Derivatives   Formulas.

$$\text{ARC} = \frac{f(b) - f(a)}{b - a} \quad \text{or} \quad \text{ARC} = \frac{f(x+h) - f(x)}{h} \qquad \text{IRC} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Type	Function	Derivative	Example
1. Constant	$f(x) = c$	$f'(x) = 0$	$f(x) = y = 3$ $f'(x) = y' = 0$
Line	$f(x) = a\,x$	$f'(x) = a$	$f(x) = y = 5x$ $f'(x) = y' = 5$
Power	$f(x) = a\,x^n$	$f'(x) = a\,n\,x^{n-1}$	$f(x) = y = 2x^4$ $f'(x) = y' = (2)x^{4-1} = 8x^3$
	$y = f(x) + g(x)$	$y' = f'(x) + g'(x)$	$y = 3 + 5x + 2x^4$ $y' = 0 + 5 + 8x^3 = 5 + 8x^3$
Product	$f(x) = V\,W$	$f'(x) = V'\,W + V\,W'$	$y = (3+5x)(6x+2x^4)$ $y' = (0+5)(6x+2x^4) + (3+5x)(6+8x^3)$
Quotient	$f(x) = \dfrac{V}{W}$	$f'(x) = \dfrac{V'\,W - V\,W'}{W^2}$	$y = \dfrac{(3+5x)}{(6x+2x^4)}$ $y' = \dfrac{(0+5)(6x+2x^4) - (3+5x)(6+8x^3)}{(6x+2x^4)^2}$
Chain	$f(x) = a\,u^n$	$f'(x) = a\,n\,u'\,u^{n-1}$	$y = 4(3+5x)^6$ $y' = 4(6)(0+5)(3+5x)^{6-1} = 120(3+5x)^5$
Exponential	$f(x) = a\,e^u$	$f'(x) = a\,u'\,e^u$	$y = 4e^{(3+5x)}$ $y' = 4(0+5)e^{(3+5x)} = 20e^{(3+5x)}$
Natural Log	$f(x) = a\,\ln u$	$f'(x) = a\,\dfrac{u'}{u}$	$y = 4\ln(3+5x)$ $y' = 4\dfrac{0+5}{3+5x} = \dfrac{20}{3+5x}$
Log	$f(x) = a\,\log_b^u$	$f'(x) = \dfrac{a}{\ln b}\,\dfrac{u'}{u}$	$y = 4\log_2^{(3+5x)}$ $y' = \dfrac{4}{\ln 2}\dfrac{0+5}{3+5x} = \dfrac{20}{(3+5x)\ln 2}$

Practice

a) Find the derivatives $y' = f'(x) =$ of the following functions. b) and find $f'(1) =$

1) $y = -2$

2) $y = 8x$

3) $y = -21x^2$

4) $y = -2x + 9$

5) $y = 8x - 3x^5$

6) $y = -3x^2 + 5x - 4$

7) $y = x^4 + 4x^2 + 7x - 2$

8) $y = -8x - 2x^{-3}$

9) $y = -8x - 2\dfrac{3}{x^3}$

10) $y = \sqrt{x}$

11) $y = 8\sqrt{x}$

12) $y = 8\sqrt{x^3}$

13) $y = 5x^6 + \dfrac{3}{2}x^2 - 4 - \dfrac{4}{x^4} + 2\sqrt{x} - \dfrac{5}{6\sqrt{x}}$

14) $y = -3x^2 + 6x - 4 - \dfrac{3}{x^2} + 8\sqrt{x} - \dfrac{3}{8\sqrt{x}}$

15) $y = (-2x + 9)(8x - 3x^4)$

16) $y = (-2x + 9x^2)(-3x^2 + 6x - 4)$

17) $y = (x + 9x)(3x^2 - 5x^3)$

18) $y = (-x + 2x^2)(-3x^3 - 8x - 2)$

19) $y = \dfrac{-2x + 9}{8x - 3x^4}$

20) $y = \dfrac{8x - 3x^4}{-2x + 9}$

21) $y = \dfrac{x^4 + 4x^2 + 7x - 2}{-3x^2 + 5x - 4}$

22) $y = \dfrac{-3x^2 + 5x - 4}{x^4 + 4x^2 + 7x - 2}$

Practice

b) Find the derivatives $y' = f'(x) =$ of the following functions. b) and find $f'(1) =$

