

*If you want to increase your success rate, double your failure rate. T. Watson*

Rules to reduce a radical:

1. The power inside should be the **same or larger than the root**  $\sqrt{x^2}$ ,  $\sqrt{x^5}$ ,  $\sqrt[3]{x^6}$
2. If the **root is even** and the **inside number is negative** there is **no** real number  $\sqrt{-4}$ ,  $\sqrt[4]{-9}$
3. If the **root is odd** and the **inside number is negative** the answer is **negative**.  $\sqrt[3]{-8} = -2$ ,  $\sqrt[5]{-32} = -2$

$$\mathbf{E-1.} \quad \sqrt{x^{11}}, \quad \frac{11}{2} = 5\frac{1}{2}, \quad x^5 \sqrt{x^1} = x^5 \sqrt{x} \qquad \mathbf{E-2.} \quad \sqrt[3]{x^{20}}, \quad \frac{20}{3} = 6\frac{2}{3}, \quad \sqrt[3]{x^{20}} = x^6 \sqrt[3]{x^2}$$

$$\mathbf{E-3.} \quad \sqrt{24}, \quad \frac{24 \cdot 12 \cdot 6 \cdot 3 \cdot 1}{2 \cdot 2 \cdot 2 \cdot 3} = 2^3 \cdot 3, \quad \sqrt{24} = \sqrt{2^3 \cdot 3} = 2\sqrt{2 \cdot 3} = 2\sqrt{6}$$

$$\mathbf{E-4} \quad \sqrt{72}, \quad \frac{72 \cdot 36 \cdot 18 \cdot 6 \cdot 3 \cdot 1}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} = 2^3 \cdot 3^2, \quad \sqrt{72} = \sqrt{2^3 \cdot 3^2} = 2 \cdot 3\sqrt{2} = 6\sqrt{2}$$

$$\mathbf{E-5} \quad \sqrt{98}, \quad \frac{98 \cdot 49 \cdot 7 \cdot 1}{2 \cdot 7 \cdot 7} = 2 \cdot 7^2, \quad \sqrt{98} = 7\sqrt{2}$$

**Operation with radicals:** To add or subtract the **index number** and the **inside** of radicals should be the **same**, for some problems the inside needs to be simplified first.

$$15\sqrt{6} - 2\sqrt{6} = 13\sqrt{6}, \quad \mathbf{or} \quad 4\sqrt[3]{7} + 11\sqrt[3]{7} = 15\sqrt[3]{7} \quad \text{but we can not add} \quad 4\sqrt[3]{7} + 11\sqrt[3]{17}$$

**Multiplication:** If the indices are the same we just multiply the insides.

$$3\sqrt{7}(2\sqrt{5}) = 6\sqrt{35}, \quad \mathbf{or} \quad 5\sqrt{14}(2\sqrt{2}) = 10\sqrt{28} = 10(2\sqrt{7}) = 20\sqrt{7}$$

A number **multiply** by its **conjugate** is always a **real number!**  $(a-b)(a+b) = a^2 - b^2$

$$\mathbf{E-6} \quad (\sqrt{2} + 3)(\sqrt{2} - 3) = 2 - 9 = -7 \qquad \mathbf{E-7} \quad (\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5}) = 7 - 5 = 2$$

**Conjugate** of  $\sqrt{2} + 3$  is  $\sqrt{2} - 3$ , and conjugate of  $7 - \sqrt{2}$  is  $7 + \sqrt{2}$

To **rationalize**: Multiply by the same radicals or its conjugate

$$\text{E.8 } \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \quad \text{or} \quad \text{E.9 } \frac{5}{\sqrt{11}-3} = \frac{5}{\sqrt{11}-3} \cdot \frac{\sqrt{11}+3}{\sqrt{11}+3} = \frac{5\sqrt{11}+15}{11-9} = \frac{5\sqrt{11}+15}{2}$$

To **solve** an **equation** with the **radicals**

**Rules:** 1. One radical per side   2. Square both sides   3. Simplify and solve

$$\text{E.10 } \sqrt{2x-5}-9=2, \quad \sqrt{2x-5}=11, \quad (\sqrt{2x-5})^2=11^2 \quad 2x-5=121, \quad 2x=126 \quad x=63$$

**Find the distance between each two points;**  $(x_1, y_1), (x_2, y_2)$   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

## Practice Problems

**Simplify each radical,**

1.  $\sqrt{x^5}$

2.  $\sqrt{x^{13}}$

3.  $\sqrt[3]{x^{13}}$

4.  $\sqrt[4]{x^{28}}$

5.  $\sqrt[3]{x^7 y^{11}}$

6.  $\sqrt{x^{11} y^9}$

7.  $\sqrt[4]{x^{10} y^{14}}$

8.  $\sqrt[3]{192}$

9.  $\sqrt{128x^4 y^7}$

10.  $\sqrt[4]{80x^7 y^8}$

11.  $\frac{\sqrt{45x^7}}{\sqrt{5x}}$

12.  $\sqrt{\frac{6x}{27x^2}}$

13.  $\frac{\sqrt{160}}{\sqrt{5}}$

**Use a calculator to find each root;**

14.  $\sqrt{571}$

15.  $\sqrt{3.94}$

16.  $\sqrt{.00895}$

17.  $\sqrt{5^2 + 12^2}$

18.  $-\sqrt{500}$

$$19. \sqrt{-500} \qquad 20. \frac{-3+8\sqrt{1540}}{2} \qquad 21. \sqrt{\frac{10}{14}} \qquad 22. \sqrt[3]{251.8} \qquad 23. \sqrt[3]{-95}$$

