

**Not to know is bad; not to wish to know is worse.**

- African proverb

**Basic:** First factor any **common Number**, **Variable** or **Expression** with the **smallest exponent**.

$$18x + 36 = 18(x + 2)$$

$$65y^{10} + 35y^6 = 5y^6(13y^4 + 7)$$

$$8m^3n + 24mn^3 = 8mn(m^2 + 3n^2)$$

$$13y^8 + 26y^4 - 39y^2 = 13y^2(y^6 + 2y^2 - 3)$$

$$p(p+4) + 3(p+4) = (p+4)(p+3)$$

**Grouping:** Take **every two terms**, factor each, and then **factor** each expression in ( )

$$18r^2 + 12ry - 3xr - 2xy = 6r(3r + 2y) - x(3r + 2y) = (3r + 2y)(6r - x)$$

**2 Terms**     $a^2 - b^2 = (a+b)(a-b)$  Both squared, and the **sign** between two terms **must** be **negative**

$$9x^2 - 25y^2 = (3x + 5y)(3x - 5y) \quad 16k^4 - 1 = (4k^2 + 1)(4k^2 - 1) = (4k^2 + 1)(2k + 1)(2k - 1)$$

$9x^2 + 25y^2$  = Can **not be factored** because of the positive **sign** in between is **positive**.

**2 Terms both cubed**     $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$     or     $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

$$8a^3 + 27 = (2a + 3)(4a^2 + 9 - 6a)$$

$$125a^3 - 64b^3 = (5a - 4b)(25a^2 + 16b^2 + 20b)$$

**3 Terms**     $x^2 + bx + c$     when    Sum =  $b$     and    Product =  $c$

$$\begin{aligned} x^2 + 5x + 6 &\quad S = 5 \quad P = 6 \quad 2, 3 \\ &(x+2)(x+3) \end{aligned}$$

$$\begin{aligned} x^2 - 2x - 15 &\quad S = -2 \quad P = -15 \quad 3, -5 \\ &(x+3)(x-5) \end{aligned}$$

**3 Terms**  $ax^2 + bx + c$     when    Sum =  $b$     and    Product =  $ac$

$$21m^2 + 13m + 2 \quad S = 13 \quad \text{and} \quad P = 21(2) = 42 \quad 7, 6$$

$$21m^2 + 7m + 6m + 2 = 7m(3m+1) + 2(3m+1) = (3m+1)(7m+2)$$

$$4x^2 + 3x - 10 \quad S = 3 \quad \text{and} \quad P = 4(-10) = -40 \quad 8, -5$$

$$4x^2 + 8x - 5x - 10 = 4x(x+2) - 5(x+2) = (x+2)(4x-5)$$

**3 Terms**  $ax^2 + 2abx + c^2$     **Two Squared Terms (Check doubling effect)**

$$x^2 + 10x + 25 = (x+5)^2 \quad 3x^2 - 48x + 192 = 3(x^2 - 16x + 64) = 3(x-8)^2$$

$$49x^2 - 28xy + 9y^2 = (7x-3y)^2 \quad m^2 + \frac{2}{3}m + \frac{1}{9} = \left(m + \frac{1}{3}\right)^2$$

**ZFP (Zero Factor Property)** to solve equations that can be factored.

1. Have zero on one side.
2. Factor the other side.
3. Let each factor = 0 and solve.

$$x^2 - 3x = 4, \quad x^2 - 3x - 4 = 0 \quad 2. (x-4)(x+1) = 0, \quad (x-4) = 0, \quad (x+1) = 0 \\ x = 4, \quad x = -1$$

### Practice Problems

1.  $15x^3 - 30x^2$

2.  $100a^5 + 16a^3$

3.  $x^6 y^2 + 5x^4 y^3 - 6xy^4 + 10xy$

4.  $3a^3 + 3ab^2 + 2a^2b + 2b^3$

5.  $c(x+2) - d(x+2)$

6.  $x^2 + 15x + 44$

$$7. x^2 + 6x - 27$$

$$8. x^2 - 14x + 24$$

$$9. n^2 - 12n - 35$$

$$10. x^2 - 2x - 3$$

$$11. 7x^2 + 8x + 1$$

$$12. 21m^2 + 13m + 2$$

$$13. 10x^2 + 11x - 6$$

$$14. 2m^3 + 2m^2 - 40m$$

$$15. 6x^2 - 7x - 5$$

$$16. 6x^2 - 7x - 5y^2$$

$$17. x^2 - 10x + 25$$

$$18. 2x^2 + 24x + 72$$

$$19. 49x^2 - 28x - 4y^2$$

$$20. 16x^2 - 40x + 25$$

$$21. 4x^2 - 9y^2$$

$$22. 36x^2 - 1$$

Solve each equation by **ZFP** Method

$$23. x^2 + 6x - 27 = 0$$

$$24. 2y^2 - 18 = 0$$

$$\mathbf{25. } 3x^2 + 5x - 2 = 0$$

$$\mathbf{26. } x^2 = -5x + 24$$

$$\mathbf{27. } m^2 + 8m = -16$$

$$\mathbf{28. } 36x^2 + 60x = -25$$

## Answers

$$\mathbf{1. } 15x^2(x-2)$$

$$\mathbf{2. } 4a^3(25a^2 + 4)$$

$$\mathbf{3. } xy(x^5y + 5x^3y^2 - 6y^3 + 10)$$

$$\mathbf{4. } 3a(a^2 + b^2) + 2b(a^2 + b^2) = (a^2 + b^2)(3a + 2b)$$

$$\mathbf{5. } (x+2)(c-d)$$

$$\mathbf{6. } (x+4)(x+11)$$

$$\mathbf{7. } (x+9)(x-3)$$

$$\mathbf{8. } (x-12)(x-2)$$

**9.** Prime

$$\mathbf{10. } (x-3)(x+1)$$

$$\mathbf{11. } (7x+1)(x+1)$$

$$\mathbf{12. } (3m+1)(7m+2)$$

$$\mathbf{13. } (2x+3)(5x-2)$$

$$\mathbf{14. } 2m(m-4)(m+5)$$

$$\mathbf{15. } (3x-5)(2x+1)$$

$$\mathbf{16. } (3x-5y)(2x+y)$$

$$\mathbf{17. } (x-5)^2$$

$$\mathbf{18. } 2(x+6)^2$$

$$\mathbf{19. } (7x-2y)^2$$

$$\mathbf{20. } (4x-5)^2$$

$$\mathbf{21. } (2x+3y)(2x-3y)$$

$$\mathbf{22. } (6x+1)(6x-1)$$

$$\mathbf{23. } (x+9)(x-3) = 0, \quad x = -9, 3$$

$$\mathbf{24. } 2(y+3)(y-3) = 0 \quad y = -3, 3$$

$$\mathbf{25. } (3x-1)(x+2) = 0 \quad x = \frac{1}{3}, -2$$

$$\mathbf{26. } (x+8)(x-3) = 0 \quad x = -8, 3$$

$$\mathbf{27. } (m+4)^2 = 0 \quad m = -4$$

$$\mathbf{28. } (6x+5)^2 = 0 \quad x = -\frac{5}{6}$$