1.1 Basic Concepts

Objectives

1 Write sets using set notation.

2 Use number lines.

3 Know the common sets of numbers.

4 Find the additive inverses.

5 Use absolute value.

6 Use inequality symbols.

7 Graph sets of real numbers.

Write sets using set notation.

A **set** is a collection of objects called the **elements** or **members** of the set.

In algebra, the elements of a set are usually numbers. Set braces, { }, are used to enclose the elements.

Since we can count the number of elements in the set {1, 2, 3}, it is a **finite set.**

The set ***N* = {1, 2, 3, 4, 5, 6…}** is called the **natural numbers,** or **counting numbers**.

The three dots (***ellipsis* points**) show that the list continues in the same pattern indefinitely.

We cannot list all the elements of the set of natural numbers, so it is an **infinite set.**

Write sets using set notation.

When 0 is included with the set of natural numbers, we have the set of **whole numbers**, written

***W* = {0, 1, 2, 3, 4, 5, 6…}.**

The set containing no elements, such as the set of whole numbers less than 0, is called the **empty set,** or **null set**, usually written ∅ or { }.

To write the fact that 2 is an element of the set {1, 2, 3}, we use the symbol ∈ (read "is an element of").

$$2 \in \{1, 2, 3\}$$

CAUTION Do not write {∅} for the empty set. {∅} is a set with one element: ∅. Use the notation ∅ or { } for the empty set.

Write sets using set notation.

Two sets are equal if they contain exactly the same elements. For example, {1, 2} = {2, 1} (Order does not matter.)

{0, 1, 2} ≠ {1, 2} (≠ means "is not equal to"), since one set contains the element 0 while the other does not.

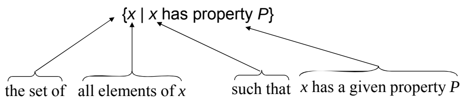
Letters called **variables** are often used to represent numbers or to define sets of numbers. For example,

{*x* | *x* is a natural number between 3 and 15}

(read "the set of all elements *x* such that *x* is a natural number between 3 and 15"} defines the set {4, 5, 6, 7, …14}.

Write sets using set notation.

The notation {*x* | *x* is a natural number between 3 and 15} is an example of **set-builder notation.**



{*x* | *x* has property *P*}

the set of

all elements of *x*

such that

x has a given property *P*

CLASSROOM EXAMPLE 1

Listing the Elements in Sets

List the elements in { *x* | *x* is a natural number greater than 12 }.

Solution:

{13, 14, 15, …}

CLASSROOM EXAMPLE 2

Using Set-Builder Notation to Describe Sets

Use set builder notation to describe the set.

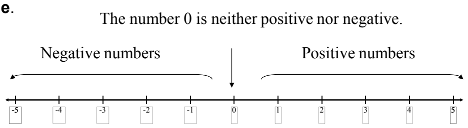
{ 0, 1, 2, 3, 4, 5 }

Solution:

{ *x* | *x* is a whole number less than 6 }

Use number lines.

A good way to get a picture of a set of numbers is to use a **number line**.

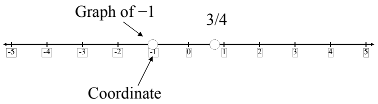


The set of numbers identified on the number line above, including positive and negative numbers and 0, is part of the set of **integers**, written

***I* = {…, –3, –2, –1, 0, 1, 2, 3…}.**

Use number lines.

Each number on a number line is called the **coordinate** of the point that it labels while the point is the **graph** of the number.



The fraction ¾ graphed on the number line is an example of a ***rational number***. A **rational number** can be expressed as the quotient of two integers, with denominator not 0. The set of all rational numbers is written

$$\left\{\frac{p}{q} \mid p \text{ and } q \text{ are integers, } q \neq 0\right\}.$$

Use number lines.

The set of rational numbers includes the natural numbers, whole numbers, and integers, since these numbers can be written as fractions.

For example, $20 = \frac{20}{1}.$

A rational number written as a fraction, such as ½ or 1/8, can also be expressed as a decimal by dividing the numerator by the denominator.

Decimal numbers that neither terminate nor repeat are ***not*** rational numbers and thus are called **irrational numbers.**

For example,

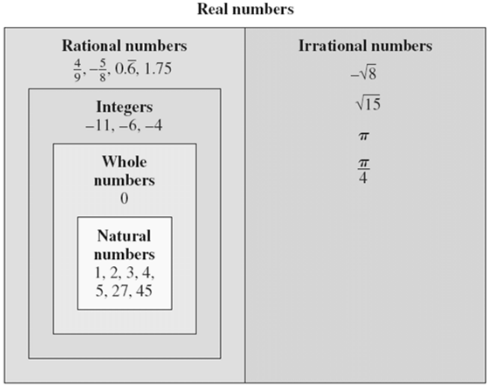
$$\sqrt{2} = 1.414213562\ldots \text{ and } -\sqrt{7} = -2.6457513\ldots$$

Know the common sets of numbers.

