3.2 Product and Quotient Rule

NAME:_____

Corrective Assignment

DATE:_____

Find the derivative of the following.

1. $f(x) = (x^3 + 5)(x^2 - 1)$

2. $y = \frac{x-2}{3x^2+1}$

3. $y = \frac{5x^2+7x-2}{2x}$

4. $g(x) = (x^2 - 3x + 2)(3x + 1)$

5. $h(x) = \frac{x^2-1}{x+2}$

6. $y = 3x(x^2 - 2x + 1)$

7. $f(x) = (x^3 + 5x - 2)(x - 3)$

8. $y = \frac{3x-2}{x+1}$

Write the equation of the tangent line and the normal line at the point given.

9. $f(x) = \frac{x-3}{2x+1}$ at $x = -1$

10. $f(x) = (x^2 + 3x - 2)(x + 1)$ at $x = 0$

11. $f(x) = \frac{2x-1}{3x+1}$ at $x = 3$

Find $f'(1)$ given the following.

	12. $f(x) = \frac{g(x)}{h(x)}$	13. $f(x) = g(x)h(x)$
$g(1) = 0$ and $g'(1) = 2$ $h(1) = -1$ and $h'(1) = 5$		

Find $f'(-2)$ given the following.

	14. $f(x) = g(x)h(x)$	15. $f(x) = \frac{g(x)}{h(x)}$
$g(-2) = -3$ and $g'(-2) = 5$ $h(-2) = 1$ and $h'(-2) = -4$		

Find $f'(0)$ given the following.

|	16. $f(x) = \frac{h(x)}{g(x)}$	17. $f(x) = h(x)g(x)$
$g(0) = 7$ and $g'(0) = 3$ $h(0) = -2$ and $h'(0) = -8$		

ANSWERS TO CORRECTIVE ASSIGNMENT

| 1. $5x^4 - 3x^2 + 10x$	2. $\frac{-3x^2+12x+1}{(3x^2+1)^2}$	3. $\frac{5}{2} + \frac{1}{x^2}$	4. $9x^2 - 16x + 3$
5. $\frac{x^2+4x+1}{(x+2)^2}$	6. $9x^2 - 12x + 3$	7. $4x^3 - 9x^2 + 10x - 17$	8. $\frac{5}{(x+1)^2}$
9. $y - 4 = 7(x + 1)$ Tangent $y - 4 = -\frac{1}{7}(x + 1)$ Normal	10. $y + 2 = 1(x)$ Tangent $y + 2 = -1(x)$ Normal		11. $y - \frac{1}{2} = \frac{1}{20}(x - 3)$ Tangent $y - \frac{1}{2} = -20(x - 3)$ Normal
12. -2	13. -2	14. 17	15. -7	16. $-\frac{50}{49}$	17. -62

Quotient Rule Problems. Find the derivative of each of the following.

	Problems	Answers
1	$y = \dfrac{x-1}{x+1}$	$f'(x) = \dfrac{2}{(x+1)^2}$
2	$y = \dfrac{2x-1}{x-1}$	$f'(x) = \dfrac{-1}{(x-1)^2}$
3	$f(x) = \dfrac{3x}{2x-5}$	$f'(x) = \dfrac{-15}{(2x-5)^2}$
4	$f(x) = \dfrac{3x-2}{4x+1}$	$f'(x) = \dfrac{11}{(4x+1)^2}$
5	$f(x) = \dfrac{x^3+1}{x^2+x}$	$f'(x) = \dfrac{x^4+2x^3-2x-1}{(x^2+x)^2}$
6	$y = \dfrac{2x^2-3x+1}{2x+1}$	$y' = \dfrac{4x^2+4x-5}{(2x+1)^2}$
7	$f(x) = \dfrac{\sqrt{x}}{x^3+2}$	$f'(x) = \dfrac{-5x^3+2}{2x^{1/2}(x^3+2)^2}$
8	$f(x) = \dfrac{\sqrt[3]{x}}{x^2-x}$	$f'(x) = \dfrac{-5x^{4/3}+2x^{1/3}}{3(x^2-x)^2}$
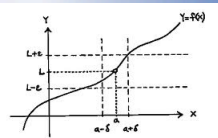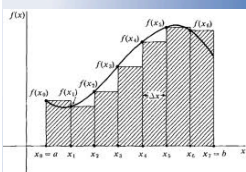
Higher Order Derivatives

$$f'(x), f''(x), f'''(x), f^{iv}(x)$$

$$\frac{dy}{dx}, \quad \frac{d^2 y}{dx^2}, \quad \frac{d^3 y}{dx^3}, \quad \frac{d^4 y}{dx^4}$$

$$y', \quad y'', \quad y''', \quad y^{(4)}$$

$$D_x(y), \quad D_x^2(y), \quad D_x^3(y) \quad D_x^4(y)$$

$$Note \; that \; \frac{d^2 y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) or \; \frac{dy'}{dx}$$

