10.4 Properties of Logarithms

Objectives

1 Use the product rule for logarithms.

2 Use the quotient rule for logarithms.

3 Use the power rule for logarithms.

4 Use properties to write alternative forms of logarithmic expressions.

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Product Rule for Logarithms

If x, y, and b are positive real numbers, where $b \neq 1$, then the following are true.

$$\log_b xy = \log_b x + \log_b y$$

That is, the logarithm of a product is the sum of the logarithm of the factors.

The word statement of the product rule can be restated by replacing "logarithm" with "exponent." The rule then becomes the familiar rule for multiplying exponential expressions: The ***exponent*** of a product is the sum of the ***exponents*** of the factors.

CLASSROOM EXAMPLE 1 **Using the Product Rule**

Use the product rule to rewrite each logarithm.

Solution:

$\log_4(3\cdot 7) \quad = \log_4 3 + \log_4 7$

$\log_8 10 + \log_8 3 \quad = \log_8(10\cdot 3) \quad = \log_8 30$

$\log_8 8k, \quad k>0 \quad = \log_8 8 + \log_8 k \quad = 1 + \log_8 k$

$\log_5 m^2, \quad m>0 \quad = \log_5(m\cdot m)$

$= \log_5 m + \log_5 m$

$= 2\log_5 m$

Use the quotient rule for logarithms.

Quotient Rule for Logarithms

If x, y, and b are positive real numbers, where $b \neq 1$, then the following is true.

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

That is, the logarithm of a quotient is the difference between the logarithm of the numerator and the logarithm of the denominator.

CLASSROOM EXAMPLE 2 **Using the Quotient Rule**

Use the quotient rule to rewrite each logarithm.

Solution:

$\log_7 \frac{9}{4} \quad = \log_7 9 - \log_7 4$

$\log_3 p - \log_3 q, \quad \text{p>0, q>0} \quad = \log_3 \frac{p}{q}$

$\log_4 \frac{3}{16} \quad = \log_4 3 - \log_4 16$

$= \log_4 3 - 2$

Objective 3

Use the power rule for logarithms.

Use the power rule for logarithms.

Power Rule for Logarithms

If *x* and *b* are positive real numbers, where $b \neq 1$, and if *r* is any real number then the following is true.

$$\log_b x^r = r \log_b x$$

That is, the logarithm of a number to a power equals the exponent times the logarithm of the number.

CLASSROOM EXAMPLE 3 **Using the Power Rule**

Use the power rule to rewrite each logarithm. Assume $a > 0$, $b > 0$, $x > 0$, $a \neq 1$, and $b \neq 1$.

Solution:

$$\log_3 5^2 = 2\log_3 5$$

$$\log_a x^4 = 4\log_a x$$

$$\log_b \sqrt{8} = \log_b 8^{1/2} = \frac{1}{2}\log_b 8$$

$$\log_2 \sqrt[7]{x^4} = \log_2 x^{4/7} = \frac{4}{7}\log_2 x$$

Use the power rule for logarithms.

Special Properties

If $b > 0$ and $b \neq 1$, then the following are true.

$$b^{\log_b x} = x, \quad x > 0 \quad \text{and} \quad \log_b b^x = x$$

CLASSROOM EXAMPLE 4 **Using the Special Properties**

Use the special properties to find each value.

Solution:

$$\log 10^5 = 5$$

$$\log_2 8 = \log_2 2^3 = 3$$

$$5^{\log_5 9} = 9$$

Use the power rule for logarithms.

Properties of Logarithms

If *x*, *y*, and *b* are positive real numbers, where $b \neq 1$, and *r* is any real number, then the following are true.

Product Rule	$\log_b xy = \log_b x + \log_b y$
Quotient Rule	$\log_b \frac{x}{y} = \log_b x - \log_b y$
Power Rule	$\log_b x^r = r\log_b x$
Special Properties	$b^{\log_b x} = x$ and $\log_b b^x = x$

Objective 4

Use properties to write alternative forms of logarithmic expressions.

CLASSROOM EXAMPLE 5 — **Writing Logarithms in Alternative Forms**

Use the properties of logarithms to rewrite each expression if possible. Assume all variable represent positive real numbers.

Solution:

$$\log_6 36m^5 = \log_6 36 + \log_6 m^5 = 2 + 5\log_6 m$$

$$\log_2 \sqrt{9z} = \tfrac{1}{2}\log_2(9z) = \tfrac{1}{2}(\log_2 9 + \log_2 z) = \tfrac{1}{2}\log_2 3^2 + \tfrac{1}{2}\log_2 z = \tfrac{1}{2}(2\log_2 3) + \tfrac{1}{2}\log_2 z = \log_2 3 + \tfrac{1}{2}\log_2 z$$

CLASSROOM EXAMPLE 5 — **Writing Logarithms in Alternative Forms (cont'd)**

Use the properties of logarithms to rewrite the expression if possible. Assume all variable represent positive real numbers.

$$\log_b \frac{8r^2}{m-1} \qquad (m > 1, b \neq 1)$$

Solution:

$= \log_b(8r^2) - \log_b(m-1)$ **Quotient rule**

$= \log_b 8 + \log_b r^2 - \log_b(m-1)$ **Product rule**

$= \log_q 8 + 2\log_q r - \log_q(m-1)$ **Power rule**

CLASSROOM EXAMPLE 5 — **Writing Logarithms in Alternative Forms (cont'd)**

Use the properties of logarithms to rewrite each expression if possible. Assume all variable represent positive real numbers.

$$2\log_b x + 3\log_b y \qquad (b \neq 1)$$

Solution:

$= \log_b x^2 + \log_b y^3$ **Power rule**

$= \log_b x^2 y^3$ **Product rule**

$$\log_4(3x + y)$$

Cannot be rewritten.

CLASSROOM EXAMPLE 6 — **Using the Properties of Logarithms with Numerical Values**

Given that $\log_2 5 = 2.3219$ and $\log_2 3 = 1.5850$, evaluate the following.

Solution:

$$\log_2 25 = \log_2(5 \cdot 5) = \log_2 5 + \log_2 5 = 2.3219 + 2.3219 = 4.6438$$

$$\log_2 \frac{5}{3} = \log_2 5 - \log_2 3 = 2.3219 - 1.5850 = 0.7369$$

CLASSROOM EXAMPLE 6 — **Using the Properties of Logarithms with Numerical Values (cont'd)**

$$\log_2 75$$

Solution:

$$= \log_2(3 \cdot 5^2) = \log_2 3 + \log_2 5^2 = \log_2 3 + 2\log_2 5 = 1.5850 + 2(2.3219) = 6.2288$$

CLASSROOM EXAMPLE 7 — **Deciding Whether Statements about Logarithms are True**

Decide whether each statement is ***true*** or ***false.***

Solution:

$$\log_6 36 - \log_6 6 = \log_6 30$$

Left side $= \log_6 36 - \log_6 6 = \log_6 \frac{36}{6} = \log_6 6 = 1$

Right side $= \log_6 30$ **False** $\log_6 30 \neq 1.$

$$\log_4(\log_2 16) = \frac{\log_6 6}{\log_6 36}$$

Left side $= \log_4(\log_2 16) = \log_4(\log_2 2^4) = \log_4 4 = 1$

Right side $= \dfrac{\log_6 6}{\log_6 36} = \dfrac{1}{\log_6 6^2} = \dfrac{1}{2}$ **False 1 ≠ ½.**