

Find the relative extrema of the function, if they exist.

1) $f(x) = x^2 - 4x + 7$

1) _____

2) $s(x) = -x^2 - 20x - 19$

2) _____

3) $f(x) = -6x^2 - 2x - 7$

3) _____

4) $f(x) = 0.2x^2 - 2.4x + 5.9$

4) _____

5) $f(x) = x^3 - 3x^2 + 1$

5) _____

6) $f(x) = \frac{2}{3}x^3 - \frac{3}{2}x^2 - 9x + 2$

6) _____

7) $f(x) = 3x^4 + 16x^3 + 24x^2 + 32$

7) _____

8) $f(x) = x^3 - 3x^4$

8) _____

9) $f(x) = \frac{4}{x^2 - 1}$

9) _____

10) $f(x) = \frac{8x}{x^2 + 1}$

10) _____

11) $f(x) = \sqrt[3]{x + 8}$

11) _____

12) $f(x) = \sqrt{x^2 + 12x + 72}$

12) _____

Find the relative extrema of the function and classify each as a maximum or minimum.

13) $f(x) = 3x^2 - 24x + 49$

13) _____

14) $f(x) = x^4 - 18x^2 - 1$

14) _____

15) $s(x) = -x^2 - 18x - 32$

15) _____

16) $y = x^3 - 3x^2 + 4x - 4$

16) _____

17) $f(x) = x^3 - 4x^4$

17) _____

18) $f(x) = (x + 3)^3$

18) _____

19) $f(x) = 20x^3 - 3x^5$ 19) _____

Solve the problem.

20) A firm estimates that it will sell N units of a product after spending x dollars on advertising where 20) _____

$N(x) = -x^2 + 500x + 11, \quad 0 \leq x \leq 500,$

and x is in thousands of dollars. Find the relative extrema of the function.

21) Assume that the temperature of a person during an illness is given by 21) _____

$T(t) = -0.1t^2 + 1.4t + 98.6, \quad 0 \leq t \leq 14,$

where T = the temperature ($^{\circ}\text{F}$) at time t , in days. Find the relative extrema of the function.

22) The Olympic flame at the 1992 Summer Olympics was lit by a flaming arrow. As the arrow moved d feet horizontally from the archer, assume that its height h , in feet, was approximat the function 22) _____

$h = -0.002d^2 + 0.6d + 6.4.$

Find the relative maximum of the function.

Graph the function by first finding the relative extrema.

23) $f(x) = x^2 - 10x + 25$ 23) _____

24) $f(x) = 6 + 4x - x^2$ 24) _____

25) $f(x) = x^3 + 2x^2 - x - 2$ 25) _____

26) $f(x) = x^3 + 3x$ 26) _____

Draw a graph to match the description. Answers will vary.

27) $f(x)$ is decreasing over $(-\infty, 4]$ and increasing over $[4, \infty)$. 27) _____

28) $f(x)$ has a negative derivative over $(-\infty, -9)$ and a positive derivative over $(-9, \infty)$. 28) _____

29) $G(x)$ has a positive derivative over $(-\infty, -7)$ and $(-3, 7)$ and a negative derivative over $(-7, -3)$ and $(7, \infty)$. 29) _____

Graph the function.

30) $f(x) = x^3 + x^2 - 5x - 5$

30) _____

31) $f(x) = -x^3 - 4x^2 + 5$

31) _____

32) $f(x) = x^3 - 48x$

32) _____

Find the points of inflection.

33) $f(x) = 5x^3 + 2x + 5$

33) _____

34) $f(x) = -x^3 + 9x + 3$

34) _____

35) $f(x) = 4x - x^3$

35) _____

36) $f(x) = \frac{4}{3}x^3 - 12x^2 + 10x + 46$

36) _____

37) $f(x) = \frac{1}{4}x^4 - x^3 + 10$

37) _____

Determine where the given function is increasing and where it is decreasing.

38) $s(x) = -x^2 - 4x + 60$

38) _____

39) $f(x) = -4x^2 - 2x - 12$

39) _____

40) $y = x^3 - 3x^2 + 6x - 8$

40) _____

41) $f(x) = x^4 - 18x^2 + 4$

41) _____

Determine where the given function is concave up and where it is concave down.

