10.3 Logarithmic Functions

Objectives

1. Define a logarithm.
2. Convert between exponential and logarithmic forms.
3. Solve logarithmic equations of the form $\log_a b = k$ for a, b, or k.
4. Define and graph logarithmic functions.
5. Use logarithmic functions in applications involving growth or decay.

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Define a logarithm.

Logarithm

For all positive numbers a, with $a \neq 1$, and all positive numbers x,
$y = \log_a x$ means the same as $x = a^y$.

Exponent

Logarithmic form: $y = \log_a x$ ← Base

Exponent

Exponent form: $x = a^y$ ← Base

Define a logarithm.

Meaning of $\log_a x$

A logarithm is an exponent. ***The expression $\log_a x$ represents the exponent to which the base a must be raised to obtain x.***

Convert between exponential and logarithmic forms.

The table shows several pairs of equivalent forms.

Exponential Form	Logarithmic Form
$3^2 = 9$	$\log_3 9 = 2$
$\left(\frac{1}{5}\right)^{-2} = 25$	$\log_{1/5} 25 = -2$
$10^5 = 100,000$	$\log_{10} 100,000 = 5$
$4^{-3} = \frac{1}{64}$	$\log_4 \frac{1}{64} = -3$

CLASSROOM EXAMPLE 1 **Converting Between Exponential and Logarithmic Forms**

Fill in the blanks with the equivalent forms.

Solution:

Exponential Form	Logarithmic Form
$2^5 = 32$	$\log_2 32 = 5$
$100^{\frac{1}{2}} = 10$	$\log_{100} 10 = \frac{1}{2}$
$8^{\frac{2}{3}} = 4$	$\log_8 4 = \frac{2}{3}$
$6^{-4} = \frac{1}{1296}$	$\log_6 \frac{1}{1296} = -4$

CLASSROOM EXAMPLE 2 **Solving Logarithmic Equations**

Solve logarithmic equations of the form $\log_a b = k$ for a, b, or k

$\log_3 x = -3$ Solve the equation.

Solution:

$$x = 3^{-3}$$
$$x = \frac{1}{3^3} = \frac{1}{27}$$

The input of the logarithm must be a positive number.

The solution set is $\left\{\frac{1}{27}\right\}$.

CLASSROOM EXAMPLE 2 Solving Logarithmic Equations (cont'd)

Solve the equation.

$\log_{1/3}(2x+5) = 2$

Solution:

$$2x+5 = \left(\frac{1}{3}\right)^2$$
$$2x+5 = \frac{1}{9}$$
$$18x+45 = 1$$
$$18x = -44$$
$$x = -\frac{44}{18} = -\frac{22}{9}$$

The solution set is $\left\{-\frac{22}{9}\right\}$.

CLASSROOM EXAMPLE 2 Solving Logarithmic Equations (cont'd)

Solve the equation.

$\log_x 5 = 4$

Solution:

$$x^4 = 5$$
$$x = \pm\sqrt[4]{5}$$

Reject the negative solution since the base of a logarithm must be positive and not equal to 1. The solution set is $\left\{\sqrt[4]{5}\right\}$.

CLASSROOM EXAMPLE 2 Solving Logarithmic Equations (cont'd)

Solve the equation.

$\log_{36}\sqrt[4]{6} = x$

Solution:

$$36^x = \sqrt[4]{6}$$
$$\left(6^2\right)^x = 6^{1/4}$$
$$6^{2x} = 6^{1/4}$$
$$2x = \frac{1}{4}$$
$$x = \frac{1}{8}$$

The solution set is $\left\{\frac{1}{8}\right\}$.

Solve logarithmic equations of the form $\log_a b = k$ for *a, b,* or *k.*

Properties of Logarithms

For any positive real number *b*, with $b \neq 1$, the following are true

$\log_b b = 1$ and $\log_b 1 = 0$.

CLASSROOM EXAMPLE 3 Using Properties of Logarithms

Evaluate each logarithm.

Solution:

$\log_{12} 12 = 1$

$\log_{\sqrt{3}} \sqrt{3} = 1$

$\log_3 1 = 0$

$\log_{0.4} 1 = 0$

Define and graph logarithmic functions.

Logarithmic Function

If *a* and *x* are positive numbers, with $a \neq 1$, then

$g(x) = \log_a x$

defines the **logarithmic function with base *a*.**

CLASSROOM EXAMPLE 4 — **Graphing a Logarithmic Function ($a > 1$)**

Graph $f(x) = \log_4 x$.

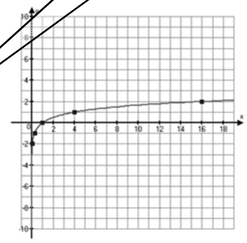
Solution:

Write in exponential form as $x = 4^y$.

Make a list of ordered pairs.

Be sure to write the *x* and *y* values in the correct order.

$x = 4^y$	y
1/16	–2
1/4	–1
1	0
4	1
16	2



CLASSROOM EXAMPLE 5 — **Graphing a Logarithmic Function ($0 < a < 1$)**

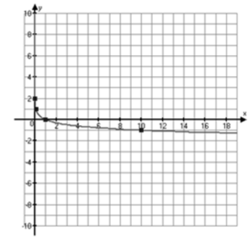
Graph $f(x) = \log_{1/10} x$.

Solution:

Write in exponential form as $x = \left(\frac{1}{10}\right)^y$.

Make a list of ordered pairs.

$x = (1/10)^y$	y
10	–1
1	0
1/10	1
1/100	2



Define and graph logarithmic functions.

Characteristics of the Graph of $g(x) = \log_a x$

1. The graph contains the point (1, 0).

2. The function is one-to-one. When $a > 1$, the graph will ***rise*** from left to right, from the fourth quadrant to the first. When $0 < a < 1$, the graph will ***fall*** from left to right, from the first quadrant to the fourth quadrant.

3. The graph will approach the *y*-axis, but never touch it. (The *y*-axis is an asymptote.)

4. The domain is $(0, \infty)$, and the range is $(-\infty, \infty)$.

CLASSROOM EXAMPLE 6 — **Solving an Application of Logarithmic Function**

Use logarithmic functions in applications involving growth or decay.

A population of mites in a laboratory is growing according to the function defined by $P(t) = 80 \log_{10}(t + 10)$, where t is the number of days after a study is begun.

Find the number of mites at the beginning of the study.

Find the number present after 90 days.

CLASSROOM EXAMPLE 6 — **Solving an Application of Logarithmic Function (cont'd)**

$P(t) = 80 \log_{10}(t + 10)$

Find the number of mites at the beginning of the study.

Solution:

$$\begin{aligned} P(0) &= 80\log_{10}(0+10) \\ &= 80\log_{10} 10 \\ &= 80(1) \\ &= 80 \end{aligned}$$

The number of mites at the beginning of the study is 80.

CLASSROOM EXAMPLE 6 — **Solving an Application of Logarithmic Function (cont'd)**

$P(t) = 80 \log_{10}(t + 10)$

Find the number present after 90 days.

Solution:

$$\begin{aligned} P(90) &= 80\log_{10}(90+10) \\ &= 80\log_{10} 100 \\ &= 80(2) \\ &= 160 \end{aligned}$$

The population after 90 days is 160 mites.