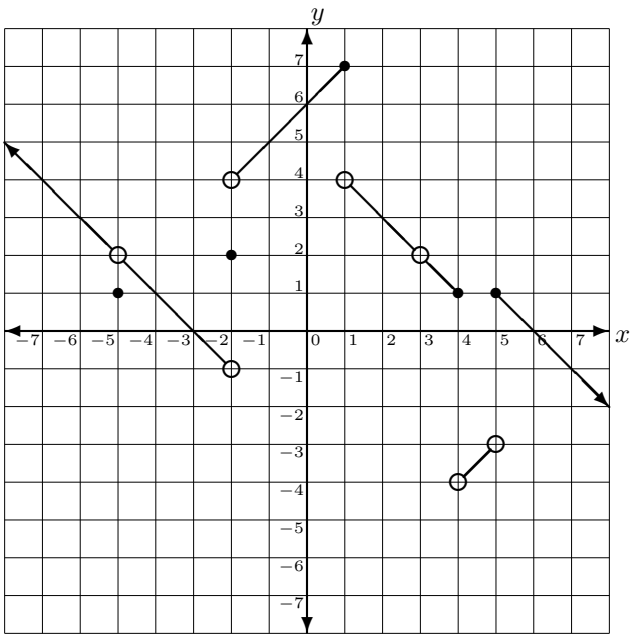
EXERCISES

For the function f graphed below, find the following:



1. $\lim_{x\to -5^-} f(x) =$
2. $\lim_{x\to -5^+} f(x) =$
3. $\lim_{x\to -5} f(x) =$
4. $f(-5) =$
5. $\lim_{x\to -2^-} f(x) =$
6. $\lim_{x\to -2^+} f(x) =$
7. $\lim_{x\to -2} f(x) =$
8. $f(-2) =$
9. $\lim_{x\to 0^-} f(x) =$
10. $\lim_{x\to 0^+} f(x) =$
11. $\lim_{x\to 0} f(x) =$
12. $f(0) =$
13. $\lim_{x\to 1^-} f(x) =$
14. $\lim_{x\to 1^+} f(x) =$
15. $\lim_{x\to 1} f(x) =$
16. $f(1) =$
17. $\lim_{x\to 3^-} f(x) =$
18. $\lim_{x\to 3^+} f(x) =$
19. $\lim_{x\to 3} f(x) =$
20. $f(3) =$
21. $\lim_{x\to 4^-} f(x) =$
22. $\lim_{x\to 4^+} f(x) =$
23. $\lim_{x\to 4} f(x) =$
24. $f(4) =$
25. $\lim_{x\to 5^-} f(x) =$
26. $\lim_{x\to 5^+} f(x) =$
27. $\lim_{x\to 5} f(x) =$
28. $f(5) =$
29. List the value(s) of x at which f is discontinuous.

ANSWERS

1. 2
2. 2
3. 2
4. 1
5. -1
6. 4
7. Does not exist
8. 2
9. 6
10. 6
11. 6
12. 6
13. 7
14. 4
15. Does not exist
16. 7
17. 2
18. 2
19. 2
20. Undefined
21. 1
22. -4
23. Does not exist
24. 1
25. -3
26. 1
27. Does not exist
28. 1
29. $x = -5, -2, 1, 3, 4, 5$

Finding Limits Algebraically - Classwork

We are going to now determine limits without benefit of looking at a graph, that is $\lim_{x\to a} f(x)$.

There are three steps to remember:
1) plug in a
2) Factor/cancel and go back to step 1
3) ∞, $-\infty$, or DNE

Example 1) find $\lim_{x\to -2} x^2 - 4x + 1$

You can do this by plugging in.

$(-2)^2 - 4(-2) + 1$
$4 + 8 + 1 = (13)$

Example 2) find $\lim_{x\to -2} \frac{2x-6}{x-2}$

You can also do this by plugging in.

$\frac{2(-2)-6}{-2-2} = \frac{-10}{-4} = (5/2)$

Example 3) find $\lim_{x\to -2} \frac{x^2-2x-8}{x^2-4}$

Plug in and you get $\frac{0}{0}$ - no good

So attempt to factor and cancel

$\frac{(x-4)(x+2)}{(x-2)(x+2)} = \frac{x-4}{x-2} = \frac{-2-4}{-2-2} = \frac{-6}{-4}$
$= (3/2)$

Example 4) find $\lim_{x\to 1} \frac{x^2-2x+1}{x^3-1}$

Plug in and you get $\frac{0}{0}$ - no good

So attempt to factor and cancel

$\frac{(x-1)(x-1)}{(x-1)(x^2+x+1)} = \frac{x-1}{x^2+x+}$
$= \frac{0}{3} = ($

If steps 1 and 2 do not work (you have a zero in the denominator, your answer is one of the following:

∞ $-\infty$ Does Not Exist (DNE)

To determine which, you must split your limit into two separate limits.: $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a+} f(x)$. Make a sign chart by plugging in a number close to a on the left side and determining its sign. You will also plug in a number close to a on the right side and determine its sign. **Each of these will be some form of ∞, either positive or negative.** Only if they are the same will the limit be ∞ or $-\infty$.

What this says is that in this case, $\lim_{x\to a^-} f(x)$ = some form of ∞ and $\lim_{x\to a+} f(x)$ = some form of ∞

You need to check whether they are the same.

Example 5) find $\lim_{x\to 2} \frac{2x+5}{x-2}$

Step 1) Plug in $-\frac{9}{0}$ - no good Step 2) - No factoring/cancel So your answer is ∞, $-\infty$ or DNE

$\lim_{x\to 2^-}$

X	Y
1	-7
1.5	-16
1.9	-88
	$-\infty$

$\lim_{x\to 2^+}$

X	Y
2.5	20
3	11
2.1	92
	$+\infty$

So $\lim_{x\to 2}$ DNE

Finding Limits Algebraically - Homework

1) $\lim_{x\to 5} 12$ (12)

2) $\lim_{x\to 0} \pi$ (π)

3) $\lim_{x\to 2} 4x$ (8)

4) $\lim_{x\to 5} 3x^2 - 4x - 1$ (54)

5) $\lim_{x\to 0^-} 5x^3 - 7x^2 + 2^x - 2$ $0 \quad 0 \quad 1 \quad -2$ (-1)

6) $\lim_{y\to -1} 3y^4 - 6y^3 - 2y$ $3 + 6 + 2$ (11)

7) $\lim_{x\to 4} \frac{2x-4}{x-1}$ $\frac{4}{3}$

8) $\lim_{x\to -2} \frac{x^2+4x+4}{x^2}$ $\frac{4-8+4}{4} = 0$

9) $\lim_{x\to 1} \frac{2x-2}{x-1}$ $\frac{2(x-1)}{(x-1)} = 2$

10) $\lim_{x\to 4} \frac{x^2-16}{x-4}$ $\frac{(x+4)(x-4)}{(x-4)}$ $= x+4$ $= 8$

11) $\lim_{t\to -2} \frac{t^3+8}{t+2}$ $\frac{(t+2)(t^2-2t+4)}{t+2}$ $= t^2 - 2t + 4$ $= 4 + 4 + 4 = 12$

12) $\lim_{x\to 2} \frac{x^2-4x+4}{x^2+x-6}$ $\frac{(x-2)(x-2)}{(x-2)(x+3)}$ $= \frac{x-2}{x+3} = \frac{0}{5}$ (0)

13) $\lim_{x\to -1} \frac{x^2+6x+5}{x^2-3x-4}$ $\frac{(x+1)(x+5)}{(x-4)(x+1)}$ $= \frac{x+5}{x-4} = \frac{4}{-5}$

14) $\lim_{x\to 1} \frac{x^3+x^2-5x+3}{x^2-3x+2}$ $\frac{(x+3)(x-1)(x-1)}{(x-1)(x-2)} = 0$

15) $\lim_{x\to 3} \frac{x}{x-3}$ $x \to 3^-$ $-\infty$ $x \to 3^+$ $+\infty$ DNE

16) $\lim_{x\to 5} \frac{x}{x^2-25}$ $x \to 5^-$ $-\infty$ $x \to 5^+$ $+\infty$ DNE

17) $\lim_{y\to 6} \frac{y+6}{y^2-36}$ $(5.99, -100)$ $-\infty$ $(6.01, 100)$ $+\infty$ DNE

18) $\lim_{x\to 4} \frac{3-x}{x^2-2x-8}$ $\frac{-(x-3)}{(x-4)(x+2)}$ $(3.99, 16.5)$ $+\infty$ $(4.01, -16.8)$ $-\infty$ DNE

19) $\lim_{x\to 1} \frac{4}{x^2-2x+1}$ $\frac{4}{(x-1)(x-1)}$ $(.99, 40{,}000)$ $(1.01, 40{,}400)$ ∞

20) $\lim_{x\to 5} \frac{x}{|x-5|}$ $(4.99, 499)$ $(5.01, 501)$ ∞

21) $\lim_{x\to 3} \frac{-x^2}{x^2-6x+9}$ $\frac{-x^2}{(x-3)(x-3)}$ $(2.99, -89401)$ $-\infty$ $(3.01, -90601)$ $-\infty$ $-\infty$