Higher Order Derivatives

Derivative	f' notation	y' notation	D_x notation	Leibniz notation
First	$f'(x)$	y'	$D_x(f)$	$\dfrac{dy}{dx}$
Second	$f''(x)$	y''	$D_x^2(f)$	$\dfrac{d^2y}{dx^2}$
Third	$f'''(x)$	y'''	$D_x^3(f)$	$\dfrac{d^3y}{dx^3}$
Fourth	$f^{(4)}(x)$	$y^{(4)}$	$D_x^4(f)$	$\dfrac{d^4y}{dx^4}$
Fifth	$f^{(5)}(x)$	$y^{(5)}$	$D_x^5(f)$	$\dfrac{d^5y}{dx^5}$
nth	$f^{(n)}(x)$	$y^{(n)}$	$D_x^n(f)$	$\dfrac{d^ny}{dx}$

$\dfrac{d^2(y)}{dx^2}$

$y^4 = y$ to the 4th power ≠ $y^{(4)}$ is the fourth derivative of y

EX 1   Find $f'''(x)$ for $f(x) = (3-5x)^5$ ✓

$$f'(x) = 5(3-5x)^4(-5) = -25(3-5x)^4$$

$$f''(x) = -25(4)(3-5x)^3(-5) = 500(3-5x)^3$$

$$f'''(x) = 500(3)(3-5x)^2(-5)$$
$$= -7500(3-5x)^2$$

Higher Order Derivatives

Name_____

Date_____ Period_____

For each problem, find the indicated derivative with respect to x.

1) $y = -x^2$ Find $\dfrac{d^2y}{dx^2}$

2) $f(x) = 4x^3$ Find f''

3) $y = -4x$ Find $\dfrac{d^3y}{dx^3}$

4) $f(x) = 5x^4$ Find f'''

5) $y = -5x^4$ Find $\dfrac{d^4y}{dx^4}$

6) $y = 3x^5 - 2x$ Find $\dfrac{d^3y}{dx^3}$

7) $y = -2x^3 - 4x^{-3}$ Find $\dfrac{d^3y}{dx^3}$

8) $y = -x^2 + 2\sqrt[5]{x^2}$ Find $\dfrac{d^3y}{dx^3}$

Critical thinking questions. Find the indicated derivatives with respect to x.

9) $y = 99x^{99}$ Find $\dfrac{d^{100}y}{dx^{100}}$

10) $f(x) = x^{99}$ Find $f^{(99)}$

Higher Order Derivatives

Name_____

For each problem, find the indicated derivative with respect to x.

1) $y = -x^2$ Find $\dfrac{d^2y}{dx^2}$

$$\dfrac{d^2y}{dx^2} = -2$$

2) $f(x) = 4x^3$ Find f''

$$f''(x) = 24x$$

3) $y = -4x$ Find $\dfrac{d^3y}{dx^3}$

$$\dfrac{d^3y}{dx^3} = 0$$

4) $f(x) = 5x^4$ Find f'''

$$f'''(x) = 120x$$

5) $y = -5x^4$ Find $\dfrac{d^4y}{dx^4}$

$$\dfrac{d^4y}{dx^4} = -120$$

6) $y = 3x^5 - 2x$ Find $\dfrac{d^3y}{dx^3}$

$$\dfrac{d^3y}{dx^3} = 180x^2$$

7) $y = -2x^3 - 4x^{-3}$ Find $\dfrac{d^3y}{dx^3}$

$$\dfrac{d^3y}{dx^3} = -12 + \dfrac{240}{x^6}$$

8) $y = -x^2 + 2\sqrt[5]{x^2}$ Find $\dfrac{d^3y}{dx^3}$

$$\dfrac{d^3y}{dx^3} = \dfrac{96}{125x^{\frac{13}{5}}}$$

Critical thinking questions. Find the indicated derivatives with respect to x.

9) $y = 99x^{99}$ Find $\dfrac{d^{100}y}{dx^{100}}$

The 99th derivative is a constant, so 100th derivative is 0.

10) $f(x) = x^{99}$ Find $f^{(99)}$

99! (Made easy by factorial notation)