10.1 Inverse Functions

Objectives

1. Decide whether a function is one-to-one and, if it is, find its inverse.
2. Use the horizontal line test to determine whether a function is one-to-one.
3. Find the equation of the inverse of a function.
4. Graph f^{-1} given the graph of f.

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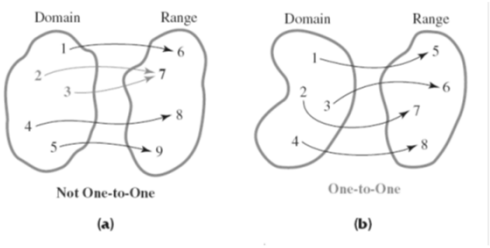
Decide whether a function is one-to-one and, if it is, find its inverse.

One-to-One Function

In a **one-to-one function,** each x-value corresponds to only one y-value, and each y-value corresponds to only one x-value.

Decide whether a function is one-to-one and, if it is, find its inverse.



Decide whether a function is one-to-one and, if it is, find its inverse.

Inverse of a Function

The **inverse** of a one-to-one function f, written f^{-1}, is the set of all ordered pairs of the form (y, x), where (x, y) belongs to f. Since the inverse is formed by interchanging x and y, the domain of f becomes the range of f^{-1} and the range of f becomes the domain of f^{-1}.

CAUTION The symbol $f^{-1}(x)$ does **not** represent $\dfrac{1}{f(x)}$.

CLASSROOM EXAMPLE 1 **Finding Inverses of One-to-One Functions**

Decide whether each function is one-to-one. If it is, find the inverse.

{(2, 5), (3, 6), (4, 8), (8, 7)}

Solution:

Each x-value corresponds to only one y-value, and every y-value corresponds to only one x-value. The function is one-to-one.

{(5, 2), (6, 3), (8, 4), (7, 8)}

{(0, 3), (−1, 2), (1, 3)}

Each x-value corresponds to just one y-value. However, the y-value 3 corresponds to both 0 and 1. The function is not one-to-one.

Use the horizontal line test to determine whether a function is one-to-one.

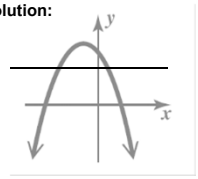
Horizontal Line Test

A function is one-to-one if every horizontal line intersects the graph of the function at most once.

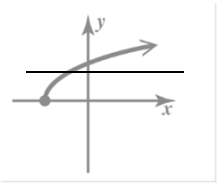
CLASSROOM EXAMPLE 2 **Using the Horizontal Line Test**

Use the horizontal line test to determine whether each graph is the graph of a one-to-one function.

Solution:



not a one-to-one function



one-to-one function

Find the equation of the inverse of a function.

Finding the Equation of the Inverse of $y = f(x)$

For a one-to-one function f defined by an equation $y = f(x)$, find the defining equation of the inverse as follows.

Step 1 Interchange x and y.

Step 2 Solve for y.

Step 3 Replace y with $f^{-1}(x)$.

CLASSROOM EXAMPLE 3 **Finding Equations of Inverses**

Decide whether the equation defines a one-to-one function. If so, find the equation that defines the inverse.

$f(x) = 3x - 4$

Solution:

The graph of $y = 3x - 4$ is a nonvertical line, so by the horizontal line test, f is a one-to-one function.

$$y = 3x - 4$$
$$x = 3y - 4$$
$$3y = x + 4$$
$$y = \frac{x+4}{3} \longrightarrow f^{-1}(x) = \frac{x+4}{3}$$

CLASSROOM EXAMPLE 3 **Finding Equations of Inverses (cont'd)**

Decide whether each equation defines a one-to-one function. If so, find the equation that defines the inverse.

$f(x) = x^3 + 1$

Solution:

The graph of a cubic equation is one-to-one.

$$y = x^3 + 1$$
$$x = y^3 + 1$$
$$y^3 = x - 1$$
$$y = \sqrt[3]{x-1} \longrightarrow f^{-1}(x) = \sqrt[3]{x-1}$$

$f(x) = (x - 3)^2$

The graph is a vertical parabola. The function is not one-to-one.

Objective 4

Graph f^{-1} from the graph of f.

CLASSROOM EXAMPLE 4 **Graphing the Inverse**

Use the given graph to graph the inverse of f.

Solution: