

Practice Problems in using the definition of derivatives.

a. Given $f(x) = 10$ Find $f'(3)$ by using $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$ Answer: $f'(3) = 0$

b. Given $f(x) = -4x$ Find $f'(3)$ by using $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$ Answer: $f'(3) = -4$

c. Given $f(x) = -4x + 10$ Find $f'(3)$ by using $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$ Answer: $f'(3) = -4$

d. Given $f(x) = x^2$ Find $f'(3)$ by using $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$ Answer: $f'(3) = 6$

e. Given $f(x) = x^2 - 4x + 10$ Find $f'(3)$ by using $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$ Answer: $f'(3) = 2$

f. Given $f(x) = 3x^2 - 5x + 2$ Find $f'(1)$ by using $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$ Answer: $f'(1) = 1$

Practice Problems in using the definition of derivatives.

g. Given $f(x) = 10$ Find $f'(x)$ by using $\lim_{h \rightarrow 0} \frac{f(x+h) - f(h)}{h}$ Answer: $f'(x) = 0$

h. Given $f(x) = -4x$ Find $f'(x)$ by using $\lim_{h \rightarrow 0} \frac{f(x+h) - f(h)}{h}$ Answer: $f'(x) = -4$

i. Given $f(x) = -4x + 10$ Find $f'(x)$ by using $\lim_{h \rightarrow 0} \frac{f(x+h) - f(h)}{h}$ Answer: $f'(3) = -4$

j. Given $f(x) = x^2$ Find $f'(x)$ by using $\lim_{h \rightarrow 0} \frac{f(x+h) - f(h)}{h}$ Answer: $f'(x) = 2x$

k. Given $f(x) = x^2 - 4x + 10$ Find $f'(x)$ by using $\lim_{h \rightarrow 0} \frac{f(x+h) - f(h)}{h}$ Answer: $f'(x) = 2x - 4$

l. Given $f(x) = 3x^2 - 5x + 2$ Find $f'(x)$ by using $\lim_{h \rightarrow 0} \frac{f(x+h) - f(h)}{h}$ Answer: $f'(x) = 6x - 5$

2.2 Worksheet #2 - More Power Rule Practice

Compute the derivatives of the following functions.