42) $f(x) = x^2 - 20x + 104$

42) _____

43) $q(x) = 7x^3 + 2x + 5$

43) _____

44) $f(x) = 6x - x^3$

44) _____

45) $f(x) = x^3 + 3x^2 - x - 24$

45) _____

Draw a graph to match the description. Answers will vary.

46) $f(x)$ is decreasing and concave up on $(-\infty, -10)$; $f(x)$ is decreasing and concave down on $(-10, \infty)$. 46) _____

47) $f(x)$ is increasing and concave up on $(-\infty, -5)$; $f(x)$ is increasing and concave down on $(-5, \infty)$. 47) _____

48) $f(x)$ is decreasing and concave down on $(-\infty, 10)$; $f(x)$ is decreasing and concave up on $(10, \infty)$. 48) _____

49) $f(x)$ is increasing and concave down on $(-\infty, 8)$; $f(x)$ is increasing and concave up on $(8, \infty)$. 49) _____

Solve the problem.

50) The annual revenue and cost functions for a manufacturer of grandfather clocks are approximately $R(x) = 500x - 0.01x^2$ and $C(x) = 160x + 100,000$, where x denotes the number of clocks made. What is the maximum annual profit? 50) _____

51) The annual revenue and cost functions for a manufacturer of precision gauges are approximately $R(x) = 500x - 0.03x^2$ and $C(x) = 160x + 100,000$, where x denotes the number of gauges made. What is the maximum annual profit? 51) _____

52) Because of material shortages, it is increasingly expensive to produce 6.0L diesel engines. In fact, the profit in millions of dollars from producing x hundred thousand engines is approximated by $P(x) = -x^3 + 14x^2 + 15x - 55$, where $0 \leq x \leq 20$. Find the inflection point of this function to determine the point of diminishing returns. 52) _____

53) The function $R(x) = 5000 - x^3 + 30x^2 + 600x$, $0 \leq x \leq 20$, represents revenue in thousands of dollars where x represents the amount spent on advertising in tens of thousands of dollars. Find the inflection point for the function to determine the point of diminishing returns. 53) _____

Find the absolute maximum and absolute minimum values of the function, if they exist, over the indicated interval.

When no interval is specified, use the real line $(-\infty, \infty)$.

54) $f(x) = -21$; $[-7, 7]$ 54) _____

55) $f(x) = 6x + 2$; $[-1, 2]$ 55) _____

56) $f(x) = 4x^2 - 5x^3$; $[0, 5]$ 56) _____

57) $f(x) = \frac{1}{3}x^3 - 4x$; $[-8, 8]$

57) _____

Find the absolute maximum and absolute minimum values of the function, if they exist, on the indicated interval.

58) $f(x) = x^3 - 4x^2 - 16x + 1$; $[-9, 0]$

58) _____

Find the absolute maximum and absolute minimum values of the function, if they exist, over the indicated interval.

When no interval is specified, use the real line $(-\infty, \infty)$.

59) $f(x) = x^3 + x^2 - 5x + 6$; $(0, \infty)$

59) _____

Solve the problem.

60) A carpenter is building a rectangular room with a fixed perimeter of 420 ft. What are the dimensions of the largest room that can be built? What is its area?

60) _____

61) Find the dimensions that produce the maximum floor area for a one-story house that is rectangular in shape and has a perimeter of 166 ft. Round to the nearest hundredth, if necessary.

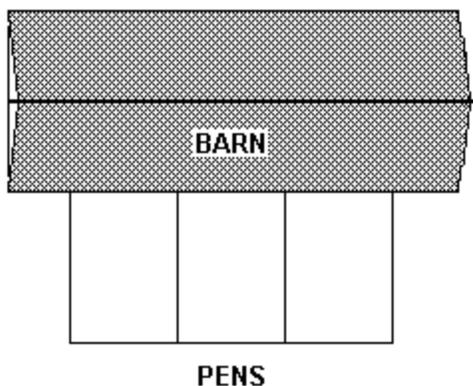
61) _____

62) An architect needs to design a rectangular room with an area of 83 ft². What dimensions should he use in order to minimize the perimeter? Round to the nearest hundredth, if necessary.

62) _____

63) A farmer decides to make three identical pens with 136 feet of fence. The pens will be next to each other sharing a fence and will be up against a barn. The barn side needs no fence.

63) _____



What dimensions for the total enclosure (rectangle including all pens) will make the area as large as possible?

64) A company wishes to manufacture a box with a volume of 40 cubic feet that is open on top and is twice as long as it is wide. Find the width of the box that can be produced using the minimum amount of material. Round to the nearest tenth, if necessary.

64) _____

65) Find the number of units that must be produced and sold in order to yield the maximum profit given the following equations for revenue and cost:

65) _____

$R(x) = 60x - 0.5x^2$

$C(x) = 4x + 7$.

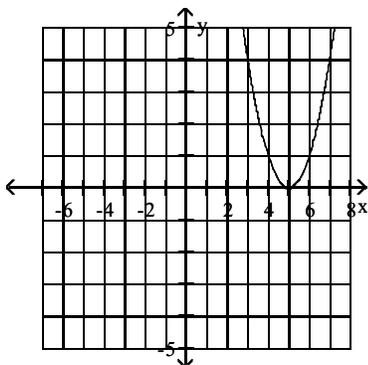
- 66) Find the number of units that must be produced and sold in order to yield the maximum profit given the following equations for revenue and cost: 66) _____
 $R(x) = 7x$
 $C(x) = 0.001x^2 + 0.8x + 80$.
- 67) An appliance company determines that in order to sell x dishwashers, the price per dishwasher must be 67) _____
 $p = 660 - 0.3x$.
It also determines that the total cost of producing x dishwashers is given by
 $C(x) = 4000 + 0.3x^2$.
How many dishwashers must the company produce and sell in order to maximize profit?
- 68) A hotel has 230 units. All rooms are occupied when the hotel charges \$110 per day for a room. For every increase of x dollars in the daily room rate, there are x rooms vacant. Each occupied room costs \$34 per day to service and maintain. What should the hotel charge per day in order to maximize daily profit? 68) _____
- 69) An outdoor sports company sells 896 kayaks per year. It costs \$14 to store one kayak for a year. Each reorder costs \$8, plus an additional \$9 for each kayak ordered. In what lot size should the store order kayaks in order to minimize inventory costs? 69) _____
- 70) A bookstore has an annual demand for 38,000 copies of a best-selling book. It costs \$0.60 to store one copy for one year, and it costs \$55 to place an order. Find the optimum number of copies per order. 70) _____
- 71) A company estimates that the daily revenue (in dollars) from the sale of x cookies is given by $R(x) = 885 + 0.02x + 0.0003x^2$. 71) _____
Currently, the company sells 900 cookies per day. Use marginal revenue to estimate the increase in revenue if the company increases sales by one cookie per day.
- 72) A grocery store estimates that the weekly profit (in dollars) from the production and sale of x cases of soup is given by 72) _____
 $P(x) = -5600 + 9.5x - 0.0017x^2$
and currently 1300 cases are produced and sold per week. Use the marginal profit to estimate the increase in profit if the store produces and sells one additional case of soup per week.
- 73) A company estimates that the daily cost (in dollars) of producing x chocolate bars is given by $C(x) = 1635 + 0.02x + 0.0004x^2$. 73) _____
Currently, the company produces 600 chocolate bars per day. Use marginal cost to estimate the increase in the daily cost if one additional chocolate bar is produced per day.
- 74) The total cost, in dollars, to produce x DVD players is $C(x) = 70 + 7x - x^2 + 2x^3$. Find the marginal cost when $x = 3$. 74) _____
- 75) The profit, in dollars, from the sale of x compact disc players is $P(x) = x^3 - 8x^2 + 9x + 7$. Find the marginal profit when $x = 10$. 75) _____

