10.2 Exponential Functions

Objectives

1. Define an exponential function.
2. Graph an exponential function.
3. Solve exponential equations of the form $a^x = a^k$ for x.
4. Use exponential functions in applications involving growth or decay.

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Define an exponential function.

Exponential Function

For $a > 0$, $a \neq 1$, and all real numbers x,

$$f(x) = a^x$$

defines the **exponential function with base *a*.**

CAUTION The graph of an exponential function approaches the x-axis, but does ***not*** touch it.

Objective 2

Graph an exponential function.

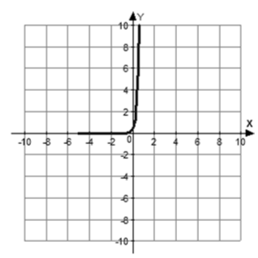
CLASSROOM EXAMPLE 1 **Graphing an Exponential Function ($a > 1$)**

Graph $f(x) = 10^x$.

Solution:

Choose values of x and find the corresponding values of y.

x	$f(x) = 10^x$
–2	0.01
–1	0.1
0	1
1	10
2	100



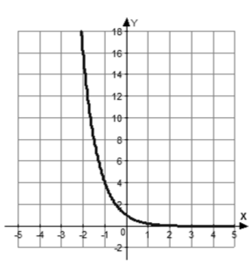
CLASSROOM EXAMPLE 2 **Graphing an Exponential Function ($0 < a < 1$)**

Graph $g(x) = (1/4)^x$.

Solution:

Choose values of x and find the corresponding values of y.

x	$g(x) = (1/4)^x$
–2	16
–1	4
0	1
1	1/4
2	1/16



Graph an exponential function.

Characteristics of the Graph of $f(x) = a^x$

1. The graph contains the point (0,1).
2. The function is one-to-one. When $a > 1$, the graph will ***rise*** from left to right. (*See* **Example 1**). When $0 < a < 1$, the graph will ***fall*** from left to right. (*See* **Example 2**). In both cases, the graph goes from the second quadrant to the first.
3. The graph will approach the x-axis, but never touch it (Recall from **Section 7.4** that such a line is called an ***asymptote*.)**
4. The domain is $(-\infty, \infty)$, and the range is $(0, \infty)$.

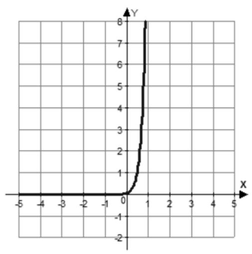
CLASSROOM EXAMPLE 3 — **Graphing a More Complicated Exponential Function**

Graph $g(x) = 2^{4x - 3}$.

Solution:

Choose values of x and find the corresponding values of y.

x	$4x - 3$	$y = 2^{4x-3}$
0	–3	1/8
3/4	0	1
1	1	2
–1	–7	1/128



Objective 3

Solve exponential equations of the form $a^x = a^k$ for x.

Solve exponential equations of the form $a^x = a^k$ for x.

An **exponential equation** is an equation that has a variable in an exponent, such as $9^x = 27$.

Property for Solving an Exponential Equation

For $a > 0$ and $a \neq 1$, if $a^x = a^y$ then $x = y$.

Solve exponential equations of the form $a^x = a^k$ for x.

Solving an Exponential Equation

Step 1 **Each side must have the same base.** If the two sides of the equation do not have the same base, express each as a power of the same base if possible.

Step 2 **Simplify exponents** if necessary, using the rules of exponents.

Step 3 **Set exponents equal** using the property given in this section.

Step 4 **Solve** the equation obtained in ***Step 3.***

CLASSROOM EXAMPLE 4 — **Solving an Exponential Equation**

Solve the equation $25^x = 125$.

Solution:

Step 1 **Write each side with the base 5.**

$$(5^2)^x = 5^3$$

Step 2 **Simplify exponents.**

$$5^{2x} = 5^3$$

Step 3 **Set the exponents equal.**

$$2x = 3$$

Step 4 **Solve.**

$$x = \frac{3}{2}$$

CLASSROOM EXAMPLE 4 — **Solving an Exponential Equation (cont'd)**

Check that the solution set is $\frac{3}{2}$ by substituting $\frac{3}{2}$ for x.

$$25^x = 25^{3/2} = \left(25^{1/2}\right)^3 = 5^3 = 125$$

The solution set is $\left\{\frac{3}{2}\right\}$.

CLASSROOM EXAMPLE 5 — **Solving Exponential Equations**

Solve the equation.

$25^{x-2} = 125^{x}$

Solution:

$(5^2)^{x-2} = (5^3)^x$

$5^{2(x-2)} = 5^{3x}$

$2(x - 2) = 3x$

$2x - 4 = 3x$

$-4 = x$

Check

$25^{-4-2} = 125^{-4}$

$25^{-6} = 125^{-4}$

$4.096 \times 10^{-9} = 4.096 \times 10^{-9}$

True

The solution set is {–4}.

CLASSROOM EXAMPLE 5 — **Solving Exponential Equations (cont'd)**

Solve the equation.

$4^x = \frac{1}{32}$

Solution:

$\left(2^2\right)^x = \frac{1}{2^5}$

$\left(2^2\right)^x = 2^{-5}$

$2x = -5$

$x = -\frac{5}{2}$

Check

$4^{-5/2} = \frac{1}{4^{5/2}} = \frac{1}{2^5} = \frac{1}{32}$

True

The solution set is $\left\{-\frac{5}{2}\right\}$.

CLASSROOM EXAMPLE 5 — **Solving Exponential Equations (cont'd)**

Solve the equation.

$\left(\frac{3}{4}\right)^x = \frac{16}{9}$

Solution:

$\left(\frac{3}{4}\right)^x = \left(\frac{4}{3}\right)^2$

$\left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^{-2}$

$x = -2$

Check

$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

True

The solution set is $\{-2\}$.

Objective 4

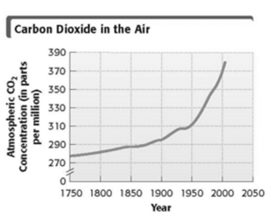
Use exponential functions in applications involving growth or decay.

CLASSROOM EXAMPLE 6 — **Solving an Application Involving Exponential Growth**

Use the function to approximate, to the nearest unit, the carbon dioxide concentration in 2007 which was 381 parts per million? (*Source:* U.S. Department of Energy.)

Solution:

The function is defined by $f(x) = 266(1.001)^x$, where x is the number of years since 1750.



Source: Sacramento Bee; National Oceanic and Atmospheric Administration.

$x = 2007 - 1750 = 257$

$f(257) = 266(1.001)^{257}$

$f(257) = 343.91$

≈ 344 parts per million

CLASSROOM EXAMPLE 7 — **Applying an Exponential Decay Function**

The atmospheric pressure (in millibars) at a given altitude x, in meters, can be approximated by the function defined by $f(x) = 1038(1.000134)^{-x}$.

Use the function to find the pressure at 8000m.

Solution:

$f(x) = 1038(1.000134)^{-x}$

$f(8000) = 1038(1.000134)^{-8000}$

≈ 355

The pressure is approximately 355 millibars.