

Factoring

Factoring is used in algebra, which is used in precalculus, which is used in calculus, which is used in just about every high-level math, which is used in real-life applications. So there you go... factoring is important to real-life applications

Main reason for factoring is to simplify algebraic expression to reduce fractions or solve equations.

$$\frac{x^2 + 4x + 3}{x + 1} = \frac{(x + 3)(x + 1)}{x + 1} = x + 3$$

$$x^3 + 4x^2 + 3x = 0$$

$$x(x^2 + 4x + 3) = x(x + 3)(x + 1) = 0$$

$$x = 0$$

$$x + 3 = 0 \quad \rightarrow \quad x = -3$$

$$x + 1 = 0 \quad \rightarrow \quad x = -1$$

Methods of Factoring

- 1) Basic(Always the first step to check) Look for greatest common Factor
- 2) Grouping
- 3) 4 terms(by grouping)
- 4) 3 terms $x^2 + bx + c$ into two binomial $(x + ?)(x + ?)$
- 5) 3 terms $ax^2 + bx + c$
- 6) Difference of 2 squares $x^2 - y^2$
- 7) **Zero Factor Property** $(x + a)(x + b) = 0$
- 8) Applications

1) Basic(look for greatest common factor)

$$\begin{array}{c} 6x^2 - 3x \\ \downarrow \\ 3x(2x - 1) \end{array}$$

$$\begin{array}{c} ax - bx \\ \downarrow \\ x(a - b) \end{array}$$

$$\begin{array}{c} 5x^6 - 10x^3 - 15x^2 \\ \downarrow \\ 5x^2(x^4 - 2x - 3) \end{array}$$

$$\begin{array}{c} 8a^4b^3 - 2a^3b^4 + 4ab \\ \downarrow \\ 2ab(4a^3b^2 - a^2b^3 + 2) \end{array}$$

$$6k^3 - 26k^4 - 48k^5 = ?$$

2) Grouping

$$\begin{array}{c} (x-1)(2x+1) + (x-1)(3x+2) \\ \downarrow \\ (x-1)(2x+1+3x+2) \end{array}$$

$$\begin{array}{c} (4x+5)(a+b) - (2x-1)(a+b) \\ \downarrow \\ (a+b)(4x+5-2x+1) \end{array}$$

$$(x-1)(5x+3)$$

$$(a+b)(2x+6) = (a+b)2(x+3) = 2(a+b)(x+3)$$

Practice

$$\begin{array}{c} 5(2-x)^2 - 2(2-x)^3 \\ \downarrow \end{array}$$

$$\begin{array}{c} (2m+3)(m+2) + (m-2)(m+2) \\ \downarrow \end{array}$$

3) 4 Terms(group two at the time)

$$\begin{array}{c} 10m + 2n + 5mk + nk \\ \downarrow \\ 2(5m+n) + k(5m+n) \\ \downarrow \\ (5m+n)(2+k) \end{array}$$

$$\begin{array}{c} 2a^3 + a^2 - 14a - 7 \\ \downarrow \\ a^2(2a+1) - 7(2a+1) \\ \downarrow \\ (2a+1)(a^2-7) \end{array}$$

Practice

$$2x - 4 + xy - 2y$$

↓

4) 3 terms $x^2 + bx + c$ can be factored into two binomial $(x + ?)(x + ?)$

a) $x^2 + 5x + 6 = (x + 3)(x + 2)$

b) $x^2 + x - 6$

c) $y^2 + 7y - 30$

d) $x^2 + 7xy - 30y^2$

e) $k^2 - 11k + 30$

f) $3x^2 - 6x - 45$

g) $x^2 + 6x + 6$

5) 3 terms $ax^2 + bx + c$ split the middle term and then factor by grouping

a) $3x^2 + 7x + 2 = 3x^2 + 6x + x + 2 = 3x(x + 2) + 1(x + 2) = (x + 2)(3x + 1)$

b) $2x^2 + 5x + 3$

c) $-6x^2 - x + 1$

d) $8x^2 + 6x - 2$

e) $12x^2 - 5x - 2$

6) Difference of 2 squares $x^2 - y^2$ factor it as $(x + y)(x - y)$

a) $x^2 - 36$

b) $x^2 - 16$

c) $2x^2 - 18$

d) $16m^2 - 49p^2$

e) $(x + y)^2 - (x - y)^2$

7) **Zero Factor Property** $(x+a)(x+b)=0$

Have zero on one side = 0

Factor the other side

Set each factor = 0 and solve (Hint **do not** put a factor that is a **number equal 0**)

$$x^2 + 5x + 6 = 0$$

$$(x+2)(x+3) = 0 \quad \rightarrow \quad (x+2) = 0 \quad (x+3) = 0$$
$$x = -2 \quad x = -3$$

