10.5 Common and Natural Logarithms

Objectives

1 Evaluate common logarithms using a calculator.

2 Use common logarithms in applications.

3 Evaluate natural logarithms using a calculator.

4 Use natural logarithms in applications.

5 Use the change-of-base rule.

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Objective 1

Evaluate common logarithms using a calculator.

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We use calculators to evaluate common logarithms.

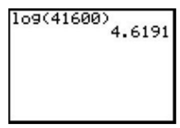
We give most approximations for logarithms to four decimal places.

CLASSROOM EXAMPLE 1 **Evaluating Common Logarithms**

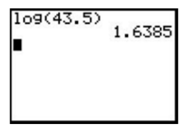
Evaluate each logarithm to four decimal places using a calculator.

Solution:

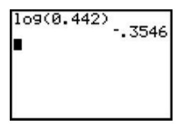
log 41,600



log 43.5



log 0.442



Objective 2

Use common logarithms in applications.

CLASSROOM EXAMPLE 2 **Using pH in an Application**

Find the pH of water with a hydronium ion concentration of 1.2×10^{-3}. If it is taken from a wetland, is the wetland a rich fen, a poor fen, or a bog?

Solution:

$$\begin{aligned} pH &= -\log\left[H_3O^+\right] \\ &= -\log(1.2 \times 10^{-3}) \\ &= -(\log 1.2 + \log 10^{-3}) \\ &= -(\log 1.2 - 3\log 10) \\ &= -[0.0792 - 3(1)] \\ &= -0.0792 + 3 \\ &= 2.9208 \approx 2.9 \end{aligned}$$

Since the pH is less than 3.0, the wetland is a bog.

CLASSROOM EXAMPLE 3 **Finding Hydronium Ion Concentration**

Find the hydronium ion concentration of a solution with pH 4.6.

Solution:

$$\text{pH} = -\log\left[H_3O^+\right]$$
$$4.6 = -\log[H_3O^+]$$
$$-4.6 = \log[H_3O^+]$$
$$-4.6 = \log_{10}[H_3O^+]$$
$$10^{-4.6} = [H_3O^+]$$
$$2.5\times10^{-5} \approx [H_3O^+]$$

Use common logarithms in applications.

The loudness of sound is measured in a unit called a **decibel,** abbreviated **dB**.

To measure with this unit, we first assign an intensity of I_0 to a very faint sound, called the **threshold sound.**

If a particular sound has intensity I, then the decibel level of this louder sound is

$$D = 10\log\left(\frac{I}{I_0}\right).$$

Use common logarithms in applications.

Decibel Level	Example
60	Normal conversation
90	Rush hour traffic, lawn mower
100	Garbage truck, chain saw, pneumatic drill
120	Rock concert, thunderclap
140	Gunshot blast, jet engine
180	Rock launching pad

Source: Deafness Research Foundation

CLASSROOM EXAMPLE 4 **Measuring the Loudness of Sound**

Find the decibel level to the nearest whole number of a whisper with intensity I of $115I_0$.

Solution:

$$D = 10\log\left(\frac{I}{I_0}\right)$$
$$= 10\log\left(\frac{115I_0}{I_0}\right)$$
$$= 10\log 115$$
$$\approx 10(2.06) \approx 21$$

The level is about 21 dB.

Evaluate natural logarithms using a calculator.

Logarithms used in applications are often **natural logarithms,** which have as base the number ***e.***

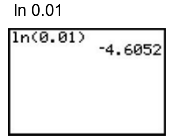
The number *e*, (≈ 2.718281828), like *π*, is a **universal constant.** Since it is an irrational number, its decimal expansion never terminates and never repeats.

CLASSROOM EXAMPLE 5 **Evaluating Natural Logarithms**

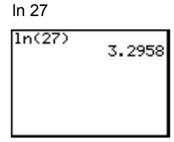
Find each logarithm to four decimal places using a calculator.

Solution:

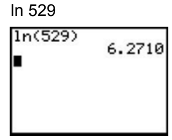
ln 0.01



ln 27



ln 529



Objective 4

Use natural logarithms in applications.

CLASSROOM EXAMPLE 6 **Applying a Natural Logarithm Function**

The altitude in meters that corresponds to an atmospheric pressure of *x* millibars is given by the logarithmic function defined by *f*(*x*) = 51,600 – 7457 ln *x*. Approximate the altitude at 700 millibars of pressure.

Solution:

$$f(x) = 51{,}600 - 7457 \ln x$$

$$f(700) = 51{,}600 - 7457 \ln 700$$

$$\approx 2748.6 \quad \approx 2700$$

Atmospheric pressure is 700 millibars at approximately 2700 m.

Objective 5

Use the change-of-base rule.

Use the change-of-base rule.

Change of Base Rule

If $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$, and $x > 0$, then the following are true.

$$\log_a x = \frac{\log_b x}{\log_b a}$$

CLASSROOM EXAMPLE 7 **Use the Change-of-Base Rule**

Evaluate $\log_3 17$ to four decimal places.

Solution:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$\log_3 17 = \frac{\log_{10} 17}{\log_{10} 3}$$

$$= \frac{\log 17}{\log 3}$$

$$\approx 2.5789$$

CLASSROOM EXAMPLE 8 **Using the Change-of-Base Rule in an Application**

Use natural logarithms in the change-of-base rule and the function *f*(*x*) = 2014 + 384.7 $\log_2 x$ to find to estimate crude oil imports into the United States in 2008. Compare this to the actual amount of 3571 million barrels.

In the equation, *x* = 1 represents 1990.

Solution:

$$f(x) = 2014 + 384.7 \log_2 x$$

$$f(19) = 2014 + 384.7 \ \log_2 19$$

$$= 2014 + 384.7\left(\frac{\log 19}{\log 2}\right)$$

$$= 2014 + 384.7(4.2478)$$

$$\approx 3648$$

The model indicates total crude oil imports of 3648 million barrels in 2006, which is greater that the actual amount of 3571.