**Add or subtract;**

$$24. \sqrt{8} - 5\sqrt{2} \qquad 25. 2\sqrt{72} - 2\sqrt{32} \qquad 26. \sqrt{20} + 2\sqrt{45} - 7\sqrt{5}$$

$$27. \sqrt{50} + \sqrt{32} - 4\sqrt{18} \qquad 28. \sqrt{50x^5} + \sqrt{32x^5} \qquad 29. 3 + \sqrt[3]{x} + 5 + 4\sqrt[3]{x}$$

**Multiply and simplify;**

$$30. \sqrt{3} \cdot \sqrt{27} \qquad 31. \sqrt{10} \cdot \sqrt{6} \qquad 32. \sqrt{6}(\sqrt{12} + 8)$$

$$33. \sqrt[4]{9} \cdot \sqrt[4]{9} \qquad 34. (\sqrt{10} + 5)(\sqrt{10} - 5) \qquad 35. (3 - \sqrt{11})(3 + \sqrt{11})$$

$$36. (2\sqrt{x} - 3)(2\sqrt{x} + 3) \qquad 37. (4 - \sqrt{12})^2 \qquad 38. \sqrt{x}(3 - \sqrt{x})$$

**Find the conjugate;**

$$39. \sqrt{8} \qquad 40. -\sqrt{8} \qquad 41. \sqrt{7} - 4 \qquad 42. \sqrt{x} - \sqrt{y} \qquad 43. \sqrt{5} + \sqrt{x} \qquad 44. 4 - \sqrt{3}$$

**Multiply each number by its conjugate;**

$$45. \sqrt{8} \qquad 46. -\sqrt{8} \qquad 47. \sqrt{7} - 4 \qquad 48. \sqrt{x} - \sqrt{y} \qquad 49. \sqrt{5} + \sqrt{x} \qquad 50. 4 - \sqrt{3}$$

**Rationalize the denominator of each expression and then simplify;**

$$51. \frac{3}{\sqrt{7}} \qquad 52. \frac{-8}{\sqrt{2}} \qquad 53. \sqrt{\frac{9}{2}} \qquad 54. \frac{\sqrt{18}}{\sqrt{2}} \qquad 55. \sqrt{\frac{1}{x}}$$

$$56. \frac{2x}{5\sqrt{2x}} \qquad 57. \frac{3}{1-\sqrt{2}} \qquad 58. \frac{7}{3-\sqrt{2}} \qquad 59. \frac{11}{2\sqrt{3}+1} \qquad 60. \frac{4}{\sqrt{x}-1}$$

**Solve each equation, Check the solution; Rules 1- One radicals per side 2. Square both sides.**

$$61. \sqrt{x+2} = 5 \quad 62. \sqrt{x+12} = 3 \quad 63. \sqrt{3x+2} = -6 \quad 64. \sqrt{2x-5} = 3$$

$$65. \sqrt{6x+4} - 3 = 5 \quad 66. \sqrt{4x-2} = \sqrt{3x-1} \quad 67. \sqrt{2x-5} = \sqrt{3x-9} \quad 68. x+1 = 2\sqrt{x+4}$$

Write each expression as radical and then simplify;

$$69. 64^{\frac{1}{2}} \quad 70. -27^{\frac{1}{3}} \quad 71. (-32)^{\frac{1}{5}} \quad 72. 81^{\frac{1}{4}} \quad 73. 243^{\frac{1}{5}}$$

Simplify each expression

$$74. \left(\frac{4}{6}\right)^{\frac{1}{2}} \quad 75. 4^{\frac{3}{2}} \quad 76. (-27)^{\frac{2}{3}} \quad 77. x^{\frac{1}{4}} \cdot x^{\frac{1}{2}} \quad 78. \frac{x^{\frac{1}{2}}}{x^{\frac{3}{2}}}$$

### Additional problems

Simplify

$$79. -\sqrt{81x^4y^3} \quad 80. \sqrt[3]{24x^6y^3} \quad 81. \sqrt{\frac{4a^5}{9b^{10}}} \quad 82. \frac{\sqrt{96}+12}{24}$$

$$83. 2\sqrt{27x^2} + 5\sqrt{48x^2} - 4\sqrt{3x^2} \quad 84. \frac{21-\sqrt{98}}{7} \quad 85. \sqrt[3]{-40} + \sqrt[3]{108}$$