$$2y^3 + 10y^2 = -12y \quad \rightarrow \quad 2y^3 + 10y^2 + 12y = 0 \quad \rightarrow \quad 2y(y^2 + 5y + 6) = y(y+2)(y+3) = 0$$

$$2y = 0 \quad y = 0$$

$$(y+2) = 0 \quad y = -2$$

$$(y+3) = 0 \quad y = -3$$

Practice: $(2x+1)(x+1) = 2(1-x) + 6$

8) Applications

- a) The area of a rectangular window is to be 36 square inch and perimeter is 26. Find its dimensions
- b) The area of a rectangular window is to be 45 square inch and perimeter is 28. Find its dimensions
- c) How many right triangles have a hypotenuse that measures $2x + 3$ meters and legs that have $2x - 5$ meters and $x + 7$ meters? What re the dimensions of these triangles?
- d) How many right triangles have a hypotenuse that measures $4x + 5$ meters and legs that have $3x + 13$ meters and x meters? What re the dimensions of these triangles?
- e) A small rocket is launched vertically upward and its height in feet after t seconds is a function defined by $h(t) = -16t^2 + 128t$ After how long it reaches 220 above the ground?

$$h(t) = -16t^2 + 128t = 220 \quad -16t^2 + 128t - 220 = 0 \quad -4(4t^2 - 32t + 55) = 0$$

$$(2t - 5)(2t - 11) = 0$$

$$(2t - 5) = 0 \quad \rightarrow \quad t = 5/2 = 2.5 \quad \rightarrow \quad (2t - 11) = 0 \quad \rightarrow \quad t = 11/2 = 5.5$$

Practice Problems

1. $15x^3 - 30x^2$

2. $100a^5 + 16a^3$

3. $x^6y^2 + 5x^4y^3 - 6xy^4 + 10xy$

4. $3a^3 + 3ab^2 + 2a^2b + 2b^3$

5. $c(x+2) - d(x+2)$

6. $x^2 + 15x + 44$

7. $x^2 + 6x - 27$

8. $x^2 - 14x + 24$

9. $n^2 - 12n - 35$

10. $x^2 - 2x - 3$

11. $7x^2 + 8x + 1$

12. $21m^2 + 13m + 2$

13. $10x^2 + 11x - 6$

14. $2m^3 + 2m^2 - 40m$

15. $6x^2 - 7x - 5$

16. $6x^2 - 7xy - 5y^2$

17. $x^2 - 10x + 25$

18. $2x^2 + 24x + 72$

19. $49x^2 - 28xy + 4y^2$

20. $16x^2 - 40x + 25$

21. $4x^2 - 9y^2$

22. $36x^2 - 1$

Solve each equation by **Zero Factor Property Method**

23. $x^2 + 6x - 27 = 0$

24. $2y^2 - 18 = 0$

25. $3x^2 + 5x - 2 = 0$

26. $x^2 = -5x + 24$

27. $m^2 + 8m = -16$

28. $36x^2 + 60x = -25$

**Answers
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1. $15x^2(x-2)$

2. $4a^3(25a^2+4)$

3. $xy(x^5y+5x^3y^2-6y^3+10)$

4. $3a(a^2+b^2)+2b(a^2+b^2)=(a^2+b^2)(3a+2b)$

5. $(x+2)(c-d)$

6. $(x+4)(x+11)$

7. $(x+9)(x-3)$

8. $(x-12)(x-2)$

9. *Prime*

10. $(x-3)(x+1)$

11. $(7x+1)(x+1)$

12. $(3m+1)(7m+2)$

13. $(2x+3)(5x-2)$

14. $2m(m-4)(m+5)$

15. $(3x-5)(2x+1)$

16. $(3x-5y)(2x+y)$

17. $(x-5)^2$

18. $2(x+6)^2$

19. $(7x-2y)^2$

20. $(4x-5)^2$

21. $(2x+3y)(2x-3y)$

22. $(6x+1)(6x-1)$

23. $(x+9)(x-3)=0$, $x=-9, 3$

24. $2(y+3)(y-3)=0$ $y=-3, 3$

25. $(3x-1)(x+2)=0$ $x=\frac{1}{3}, -2$

26. $(x+8)(x-3)=0$ $x=-8, 3$

27. $(m+4)^2=0$ $m=-4$

28. $(6x+5)^2=0$ $x=-\frac{5}{6}$

Not to know is bad; not to wish to know is worse.