$$(1) f(x) = x^2 - 2$$

$$(2) f(x) = x - x^3$$

$$(3) f(x) = x^2 + 3x - 6$$

$$(4) f(x) = 2x^2 - 4$$

$$(5) f(x) = \frac{2}{x}$$

$$(6) f(x) = \frac{4}{x^2} - \frac{x^2}{4}$$

$$(7) f(x) = 2x^{10} - 4x^2$$

$$(8) f(x) = 3\sqrt{x}$$

$$(9) f(x) = x\sqrt{3}$$

$$(10) f(x) = \frac{x^4}{4} + x - 2$$

$$(11) f(x) = x(x + 1)$$

$$(12) f(x) = x^2 - e^2$$

$$(13) f(x) = 5x^3 - \frac{5}{x^3}$$

$$(14) f(x) = (6x + 5) - (3x + x^2)$$

$$(15) f(x) = 2x^2 - 5x + 10$$

$$(16) f(x) = x - \frac{1}{x}$$

$$(17) f(x) = 4x^{\frac{5}{2}}$$

$$(18) f(x) = 1 - 5$$

$$(19) f(x) = \frac{1}{3x}$$

$$(20) f(x) = \frac{x^2}{2} - 3x$$

$$(21) f(x) = 5^2$$

$$(22) f(x) = (x^2 + 1)^2$$

$$(23) f(x) = x^{1000}$$

$$(24) f(x) = \frac{1}{x^{1000}}$$

$$(25) f(x) = \frac{x^2}{\ln(2)}$$

$$(26) f(x) = \sqrt{3x}$$

$$(27) f(x) = \sqrt{7}$$

$$(28) f(x) = \frac{x^2-1}{x}$$

$$(29) f(x) = \frac{8}{\sqrt{x}} - 3x$$

$$(30) f(x) = \frac{7x+3x^2}{5\sqrt{x}}$$

Answers

- | | | |
|---------------------------|-----------------------------------|---|
| (1) $2x$ | (2) $1 - 3x^2$ | (3) $2x + 3$ |
| (4) $4x$ | (5) $-2x^{-2}$ | (6) $-8x^{-3} - \frac{x}{2}$ |
| (7) $20x^9 - 8x$ | (8) $\frac{3}{2}x^{-\frac{1}{2}}$ | (9) $\sqrt{3}$ |
| (10) $x^3 + 1$ | (11) $2x + 1$ | (12) $2x$ |
| (13) $15x^2 + 15x^{-4}$ | (14) $3 - 2x$ | (15) $4x - 5$ |
| (16) $1 + x^{-2}$ | (17) $10x^{\frac{3}{2}}$ | (18) 0 |
| (19) $-\frac{1}{3}x^{-2}$ | (20) $x - 3$ | (21) 0 |
| (22) $4x^3 + 4x$ | (23) $1000x^{999}$ | (24) $-1000x^{-1001}$ |
| (25) $\frac{2x}{\ln(2)}$ | (26) $\frac{\sqrt{3}}{2\sqrt{x}}$ | (27) 0 |
| (28) $1 + x^{-2}$ | (29) $-4x^{-\frac{3}{2}} - 3$ | (30) $\frac{7}{10}x^{-\frac{1}{2}} + \frac{9}{10}x^{\frac{1}{2}}$ |

Find the derivative of each function.

1. $y = x^8$ 2. $y = \sqrt[3]{x}$ 3. $y = x^{-\frac{2}{5}}$ 4. $v(r) = \frac{4}{3}\pi r^3$ 5. $y(t) = 6t^{-9}$
6. $f(x) = x^2 - 10x + 100$ 7. $g(x) = x^{100} + 50x + 1$ 8. $f(x) = (2x)^3$ 9. $g(x) = x^2 + \frac{1}{x^2}$
10. $s(t) = t^8 + 6t^7 - 18t^2 + 2t$ 11. $y = \frac{x^2 + 4x + 3}{x}$ 12. $f(x) = x - 3x^{\frac{1}{3}}$
13. $y = \frac{3}{4x^3} + \frac{7}{2x^9}$ 14. $y = 5x^{-4} - \frac{7}{8}x^{-2} + 3x^2 - 6$ 15. $y = \frac{x^{12} - 2x^9 + 5x^{-7}}{4}$

16. Find an equation of the tangent line to the given curve at the specified point.

$$y = x + \sqrt{x} \quad \text{at } x = 1$$

17. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent is horizontal.

(Hint: what is the slope of any horizontal line)

ANSWER

1. $y' = 8x^7$ 2. $y' = \frac{1}{3} \frac{1}{\sqrt[3]{x^2}}$ 3. $y' = -\frac{2}{5} x^{-\frac{7}{5}}$ 4. $v'(r) = 4\pi r^2$ 5. $y(t) = 54t^{-10}$
6. $f'(x) = 2x - 10$ 7. $g'(x) = 100x^{99} + 50$ 8. $f(x) = 24x^2$ 9. $g'(x) = 2x^2 + -\frac{2}{x^3}$
10. $s'(t) = 8t^7 + 42t^6 - 36t + 2$ 11. $y' = 1 - \frac{3}{x^2}$ 12. $f'(x) = 1 - x^{\frac{2}{3}}$
13. $y' = \frac{9}{4x^4} + \frac{63}{2x^{10}}$ 14. $y' = -20x^{-5} + \frac{7}{4}x^{-3} + 6x$ 15. $y' = 3x^{11} - \frac{9}{2}x^8 - \frac{35}{4}x^{-8}$

16. Find an equation of the tangent line to the given curve at the specified point.

$$x = 1 \quad y = 1 + \sqrt{1} = 2 \quad m = y' = 1 + \frac{1}{2\sqrt{x}} \quad \text{at } x = 1 \quad m = y' = 1 + \frac{1}{2\sqrt{1}} = \frac{3}{2} \quad \text{at } x = 1$$

$$y - y_1 = m(x - x_1) \quad y - 2 = \frac{3}{2}(x - 1) \quad y = \frac{3}{2}x + \frac{1}{2}$$

17. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent is horizontal.

(Hint: what is the slope of any horizontal line)

$$m = y' = 3x^2 - 2x - 1 = 0 \quad 3x^2 - 2x - 1 = (3x + 1)(x - 1) = 0 \quad x = -\frac{1}{3}, x = 1$$