Abe Mirza **Logarithms**

Logarithms are different ways of writing exponential equations.

$100 = 10^2$	$.0001 = 10^{-4}$	$128 = 2^7$	$\frac{1}{125} = 5^{-3}$
$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
$log_{10}100 = 2$	$log_{10}.0001 = -4$	$log_2 128 = 7$	$log_5 \frac{1}{125} = -3$

Rules:
Base $b > 0, \quad b \neq 1$

Rules	Name		Example	Practice
1		$log_b 1 = 0$	$log_8 1 = 0$	$log_{10} 1 = ?$
2		$log_b b = 1$	$log_{10} 10 = 1, \quad Log_{28} 28 = 1$	$log_2 2 = ?$
3	Power	$log_b a^n = n\,log_b a$	$log_{10} 8^3 = 3log_{10} 8$	$log_5 2^6 = ?$
4	Power	$log_{b^m} a^n = \frac{n}{m} log_b a$	$log_8 128 = log_{2^3} 2^7 = \frac{7}{3} log_2 2 = \frac{7}{3}(1)$	$log_{27} 243 = ?$
5	Product	$log_b a + log_b c = log_b ac$	$log_{10} 7 + log_{10} 9 = log_{10} 63$	$log_5 20 + log_5 5 = ?$
6	Quotient	$log_b a - log_b c = log_b \frac{a}{c}$	$log_{10} 12 - log_{10} 3 = log_{10} 4$	$log_5 20 - log_5 5 = ?$
7		$b^{log_b a} = a$	$5^{log_5 8} = 8$	$3^{log_3 11} = ?$
8	Base 10	When base is 10, we write base as blank.	$log_{10} 81 = log 81, \; log_{10} x^2 = 2 log\, x$	$log_{10} 20 = ?, \; log_{10}\, yz = ?$
10	Natural Base (e)	When base is (e), we write **ln** rather **log**.	$log_e 5 = \ln 5, \; log_e x = \ln x$	

Write each as an exponential.

1) $log_2 32 = 5$ **2)** $log_e x = -2$ **3)** $log_{11} \sqrt{11} = \frac{1}{2}$ **4)** $log_{10} 1000 = 3$ **5)** $log_5 125 = 3$

Write each as a logarithmic equation.

6) $10000 = 10^4$ **7)** $10^{-2} = \frac{1}{100}$ **8)** $5^{\frac{1}{2}} = \sqrt{5}$ **9)** $9^{-1} = \frac{1}{9}$ **10)** $2^{10} = 1024$

Simplify each logarithmic expression.

11) $\log_2 8$ **12)** $\log_{25} 5$ **13)** $\log_3 \frac{1}{9}$ **14)** $2^{\log_2 8}$ **15)** $\log_2 \frac{1}{32}$ **16)** $\log_2 \sqrt{2}$ **17)** $\log_2 \sqrt{32}$

18) $\log_2 \frac{1}{8} = x$ **19)** $\log_2 x = 6$ **20)** $\log_3 \frac{1}{81} = x$ **21)** $\log_{\frac{2}{3}} x = 8$ **22)** $\log_x 100 = 2$ **23)** $\log_x \sqrt{32} = 2$

Write each logarithmic as a single expression.

24) $\log_2 x + \log_2 y$ **25)** $\log_2 x - \log_2 y$ **26)** $3\log_2 x - \log_2 y$ **27)** $\log_3 2 + \log_3 10 - \log_3 5$

28) $\log_6 18 + \log_6 2 - \log_6 9$ **29)** $3\log_4 2 + 2\log_4 3$ **30)** $3\log_2 x - 4\log_2 y$ **31)** $\log_2 x - \log_2 (x+1) + \log_2 (x^2 - 2)$

Use the calculator to compute the following logarithms

32) $log 81$ **33)** $\ln 81$ **34)** $log 8100$ **35)** $\ln 8100$ **36)** $log 1000$ **37)** $log 0.0001$ **38)** $\ln e$

Solve each equation for x.

39) $\log_2 \frac{1}{8} = x$ **40)** $\log_2 x = 6$ **41)** $\log_3 \frac{1}{81} = x$ **42)** $\log_{\frac{2}{3}} x = 8$ **43)** $\log_x 100 = 2$ **44)** $\log_x \sqrt{32} = 2$

Answers

1	$32 = 2^5$	**16**	.5	**31**	$\log_2 \frac{x(x^2-2)}{(x+1)}$			**41**	
2	$x = e^{-2}$	**17**	2.5	**32**	1.908			**42**	
3	$\sqrt{11} = 11^{\frac{1}{2}}$	**18**	$x = -3$	**33**	4.934			**43**	
4	$1000 = 10^3$	**19**		**34**	4.9084			**44**	
5	$125 = 5^3$	**20**		**35**	4.3944			**45**	
6	$\log_{10} 10000 = 4$	**21**		**36**	3			**46**	
7	$\log_{10} 1/100 = -2$	**22**		**37**	-4			**47**	
8	$\log_5 \sqrt{5} = \frac{1}{2}$	**23**	2.5	**38**	1			**48**	
9	$\log_9 1/9 = -1$	**24**	$\log_2 xy$	**39**	-3			**49**	
10	$\log_2 1024 = 10$	**25**	$\log_2 x/y$	**40**				**50**	
11	3	**26**	$\log_2 x^3/y$		-4				
12	0.5	**27**	$\log_3 4$		10				
13	-2	**28**	$\log_6 4$						
14	8	**29**	$\log_4 72$						
15	-32	**30**	$\log_2 x^3/y^4$						

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Steps to solve an exponential Equation:

1. Isolate the exponential part

2. Take **ln** (natural log) from both sides

3. Apply the property of the **ln**

4. Solve for ***x*** ,

Ex 1. Solve for x, $5^x + 3 = 27$

1. Isolate the exponential part $5^x = 24$

2. Take **ln** (natural log) from both sides $\ln 5^x = \ln 24$

3. Apply the property of the **ln** $x \ln 5 = \ln 24$,

4. Solve for $\boldsymbol{x}$, $\boldsymbol{x} = \frac{\ln 24}{\ln 5} = \frac{3.178}{1.609} = 1.975$

Ex 2. Solve for x, $3^{x-4} + 4 = 62$

1. $3^{x-4} = 58$

2. $\ln 3^{x-4} = \ln 58$

3. $(x-4)\ln 3 = \ln 58$

4. $(x-4) = \frac{\ln 58}{\ln 3} = \frac{4.06}{1.099} = 3.694$, $\boldsymbol{x} = 7.694$

Ex 3. Solve for x, $4^{x+2} = 6^{x-4}$

1. $\ln 4^{x+2} = \ln 6^{x-4}$

2. $(x+2)\ln 4 = (x-4)\ln 6$

3 $(x+2)1.386 = (x-4)1.791$, $1.386x + 2.772 = 1.791x - 7.164$,

4. $4.392 = 0.405x$ $\boldsymbol{x} = 10.84$

Ex 4 Solve for x, $7\log x = 13$

1. Isolate the log part $\log x = \frac{13}{7} = 1.857$

2. Use property of log, $\boldsymbol{x} = 10^{1.857} = 71.945$

Ex 5 Solve for x, $7\ln x = 13$

1. Isolate the log part $\ln x = \frac{13}{7} = 1.857$

2. Use property of ln, $\boldsymbol{x} = e^{1.857} = 6.404$

Ex 6. Solve for x, $4\log(x+3) = 11$ **1**. $\log(x+3) = \frac{11}{4} = 2.75$ **2**. $(x+3) = 10^{2.75} = 562.34$, $\boldsymbol{x} = 559.34$

Practice Problems: Solve for x

1. $3^x - 5 = 12$ **2**. $2^x + 4 = 15$ **3**. $7^x - 35 = 12$ **4**. $3^{x-4} + 4 = 62$ **5**. $3^x = (4^{3x-5})$