Sets of Numbers

Natural numbers, or counting numbers	{1, 2, 3, 4, 5, 6, …}
Whole numbers	{0, 1, 2, 3, 4, 5, 6, …}
Integers	{…, –3, –2, –1, 0, 1, 2, 3, …}
Rational numbers	$\left\{\frac{p}{q} \mid p \text{ and } q \text{ are integers, } q \neq 0\right\}.$
Irrational numbers	{*x* | *x* is a real number that is not rational}
Real numbers	{*x* | *x* is a rational number or an irrational number}

Use number lines.



CLASSROOM EXAMPLE 3

Identifying Examples of Number Sets

Select all the sets from the following list that apply to each number.

–7 3.14 $\sqrt{4}$

Solution:

–7

whole number
[✓ integer]
[✓ rational number]
irrational number
[✓ real number]

CLASSROOM EXAMPLE 3

Identifying Examples of Number Sets (cont'd)

Select all the sets from the following list that apply to each number.

–7 3.14 $\sqrt{4}$

Solution:

3.14

whole number
integer
[✓ rational number]
irrational number
[✓ real number]

CLASSROOM EXAMPLE 3

Identifying Examples of Number Sets (cont'd)

Select all the sets from the following list that apply to each number.

–7 3.14 $\sqrt{4}$

Solution:

$\sqrt{4} = 2$

[✓ whole number]
[✓ integer]
[✓ rational number]
irrational number
[✓ real number]

CLASSROOM EXAMPLE 4

Determining Relationships Between Sets of Numbers

Decide whether the statement is ***true*** or ***false.*** If it is false, tell why.

Solution:

Some integers are whole numbers.

true

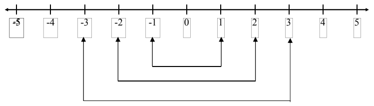
Every real number is irrational.

false; some real numbers are irrational, but others are rational numbers.

Find the additive inverses.

Additive Inverse

For any real number *a*, the number –*a* is the **additive inverse** of *a*.



Additive inverses (opposite)

Change the sign of a number to get its additive inverse. The sum of a number and it additive inverse is always 0.

Find the additive inverses.

Uses of the Symbol –

The symbol "–" is used to indicate any of the following:

1. **a negative number,** such as –9 or –15;
2. **the additive inverse of a number,** as in "– 4 is the additive inverse of 4";
3. **subtraction,** as in 12 – 3.

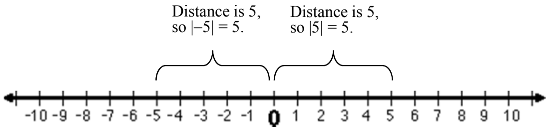
– (–*a*)

For any real number *a*, – (–*a*) = *a*.

Use absolute value.

The **absolute value** of a number *a*, written |*a*|, is the distance on a number line from 0 to *a*.

For example, the absolute value of 5 is the same as the absolute value of –5 because each number lies 5 units from 0.



Use absolute value.

Absolute Value

For any real number *a*, $$|a| = \begin{cases} a & \text{if } a \text{ is positive or } 0 \\ -a & \text{if } a \text{ is negative.} \end{cases}$$

Because absolute value represents distance, and distance is never negative, the absolute value of a number is always positive or 0.

CLASSROOM EXAMPLE 5

Finding Absolute Value

Simplify by finding each absolute value.

Solution:

$|-3|$ **$= 3$**

$-|3|$ **$= -3$**

$-|-3|$ **$= -3$**

$|8| + |-1|$ **$= 8 + 1$** **$= 9$**

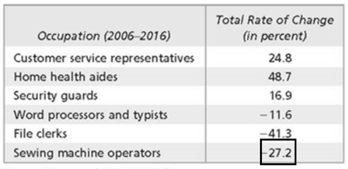
$|8 - 1|$ **$= |7|$** **$= 7$**

CLASSROOM EXAMPLE 6

Comparing Rates of Change in Industries

Of the customer service representatives and sewing machine operators, which will show the greater change (without regard to sign)?