- African proverb

Basic: First factor any common Number, Variable or Expression with the **smallest exponent**.

$$18x + 36 = 18(x + 2)$$

$$65y^{10} + 35y^6 = 5y^6(13y^4 + 7)$$

$$8m^3n + 24mn^3 = 8mn(m^2 + 3n^2)$$

$$13y^8 + 26y^4 - 39y^2 = 13y^2(y^6 + 2y^2 - 3)$$

$$p(p + 4) + 3(p + 4) = (p + 4)(p + 3)$$

Grouping: Take every two terms, factor each, and then **factor** each expression in ()

$$18r^2 + 12ry - 3xr - 2xy = 6r(3r + 2y) - x(3r + 2y) = (3r + 2y)(6r - x)$$

2 Terms $a^2 - b^2 = (a + b)(a - b)$ Both squared, and the **sign** between two terms **must** be **negative**

$$9x^2 - 25y^2 = (3x + 5y)(3x - 5y) \quad 16k^4 - 1 = (4k^2 + 1)(4k^2 - 1) = (4k^2 + 1)(2k + 1)(2k - 1)$$

$$9x^2 + 25y^2 = \text{Can not be factored because of the positive sign in between is positive.}$$

2 Terms **both cubed** $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$ **or** $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$8a^3 + 27 = (2a + 3)(4a^2 + 9 - 6a)$$

$$125a^3 - 64b^3 = (5a - 4b)(25a^2 + 16b^2 + 20b)$$

3 Terms $x^2 + bx + c$ when $Sum = b$ and $Product = c$

$$x^2 + 5x + 6 \quad S = 5 \quad P = 6 \quad 2, 3 \\ (x + 2)(x + 3)$$

$$x^2 - 2x - 15 \quad S = -2 \quad P = -15 \quad 3, -5 \\ (x + 3)(x - 5)$$

3 Terms $ax^2 + bx + c$ when $Sum = b$ and $Product = ac$

$$21m^2 + 13x + 2 \quad S = 13 \quad \text{and} \quad P = 21(2) = 42 \quad 7, 6$$

$$21m^2 + 7m + 6m + 2 = 7m(3m + 1) + 2(3m + 1) = (3m + 1)(7m + 2)$$

$$4x^2 + 3x - 10 \quad S = 3 \quad \text{and} \quad P = 4(-10) = -40 \quad 8, -5$$

$$4x^2 + 8x - 5x - 10 = 4x(x + 2) - 5(x + 2) = (x + 2)(4x - 5)$$

3 Terms $ax^2 + 2abx + c^2$ **Two Squared Terms (Check doubling effect)**

$$x^2 + 10x + 25 = (x + 5)^2$$

$$3x^2 - 48x + 192 = 3(x^2 - 16x + 64) = 3(x - 8)^2$$

$$49x^2 - 28xy + 9y^2 = (7x - 3y)^2$$

$$m^2 + \frac{2}{3}m + \frac{1}{9} = \left(m + \frac{1}{3}\right)^2$$

ZFP (Zero Factor Property) to **solve equations that can be factored.**

1. Have zero on one side.

2. Factor the other side.

3. Let each factor = 0 and solve.

$$x^2 - 3x = 4, \quad x^2 - 3x - 4 = 0$$

$$2. (x - 4)(x + 1) = 0,$$

$$(x - 4) = 0, \quad (x + 1) = 0 \\ x = 4, \quad x = -1$$

Practice Problems

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Answers

1. $15x^2(x-2)$

2. $4a^3(25a^2 + 4)$

3. $xy(x^5y + 5x^3y^2 - 6y^3 + 10)$

4. $3a(a^2 + b^2) + 2b(a^2 + b^2) = (a^2 + b^2)(3a + 2b)$

5. $(x+2)(c-d)$

6. $(x+4)(x+11)$

7. $(x+9)(x-3)$

8. $(x-12)(x-2)$

9. Prime

10. $(x-3)(x+1)$

11. $(7x+1)(x+1)$

12. $(3m+1)(7m+2)$

13. $(2x+3)(5x-2)$

14. $2m(m-4)(m+5)$

15. $(3x-5)(2x+1)$

16. $(3x-5y)(2x+y)$

17. $(x-5)^2$

18. $2(x+6)^2$

19. $(7x-2y)^2$

20. $(4x-5)^2$

21. $(2x+3y)(2x-3y)$

22. $(6x+1)(6x-1)$

23. $(x+9)(x-3) = 0$, $x = -9, 3$

24. $2(y+3)(y-3) = 0$ $y = -3, 3$

25. $(3x-1)(x+2) = 0$ $x = \frac{1}{3}, -2$

26. $(x+8)(x-3) = 0$ $x = -8, 3$

27. $(m+4)^2 = 0$ $m = -4$

28. $(6x+5)^2 = 0$ $x = -\frac{5}{6}$