1) $y = -2$ $\rightarrow$ $y' = f(x) = 0$

2) $y = 8x$ $\rightarrow$ $y' = f(x) = 8$ $\rightarrow$ $y' = f(1) = 0$

3) $y = -21x^2$ $\rightarrow$ $y' = f(x) = -21(2)x^{2-1} = -42x$ $\rightarrow$ $y' = f(1) = -42$

4) $y = -2x + 9$ $\rightarrow$ $y' = f(x) = -2 + 0 = -2$ $\rightarrow$ $y' = f(1) = -2$

5) $y = -3x^2 + 5x - 4$ $\rightarrow$ $y' = f(x) = -3(2)x^{2-1} + 5 - 0 = -6x + 5$ $\rightarrow$ $y' = f(1) = -6(1) + 5 = -1$

6) $y = 8x - 3x^5$ $\rightarrow$ $y' = f(x) = 8 - 3(5)x^{5-1} = 8 - 15x^4$ $\rightarrow$ $y' = f(1) = 8 - 15 = -7$

7) $y = x^4 + 4x^2 + 7x - 2$ $\rightarrow$ $y' = f(x) = 4x^3 + 8x^1 + 7 - 0$ $\rightarrow$ $y' = f(1) = 4 + 8 - 7 = 5$

8) $y = -8x - 2x^{-3}$ $\rightarrow$ $y' = f(x) = -8 - 2(-3)x^{-4} = -8 + 6x^{-4}$ $\rightarrow$ $y' = f(1) = -8 + 6 = -2$

9)

$y = -8x - 2\dfrac{3}{x^3}$ $\rightarrow$ $y = -8x - 2x^{-3}$ $\rightarrow$ $y' = f(x) = -8 - 2(-3)x^{-4} = -8 + 6x^{-4}$ $\rightarrow$ $y' = f(1) = -8 + 6 = -2$

10) $y = \sqrt{x}$ $\rightarrow y = x^{\frac{1}{2}}$ $\rightarrow$ $y' = f(x) = \dfrac{1}{2}x^{\frac{1}{2}-1} = \dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2}\dfrac{1}{\sqrt{x}}$ $\rightarrow$ $y' = f(1) = \dfrac{1}{2}\dfrac{1}{\sqrt{1}} = \dfrac{1}{2}$

11) $y = 8\sqrt{x}$ $\rightarrow y = 8x^{\frac{1}{2}}$ $\rightarrow$ $y' = f(x) = 8(\dfrac{1}{2}x^{\frac{1}{2}-1}) = 4x^{-\frac{1}{2}} = 4\dfrac{1}{\sqrt{x}}$ $\rightarrow$ $y' = f(1) = 4\dfrac{1}{\sqrt{1}} = 4$

12)

$y = 6\sqrt{x^3}$ $\rightarrow$ $\rightarrow y = 6x^{\frac{1}{3}}$ $\rightarrow$ $y' = f(x) = 6(\dfrac{1}{3}x^{\frac{1}{3}-1}) = 2x^{-\frac{2}{3}} = 2\dfrac{1}{\sqrt{x^3}}$ $\rightarrow$ $y' = f(1) = 2\dfrac{1}{\sqrt{1}} = \dfrac{1}{2}$

13) $y = 5x^6 + \dfrac{3}{2}x^2 - 4 - \dfrac{4}{x^4} + 2\sqrt{x} - \dfrac{5}{6\sqrt{x}}$

14) $y = -3x^2 + 6x - 4 - \dfrac{3}{x^2} + 8\sqrt{x} - \dfrac{3}{8\sqrt{x}}$

15) $y = \left(-2x + 9\right)\left(8x - 3x^4\right)$

16) $y = \left(-2x + 9x^2\right)\left(-3x^2 + 6x - 4\right)$

17) $y = \left(x + 9x\right)\left(3x^2 - 5x^3\right)$

18) $y = \left(-x + 2x^2\right)\left(-3x^3 - 8x - 2\right)$

19) $y = \dfrac{-2x + 9}{8x - 3x^4}$

20) $y = \dfrac{8x - 3x^4}{-2x + 9}$

21) $y = \dfrac{x^4 + 4x^2 + 7x - 2}{-3x^2 + 5x - 4}$

22) $y = \dfrac{-3x^2 + 5x - 4}{x^4 + 4x^2 + 7x - 2}$