Solution:



Occupation (2006–2016)	Total Rate of Change (in percent)
Customer service representatives	24.8
Home health aides	48.7
Security guards	16.9
Word processors and typists	−11.6
File clerks	−41.3
Sewing machine operators	$\boxed{-27.2}$

Source: Bureau of Labor Statistics.

Look for the number with the largest absolute value.

sewing machine operators

Use inequality symbols.

The statement

$4 + 2 = 6$

is an **equation** — a statement that two quantities are equal.

The statement

$4 \neq 6$ (read "4 is not equal to 6")

is an **inequality** — a statement that two quantities are ***not*** equal.

The symbol **<** means "is less than."

$8 < 9$, $-7 < 16$, $-8 < -2$, and $0 < 4/3$

The symbol **>** means "is greater than."

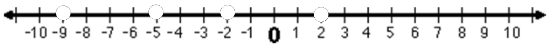
$13 > 8$, $8 > -2$, $-3 > -7$ and $5/3 > 0$

Use inequality symbols.

Inequalities on a Number Line

On a number line,
$a < b$ if *a* is to the left of *b*; **$a > b$** if *a* is to the right of *b*.

You can use a number line to determine order.



$-9 < 2$ $-2 > -5$ or $-5 < -2$

Be careful when ordering negative numbers.

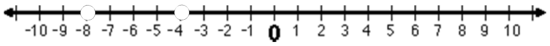
CLASSROOM EXAMPLE 7

Determining Order on a Number Line

Use a number line to determine whether each statement is ***true*** or ***false.***

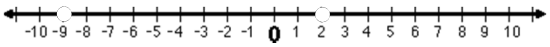
Solution:

$-8 > -4$



False

$-9 < 2$



True

Use inequality symbols.

In addition to the symbols ≠, **<**, and **>**, the symbols ≤ and ≥ are often used.

Symbol	Meaning	Example
≠	is not equal to	3 ≠ 7
<	is less than	–4 **<** –1
>	is greater than	3 **>** –2
≤	is less than or equal to	6 ≤ 6
≥	is greater than or equal to	–8 ≥ –10

CLASSROOM EXAMPLE 8

Using Inequality Symbols

Determine whether each statement is ***true*** or ***false.***

	Solution:
–2 ≤ –3	**false**
–1 ≥ –9	**true**
8 ≤ 8	**true**
3(4) > 2(6)	**false**

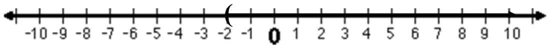
Graph sets of real numbers.

Inequality symbols and variables are used to write sets of real numbers. For example, the set

$$\{x \mid x > -2\}$$

consists of all the real numbers greater than –2.

On a number line, we graph the elements of this set by drawing an arrow from –2 to the right. We use a parenthesis at –2 to indicate that –2 is ***not*** an element of the given set.



The set of numbers greater than –2 is an example of an **interval** on the number line.

Graph sets of real numbers.

To write intervals, use **interval notation**.

The interval of all numbers greater than –2, would be (–2, ∞).

The **infinity symbol (∞)** does not indicate a number; it shows that the interval includes all real numbers greater than –2.

The left parenthesis indicated that –2 is not included.
A parenthesis is always used next to the infinity symbol.

The set of real numbers is written in interval notation as (–∞, ∞).

CLASSROOM EXAMPLE 9

Graphing an Inequality Written in Interval Notation

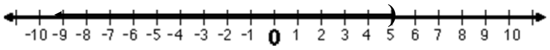
Write in interval notation and graph.

{*x* | *x* < 5}

Solution:

The interval is the set of all real numbers less than 5.

(–∞, 5)



CLASSROOM EXAMPLE 10

Graphing an Inequality Written in Interval Notation

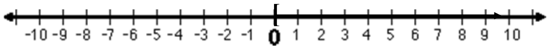
Write in interval notation and graph.

$\{x \mid x \geq 0\}$

Solution:

The interval is the set of all real numbers greater than or equal to 0. We use a square bracket [since 0 is part of the set.

$[0, \infty)$



Graph sets of real numbers.

Sometimes we graph sets of numbers that are ***between*** two given numbers.

For example: $\{x \mid 2 < x < 8\}$

This is called a **three-part inequality**, is read “2 is less than *x* and *x* is less than 8” or “*x* is between 2 and 8.”

CLASSROOM EXAMPLE 11

Graphing a Three-Part Inequality

Write in interval notation and graph.

$\{x \mid x - 4 \leq x < 2\}$

Solution:

Use a square bracket at –4.

Use a parenthesis at 2.

$[-4, 2)$