6. $3^{2x-4} - 11 = 18$ **7**. $3^{x+1} = 5^{x-1}$ **8**. $3^{x+1} = (\frac{1}{5})^{x-1}$ **9**. $5\log x = 7$ **10**. $5\ln x = 7$

11. $\log x^2 = 4.67$ **2**. $\ln x = 4.67$ **13**. $3\log(x-2) = 5.7$ **14**. $3\ln(x-2) = 5.7$

Answers

1) $\boldsymbol{x} = 2.58$ 2) $\boldsymbol{x} = 3.45$ 3) $\boldsymbol{x} = 1.979$ 4) $\boldsymbol{x} = 7.69$ 5) $\boldsymbol{x} = 2.265$ 6) $\boldsymbol{x} = 3.532$ 7) $\boldsymbol{x} = 5.2986$

8) $\boldsymbol{x} = 0.188$ 9) $\boldsymbol{x} = 25.11$ 10) $\boldsymbol{x} = 4.055$ 11) $\boldsymbol{x} = 216.27$ 12) $\boldsymbol{x} = 106.7$ 13) $\boldsymbol{x} = .81.43$ 14) $\boldsymbol{x} = 8.68$

Exponential Function

$$F = Pe^{rt}$$

$r > 0$ Exponential Growth

$r < 0$ Exponential Decay

To solve for t or r, first use $r\,t = \ln(\frac{F}{P})$ and then solve for t or r,

Ex 1: If the world population grows exponentially at a rate of 2.5% per year and at the present time the population is 6 billion, then

1. What will be world population after 10 years? $F = P\,e^{rt} = 6e^{.025(10)} = 6e^{.25} = 6(1.284) = 7.7$ *billion*

2. What will be the world population after 25 years? $F = P\,e^{rt} = 6\,e^{.025(25)} = 6\,e^{.625} = 6(1.868) = 11.21$ *billion*

3. After how long the world population will be 7 billion? $.025t = \ln(\frac{7}{6}) = 0.15415$

$.025t = 0.15415 \qquad t = 0.15415 / .025 = 6.17$ *years*

4. After how long the world population will be doubled?

$.025t = \ln(2) = 0.6931 \qquad .025t = 0.6931 \qquad t = 0.6931 / .025 = 27.73$ *years*

5. After how long the world population will be tripled?

$0.025t = \ln(3) = 1.0986 \qquad .025t = 1.0986 \qquad t = 1.0986 / .025 = 43.94$ *years*

Ex 2: If the world population was 5 billion at year 1990 and it reached 6 billion at year 2002. Assuming the world population grows exponentially then

1. What is the growth rate of the world population? $F = P\,e^{rt} \qquad 6 = 5\,e^{r\,12} \qquad \frac{6}{5} = e^{r\,12}$

$\ln(\frac{6}{5}) = 12r \qquad 0.18232 = 12r \qquad r = .01519 = 1.52\%$

2. What will be world population in year 2010?

$t = 2010 - 1990 = 20 \qquad F = P\,e^{rt} = 5e^{0.0152(20)} = 5e^{0.304} = 5(1.355) = 6.78$

3. What will be the world population in year 2015? $t = 2015 - 1990 = 25$

$F = P\,e^{rt} = 5e^{.0152(25)} = 5e^{0.38} = 5(1.462) = 7.27$

4. After how long the world population will be 7 billion?

$.0125t = \ln(\frac{7}{5}) = 0.3365 \qquad .0152t = 0.3365 \qquad t = 0.3365 / .0152 = 22.14$ *years*

5. After how long the world population will be **doubled**?

$.0152t = \ln(2) = 0.6931 \qquad .025t = 0.6931 \qquad t = 0.6931 / .0152 = 45.6$ *years*

6. After how long the world population will be **tripled**?

$.0152t = \ln(3) = 1.0986 \qquad .0152t = 1.0986 \qquad t = 1.0986 / .0152 = 72.28$ *years*