- 76) Suppose that the daily cost, in dollars, of producing x televisions is 76) _____
 $C(x) = 0.003x^3 + 0.1x^2 + 62x + 620$,
and currently 60 televisions are produced daily. Use $C(60)$ and the marginal cost to estimate the daily cost of increasing production to 63 televisions daily. Round to the nearest dollar.
- 77) A supply function for a certain product is given by 77) _____
 $S(p) = 0.08p^3 + 5p^2 + 15p + 4$,
where $S(p)$ is the number of items produced when the price is p dollars. Use $S'(p)$ to estimate how many more units a producer will supply when the price changes from \$13.00 per unit to \$13.40 per unit.
- 78) Suppose the demand for a certain item is given by 78) _____
 $D(p) = -4p^2 + 3p + 8$,
where p represents the price of the item in dollars. Currently the price of the item is \$20. Use marginal demand to estimate the change in demand when the price is increased by one dollar.
- 79) The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$ where r is the radius. A tumor 79) _____
is approximately spherical in shape. Use $V'(r)$ to estimate the increase in volume of the tumor if its radius increases from 7 mm to 8 mm. Round to the nearest 100 mm^3 .

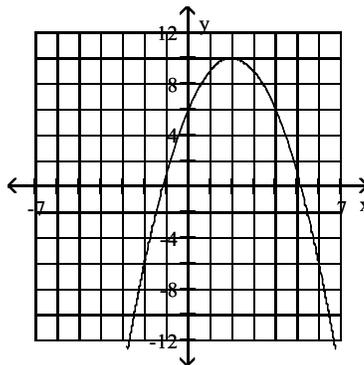
Answer Key

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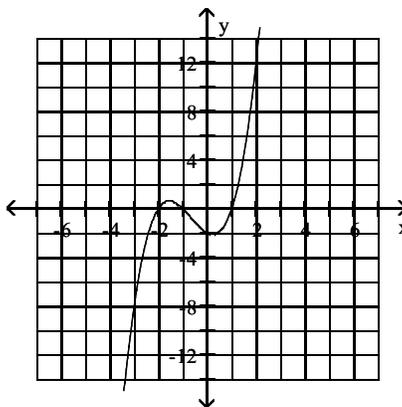
- 1) Relative minimum at (2, 3)
- 2) Relative maximum at (-10, 81)
- 3) Relative maximum at $\left(-\frac{1}{6}, -\frac{41}{6}\right)$
- 4) Relative minimum at (6, -1.3)
- 5) Relative maximum at (0, 1); relative minimum at (2, -3)
- 6) Relative maximum at $\left(-\frac{3}{2}, \frac{79}{8}\right)$; relative minimum at $\left(3, -\frac{41}{2}\right)$
- 7) Relative minimum at (0, 32)
- 8) Relative maximum at $\left(\frac{1}{4}, \frac{1}{256}\right)$
- 9) Relative maximum at (0, -4)
- 10) Relative minimum at (-1, -4); relative maximum at (1, 4)
- 11) No relative extrema exist
- 12) Relative minimum at (-6, 6)
- 13) Relative minimum: (4, 1)
- 14) Relative maximum: (0, -1); relative minima: (3, -82), (-3, -82)
- 15) Relative maximum: (-9, 49)
- 16) No relative extrema exist
- 17) Relative maximum: $\left(\frac{3}{16}, \frac{27}{16384}\right)$
- 18) Relative minimum: (-3, 0)
- 19) Relative minimum: (-2, -64), relative maximum: (2, 64)
- 20) (0, 11), (250, 62,511), (500, 11)
- 21) (0, 98.6), (7, 103.5), (14, 98.6)
- 22) (150, 51.4)
- 23)



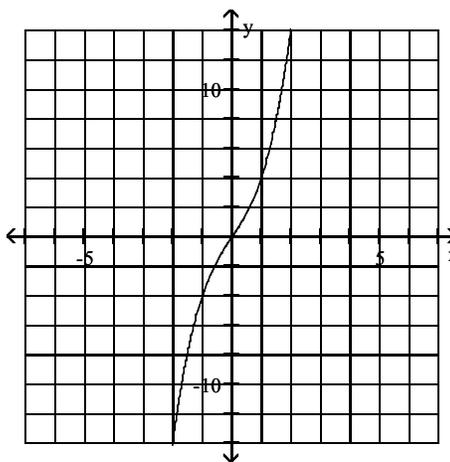
24)



25)



26)



27) Answers will vary.