Multiply and simplify

$$86. 2\sqrt{50} - \sqrt{98} + 3\sqrt{8} \quad 87. 11\sqrt{2} + \sqrt{50} \quad 88. \sqrt[3]{-54} + 5\sqrt[3]{2} \quad 89. \sqrt{24}\sqrt{6} \quad 90. \sqrt{50x^4} - 2\sqrt{8x^4}$$

$$91. (\sqrt{3}-5)(\sqrt{3}+5) \quad 92. (\sqrt{2}+3)(\sqrt{2}+5) \quad 93. (2\sqrt{x}+3)(\sqrt{x}-8)$$

Rationalize:

$$94. \sqrt{\frac{3}{20}} \quad 95. \sqrt{\frac{5}{27}} \quad 96. \frac{2}{\sqrt{3}-5} \quad 97. \frac{6}{\sqrt{5}-2}$$

Solve the following equations;

$$98. \sqrt{7x+2} = 3 \quad 99. \sqrt{4x-3} = 5 \quad 100. \sqrt{5x-3} = \sqrt{2x+3} \quad 101. 2x-5 = \sqrt{13-6x}$$

Simplify,

$$102. 16^{\frac{3}{4}} \quad 103. \left(\frac{4}{9}\right)^{-\frac{3}{2}} \quad 104. x^{\frac{3}{4}} \cdot x^{\frac{1}{2}} \quad 105. \frac{x^{\frac{3}{4}}}{x^{\frac{1}{2}}}$$

1	$x^2\sqrt{x}$	36	$4x - 9$	71	$-2$
2	$x^6\sqrt{x}$	37	$16 + 12 - 8\sqrt{12} = 28 - 16\sqrt{3}$	72	3
3	$x^4\sqrt[3]{x}$	38	$3\sqrt{x} - x$	73	3
4	$x^7$	39	$-\sqrt{8}$	74	$2/3$
5	$x^2y^3\sqrt[3]{xy^2}$	40	$\sqrt{8}$	75	8
6	$x^5y^4\sqrt{xy}$	41	$\sqrt{7} + 4$	76	9
7	$x^2y^3\sqrt[4]{x^2y^2}$	42	$\sqrt{x} + \sqrt{y}$	77	$x^{\frac{3}{4}}$
8	$4\sqrt[3]{3}$	43	$\sqrt{5} - \sqrt{x}$	78	$x^{-1/6}$
9	$8x^2y^3\sqrt{2y}$	44	$4 + \sqrt{3}$	79	$-9x^2y\sqrt{y}$
10	$2xy^2\sqrt[4]{5x^3}$	45	$-8$	80	$2x^2y^4\sqrt[3]{3}$
11	$3x^3$	46	$-8$	81	$2a^2\sqrt{a} / 3b^5$
12	$\frac{1}{3}\sqrt{\frac{2}{x}} = \frac{\sqrt{2x}}{3x}$	47	$7 - 16 = -9$	82	$\sqrt{6} + 3/6$
13	$4\sqrt{2}$	48	$x - y$	83	$22x\sqrt{3}$
14	23.896	49	$5 - x$	84	$3 - \sqrt{2}$
15	1.985	50	$16 - 3 = 13$	85	$-2\sqrt[3]{5} + 3\sqrt[3]{4}$
16	.095	51	$3\sqrt{7} / 7$	86	$9\sqrt{2}$
17	13	52	$-4\sqrt{2}$	87	$16\sqrt{2}$
18	-22.361	53	$3\sqrt{2} / 2$	88	$2\sqrt[3]{2}$
19	Not a real number	54	3	89	12
20	155.47	55	$\sqrt{x} / x$	90	$x^2\sqrt{2}$
21	0.845	56	$\sqrt{2x} / 5$	91	$-22$
22	6.315	57	$-3(1 + \sqrt{2})$	92	$17 + 8\sqrt{2}$
23	-4.563	58	$3 + \sqrt{2}$	93	$2x - 13\sqrt{x} - 24$
24	$-3\sqrt{2}$	59	$2\sqrt{3} - 1$	94	$\sqrt{15} / 10$
25	$12\sqrt{2} - 8\sqrt{2} = 4\sqrt{2}$	60	$4(\sqrt{x} + 1) / (x - 1)$	95	$\sqrt{15} / 9$
26	$2\sqrt{5} + 6\sqrt{5} - 7\sqrt{5} = \sqrt{5}$	61	$x = 23$	96	$-(\sqrt{3} + 5) / 11$
27	$5\sqrt{2} + 4\sqrt{2} - 12\sqrt{2} = -3\sqrt{2}$	62	$x = -3$	97	$6(\sqrt{5} + 2)$
28	$5x^2\sqrt{2x} + 4x^2\sqrt{2x} = 9x^2\sqrt{2x}$	63	No Solution	98	$x = 1$
29	$8 + 5\sqrt[3]{x}$	64	$x = 7$	99	$x = 7$
30	9	65	$x = 10$	100	$x = 2$
31	$2\sqrt{15}$	66	$x = 1$	101	No Solution
32	$6\sqrt{2} + 8\sqrt{6}$	67	$x = 4$	102	8
33	3	68	$x = 5$	103	$27/8$
34	-15	69	8	104	$x^{1/4}$
35	-2	70	-3	105	$x^{5/4}$