Abe Mirza **Quadratic Equations (QE)** **Algebra**

$$ax^2+bx+c=0$$

Everyone is ignorant, only on different subjects. – Will Rogers

Solving QE $ax^2+bx+c=0$ by,

1. **Factoring ZFP**,

$$(x+3)(x-5)=0,\quad (x+3)=0,(x-5)=0$$
$$x=-3,\qquad x=5$$

2. **The Square root method**

$$(x-a)^2=b,\qquad \sqrt{(x-a)^2}=\pm\sqrt{b},\quad (x-a)=\pm\sqrt{b},\quad x=a\pm\sqrt{b}$$
$$(x-5)^2=16,\qquad \sqrt{(x-5)^2}=\pm\sqrt{16},\quad (x-5)=\pm4,\quad x=5\pm4, x=9, x=1$$
$$(x+3)^2=13,\qquad \sqrt{(x+3)^2}=\pm\sqrt{13},\quad (x+3)=-3\pm\sqrt{13},\quad x=-3\pm\sqrt{13},$$

3. **Completing square(7 steps)**

		$2x^2-12x-14=0$	$3x^2+9x-15=0$	$4x^2-8x=12$
1	Have all **x's** on one side	$2x^2-12x=14$	$3x^2+9x=15$	
2	**D**ivide both sides by a if $a\neq1$	$x^2-6x=7$	$x^2+3x=5$	
3	**D**ivide $\frac{b}{2}$	$\frac{-6}{2}=-3$	$\frac{3}{2}$	
4	**S**quare $\frac{b}{2}=\left(\frac{b}{2}\right)^2=\frac{b^2}{4}$	$(-3)^2=9$	$\left(\frac{3}{2}\right)^2=\frac{9}{4}$	
5	**A**dd $\frac{b^2}{4}$ to both sides	$x^2-6x+9=7+9$	$x^2+3x+\frac{9}{4}=5+\frac{9}{4}$	
6	**C**ompleting Square	$(x-3)^2=16$	$\left(x+\frac{3}{2}\right)^2=\frac{29}{4}$	
7	Use the **Square root** method	$\sqrt{(x-3)^2}=\pm\sqrt{16},$ $(x-3)=\pm4$ $x=3\pm4,$ $x=7, x=-1$	$x+\frac{3}{2}=\pm\sqrt{\frac{29}{4}}=\pm\frac{1}{2}\sqrt{29}$ $x=-\frac{3}{2}\pm\frac{1}{2}\sqrt{29}$	

4. Quadratic Formula for $ax^2+bx+c=0$ $\quad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x^2-6x-7=0 \qquad a=1, b=-6, c=-7$

$x=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(-7)}}{2(1)}=\dfrac{6\pm\sqrt{36+28}}{2}=\dfrac{6\pm\sqrt{64}}{2}=\dfrac{6\pm 8}{2}, \; x=\dfrac{14}{2}=7, \; x=\dfrac{-2}{2}=-1$

