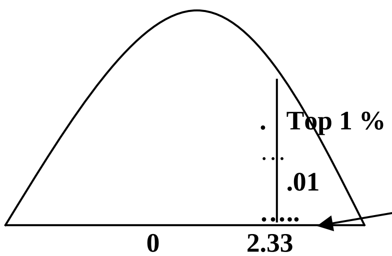


$$Z = \frac{x - \mu}{\sigma}$$



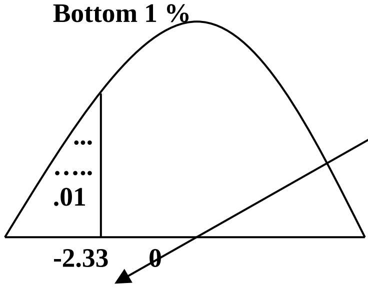
Based on Standard Normal Distribution $\mu = 0$ and $\sigma = 1$

Out Side Area



OR

Out Side Area

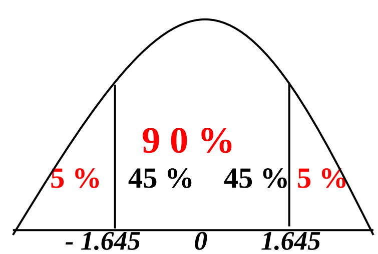


Confidence Level	Out Side Area On left or right Cut-off Point	Z - Value (±) Critical Value $= Z_{\alpha/2}$
99%	.005	± 2.5758
98%	.01	±2.3263
97%	.015	±2.1701
96%	.02	±2.0537
95%	.025	±1.9600
94%	.03	±1.8808
92%	.04	±1.7507
90%	.05	±1.6450
88%	.06	±1.5548
86%	.07	±1.4758
84%	.08	±1.4051
82%	.09	±1.3408
80%	.10	±1.2816
78%	.11	±1.2265
76%	.12	±1.1750
70%	.15	±1.0364
60%	.20	±0.8416
50%	.25	±0.6749
40%	.30	±0.5244

How to find the Z -value for different confidence intervals.

Example: Find the Z - value for 97% confidence interval

1. Divide 95% = 0.95 by 2, $\Rightarrow$ $.90/2 = 0.45$
2. Subtract 0.45 from one $\Rightarrow$ $.5 - 0.45 = .05$
3. Look for area close to 0.015from **inside** the table (page1).
4 ***Find its corresponding Z-value (- 2.17)***



TI-83/84 *2nd* $\rightarrow$ *Distr* $\rightarrow$ *Option 3* *input* ***(% ,0 ,1)***

Example: 2nd $\rightarrow$ *Distr* $\rightarrow$ *Option 3* *input* ***(.05 ,0 ,1)*** *enter , then the answer will be* ***- 1.645***

Example: 2nd $\rightarrow$ *Distr* $\rightarrow$ *Option 3* *input* ***(.95 ,0 ,1)*** *enter, then the answer will be* ***1.645***

Hint for TI *% is the area to the left of the cut off point.*