Calculus Second Derivative Test Worksheet   Name ______________________________

For the following, find all relative extrema. Use the Second Derivative Test where applicable.

1) $f(x) = 6x - x^2$

$f(x) = 6x - x^2$
$f'(x) = 6 - 2x$
$f''(x) = -2$

$6 - 2x = 0 \Rightarrow x = 3$
$f''(x) < 0,$ so $x = 3$ must be a maximum
local maximum of 9 at $x = 3$ (also an absolute maximum)

2) $f(x) = x^2 + 3x - 8$

$f(x) = x^2 + 3x - 8$
$f'(x) = 2x + 3$
$f''(x) = 2$

$2x + 3 = 0 \Rightarrow x = \frac{-3}{2}$
$f''(x) > 0,$ so $x = \frac{-3}{2}$ must be a minimum
local minimum of $-\frac{41}{4}$ at $x = \frac{-3}{2}$ (also an absolute minimum)

3) $f(x)=-(x-5)^2$

$$f(x)=-(x-5)^2$$
$$f'(x)=-2(x-5)=-2x+10$$
$$f''(x)=-2$$

$-2x+10=0 \Rightarrow x=5$

$f''(x)<0,$ so $x=5$ must be a maximum

local maximum of 0 at $x=5$ (also an absolute maximum)

4) $f(x)=x^{2/3}-3$

$$f(x)=x^{2/3}-3$$
$$f'(x)=\frac{2}{3}x^{-1/3}=\frac{2}{3x^{1/3}}$$
$$f''(x)=-\frac{2}{9}x^{-4/3}$$

$\frac{2}{3x^{1/3}}=0$ does not exist, but f'' is undefined at $x=0$

$f''(0)$ does not exist. We must use a different test.

First Derivative Test:

x	-1	0	1
f'	$-\frac{2}{3}$	DNE	$\frac{2}{3}$

Local minimum of -3 at $x = 0$ (also an absolute minimum)

5) $f(x)=\sqrt{x^2+1}$

$f(x)=\sqrt{x^2+1}$ Domain: all reals

$f'(x)=\frac{1}{2}\left(x^2+1\right)^{-1/2}\left(2x\right)=\frac{x}{\sqrt{x^2+1}}$

$f''(x)=-\frac{1}{4}\left(x^2+1\right)^{-3/2}\left(2x\right)\left(2x\right)+2\left(\frac{1}{2}\left(x^2+1\right)^{-1/2}\right)=\frac{1}{\sqrt{x^2+1}}-\frac{x^2}{\sqrt{\left(x^2+1\right)^3}}$

$\frac{x}{\sqrt{x^2+1}}=0$ at $x=0$

$f''(0)=1$, so $x=0$ is a minimum

local minimum of 1 at $x=0$ (also an absolute minimum)

6) $f(x)=x+\frac{4}{x}$

$f(x)=x+\frac{4}{x}$ Domain: all reals except $x=0$

$f'(x)=1-\frac{4}{x^2}=\frac{x^2-4}{x^2}$

$f''(x)=\frac{8}{x^3}$

$\frac{x^2-4}{x^2}=0$ at $x=2$ or $x=-2$

$f''(2)=1$, so $x=0$ is a minimum; $f''(-2)=-1$, so $x=0$ is a maximum

local minimum of 4 at $x=2$; local maximum of -4 at $x=-2$

For the following, find all relative extrema, points of inflection, and intervals of concavity. Then use a graphing calculator to graph the function and confirm you answers.

7) $f(x) = x^3 - 12x$

$f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12$

$f''(x) = 6x$

$3x^2 - 12 = 0 \Rightarrow x = \pm 2$

$f''(-2) = -12 \Rightarrow$ local max local max of 16 at $x = -2$

$f''(2) = 12 \Rightarrow$ local min local min of -16 at $x = 2$

$6x = 0 \Rightarrow x = 0$ (0,0) is a point of inflection

concave down on (-∞, 0), concave up on (0, ∞)

8) $f(x) = \frac{1}{4}x^4 - 2x^2$

$f(x) = \frac{1}{4}x^4 - 2x^2$

$f'(x) = x^3 - 4x$

$f''(x) = 3x^2 - 4$

$x^3 - 4x = 0 \Rightarrow x = 0, \ \pm 2$

$f''(-2) = 8 \Rightarrow$ local min local min of -4 at $x = -2$

$f''(0) = -4 \Rightarrow$ local max local max of 0 at $x = 0$

$f''(2) = 8 \Rightarrow$ local min local min of -4 at $x = 2$

$3x^2 - 4 = 0 \Rightarrow x = \pm\sqrt{\frac{4}{3}}$ $\left(\pm\sqrt{\frac{4}{3}}, -\frac{20}{9}\right)$ are points of inflection

concave up on $\left(-\infty, -\sqrt{\frac{4}{3}}\right)$, concave down on $\left(-\sqrt{\frac{4}{3}}, \sqrt{\frac{4}{3}}\right)$, concave up on $\left(\sqrt{\frac{4}{3}}, \infty\right)$

9) $f(x) = x\sqrt{x+3}$

$f(x) = x\sqrt{x+3}$ Domain: [-3,∞)

$$f'(x) = \sqrt{x+3} + x\left(\frac{1}{2}(x+3)^{-1/2}\right) = \frac{3x+6}{2\sqrt{x+3}}$$

$$f''(x) = \frac{\left(2\sqrt{x+3}\right)(3) - (3x+6)(x+3)^{-1/2}}{4x+12}$$

$$f'(x) = 0 \Rightarrow \frac{3x+6}{2\sqrt{x+3}} = 0 \Rightarrow x = -2$$

$f''(-2) = \dfrac{6\sqrt{1} - 0}{4} > 0$, so local min at $x = -2$ (also abs min)

$f(-3) = 0$, so local max at $x = -3$

$$f''(x) = 0 \Rightarrow \left(2\sqrt{x+3}\right)(3) - (3x+6)(x+3)^{-1/2} = 0$$

$$\Rightarrow \frac{6x+18-(3x+6)}{\sqrt{x+3}} = 0$$

$\Rightarrow 3x + 12 = 0 \Rightarrow x = -4$ (this is out of the domain)

convex up on (-3,∞)