Practice Problems

A. Solve by factoring, $3x^2-6x=0$ $\quad x=0 \quad , x=2$

B. Solve by factoring, $x^2-8x=-16$ $\quad x=4 \quad , x=4$

C. Solve by factoring, $3x^2-x-10=0$ $\quad x=-\dfrac{5}{3}, \quad x=2$

D. Solve by square root method $3x^2=108$ $\quad x=6, \quad x=-6$

E. Solve by square root method $4x^2=25$ $\quad x=\dfrac{5}{2}, \quad x=-\dfrac{5}{2}$

F. Solve by square root method $(x-9)^2=36$ $\quad x=15, \quad x=3$

G. Solve by square root method $(4x-3)^2=125$ $\quad x=\dfrac{3+5\sqrt{5}}{4}, \quad x=\dfrac{3-5\sqrt{5}}{4}$

H. Solve by square root method $(2x+1)^2=48$ $\quad x=\dfrac{-1+4\sqrt{3}}{2}, \quad x=\dfrac{-1-4\sqrt{3}}{2}$

I. Solve by completing square $x^2-10x+16=0$ $\quad x=8, \quad x=2$

J. Solve by completing square $2x^2+36x=-34$ $\quad x=1, \quad x=17$

K. Solve by completing square $x^2-4x-45=0$ $\quad x=9, \quad x=-5$

L. Solve by completing square $3x^2+x-4=0$ $\quad x=1, \quad x=-\dfrac{4}{3}$

M. Solve by completing square $x^2+5x+3=0$ $\quad x=\dfrac{-5+\sqrt{13}}{2}, \quad x=\dfrac{-5-\sqrt{13}}{2}$

N. Solve by Quadratic Formula $x^2-10x+16=0$ $\quad x=8, \quad x=2$

O. Solve by Quadratic Formula $2x^2+36x=-34$ $\quad x=1, \quad x=17$

P. Solve by Quadratic Formula $x^2 - 4x - 45 = 0$ $x = 9,$ $x = -5$

Q. Solve by Quadratic Formula $3x^2 + x - 4 = 0$ $x = 1,$ $x = -\frac{4}{3}$

R. Solve by Quadratic Formula $x^2 + 5x + 3 = 0$ $x = \frac{-5 + \sqrt{13}}{2},$ $x = \frac{-5 - \sqrt{13}}{2}$

Applications

Each side of a square is increased by 6cm. The area of the resulting square is 9 times the area of the original square **answer questions 15-18**

15. Find the length of the sides of the original square

16. Find the length of the sides of the new square

17. Find the area of the original square.

18. Find the area of the new square.

19. A side of right angle triangle is 7 feet shorter than the other, if the hypotenuse is 13 feet long, then what is the length of the longer side?

The length of a rectangle is one inch less than twice the width. The length of the diagonal is 17 inches**.**
20. Find its width

21. Find its length

22. The sum of a number and its square is 132. Find the number

A rectangle has a length 5 meters less than twice its width. If the area is 63 square meter
23. Find its width

24. Find its length

15. 3 **16**. 9 **17**. 9 **8**. 81

Parabola

$$f(x) = y = a x^2 + bx + c$$

$f(x) = y = ax^2 + bx + c$

If $a > 0$ the parabola opens up $\quad y = 3x^2 - 6x + 4$

If $a < 0$ the parabola opens down $\quad y = -4x^2 + 16x + 8$

Every parabola has a vertex, to find the coordinate of that vertex (x, y), in 2 parts

	It opens up	It open down
$f(x) = y = ax^2 + bx + c$	$y = 3x^2 - 6x + 4$	$y = -4x^2 + 16x + 8$
$x = -\dfrac{b}{2a}$	$x = -\dfrac{-6}{2(3)} = 1$	$x = -\dfrac{16}{2(4)} = -2$
$y = f(-\dfrac{b}{2a}) =$	$y = f(1) = 3(1)^2 - 6(1) + 4 = 1$	$y = f(-2) = 4(-2)^2 + 16(-2) + 8 = -8$
$\left(-\dfrac{b}{2a}, f(-\dfrac{b}{2a})\right)$	$(1,1)$	$(-2,-8)$

Find the y-intercepts by letting $x = 0$

$f(x) = y = ax^2 + bx + c$	$y = 3x^2 - 6x + 4$	$y = -4x^2 + 16x + 8$
$x = 0, \quad y = c$	$x = 0, \quad y = 4$	$x = 0, \quad y = 8$

Now graph each parabola.

Graph each parabola first by finding the coordinate of its vertex, and its y –intercept.

1. $y = x^2 - 6$ **2.** $y = x^2 - 4x + 5$ **3.** $y = -x^2 - 7x + 3$

4. $y = 3x^2 - 9x + 5$ **5.** $y = x^2 - 8x$ **6.** $y = 8 - x^2$

7. A perimeter of a rectangle is 56 feet. What are the dimensions of the rectangle with maximum area?

Answers

1. $(0,-6), \quad y = -6$ **2.** $(2,1), \quad y = 5$ **3.** $\left(-\dfrac{7}{2}, \dfrac{61}{4}\right), \quad y = 3$ **4.** $\left(\dfrac{3}{2}, -\dfrac{7}{4}\right), \quad y = 5$

5. $(4,-16), \quad y = 0$ **6.** $(0,8), \quad y = 8$ **7.** $x = y = 14$