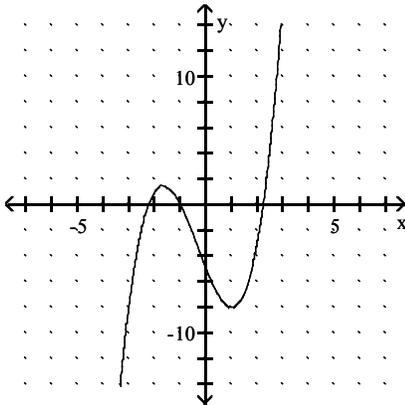
28) Answers will vary.

29) Answers will vary.

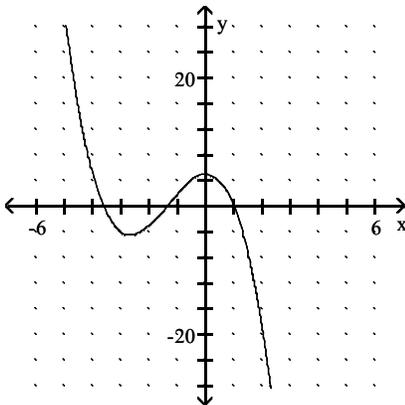
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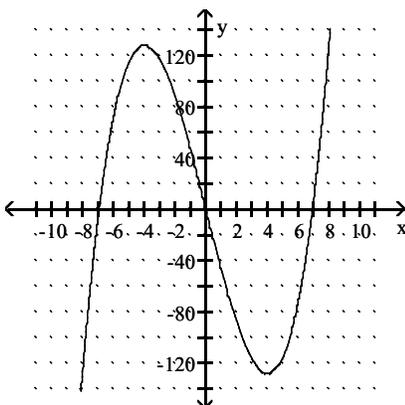
30)



31)



32)



33) (0, 5)

34) (0, 3)

35) (0, 0)

36) (3, 4)

37) (0, 10), (2, 6)

38) Increasing on $(-\infty, -2]$, decreasing on $[-2, \infty)$

39) Increasing on $(-\infty, -\frac{1}{4}]$, decreasing on $[-\frac{1}{4}, \infty)$

40) Increasing on $(-\infty, \infty)$

41) Decreasing on $(-\infty, -3]$ and $[0, 3]$, increasing on $[-3, 0]$ and $[3, \infty)$

42) Concave up for all x

43) Concave up on $(0, \infty)$, concave down on $(-\infty, 0)$

44) Concave up on $(-\infty, 0)$, concave down on $(0, \infty)$

45) Concave up on $(-1, \infty)$, concave down on $(-\infty, -1)$

46) Answers will vary.

47) Answers will vary.

48) Answers will vary.

49) Answers will vary.

50) \$2,790,000

51) \$863,333

52) (4.67, 218.26)

53) (10, 13,000)

54) Absolute maximum: -21, absolute minimum: -21

55) Absolute maximum: 14, absolute minimum: -4

56) Absolute maximum: $\frac{256}{675}$, absolute minimum: -525

57) Absolute maximum: $\frac{416}{3}$, absolute minimum: $-\frac{416}{3}$

58) Absolute maximum: $\frac{347}{27}$, absolute minimum: -908

59) No absolute maximum; absolute minimum: 3

60) 105 ft by 105 ft; 11,025 ft²

61) 41.5 ft x 41.5 ft

62) 9.11 ft x 9.11 ft

63) 17 ft by 68 ft

64) 3.2 ft

65) 56 units

66) 3100 units

67) 550

68) \$187

69) 32

70) 2639 copies

71) \$0.56

72) \$5.08

73) \$0.50

74) \$55

75) \$149

76) \$5667

77) 74

78) -157

79) 600 mm³