11.1 Additional Graphs of Functions

Objectives

1 Recognize the graphs of the elementary functions defined by $|x|$, $\frac{1}{x}$, and $\sqrt{x}$, and graph their translations.

2 Recognize and graph step functions.

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Objective 1

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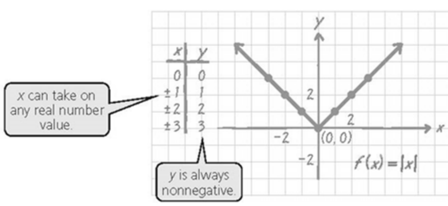
The elementary function defined by $f(x) = |x|$ is called the **absolute value function.**

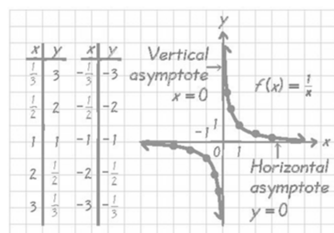


Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Recognize the graphs of the elementary functions defined by $|x|$, $\frac{1}{x}$, and $\sqrt{x}$, and graph their translation.

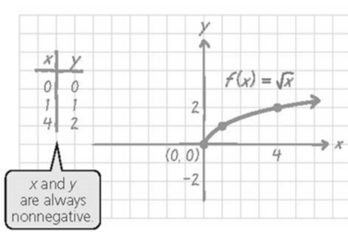
The **reciprocal function,** defined by $f(x) = 1/x$, is a rational function. The axes are **asymptotes** for the function.



Domain and Range are both $(-\infty, 0) \cup (0, \infty)$.

Recognize the graphs of the elementary functions defined by $|x|$, $\frac{1}{x}$, and $\sqrt{x}$, and graph their translation.

The **square root function,** defined by $f(x) = \sqrt{x}$. We restrict the function values to be real numbers, x cannot take on negative values.



Domain: $[0, \infty)$

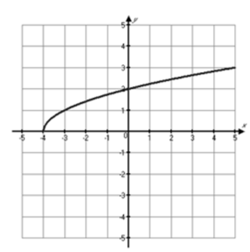
Range: $[0, \infty)$

CLASSROOM EXAMPLE 1 Applying a Horizontal Shift

Graph $f(x) = \sqrt{x+4}$. Give the domain and range.

Solution:

The graph is found by shifting the graph of the square root function, 4 units to the left.



The domain is $[-4, \infty)$.
The range is $[0, \infty)$.

CLASSROOM EXAMPLE 2 — **Applying a Vertical Shift**

Graph $f(x)=\frac{1}{x}-2$. Give the domain and range.

Solution:

The graph is found by shifting the graph of the reciprocal function 2 units down.

Since $x \neq 0$, the line $x = 0$ is a vertical asymptote.
The domain is $(-\infty, 0) \cup (0, \infty)$.

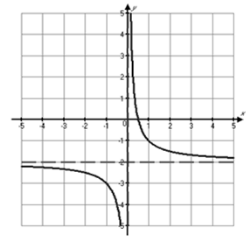
The line $y = -2$ is a horizontal asymptote.
The range is $(-\infty, -2) \cup (-2, \infty)$.

CLASSROOM EXAMPLE 2 — **Applying a Vertical Shift (cont'd)**

Graph $f(x)=\frac{1}{x}-2$.

Table of values.

x	1/*x*	*y*
–2	–1/2	–5/2
–1	–1	–3
–1/2	–2	–4
1/2	2	0
1	1	–1
2	1/2	–3/2



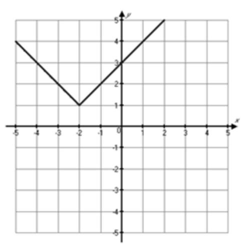
CLASSROOM EXAMPLE 3 — **Applying Both Horizontal and Vertical Shifts**

Graph $f(x)=|x+2|+1$. Give the domain and range.

Solution:

The graph is shifted 2 units left and 1 unit up.

x	*y*
–4	3
–3	2
–2	1
–1	2
0	3



The domain is $(-\infty, \infty)$.
The range is $[1, \infty)$.

Recognize and graph step functions.

$$f(x)=[\![x]\!]$$

The **greatest integer function,** written $f(x)=[\![x]\!]$, pairs every real number *x* with the greatest integer less than or equal to *x.*

CLASSROOM EXAMPLE 4 — **Finding the Greatest Integer**

Evaluate each expression.

Solution:

$[\![12]\!] = 12$ $\qquad$ $[\![-6.2]\!] = -7$

$[\![3.7]\!] = 3$ $\qquad$ $[\![1\tfrac{1}{2}]\!] = 2$

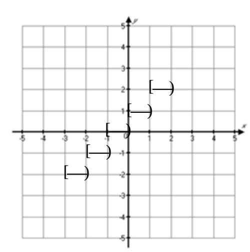
$[\![-9]\!] = -9$ $\qquad$ $[\![\pi]\!] = 3$

CLASSROOM EXAMPLE 5 — **Graphing the Greatest Integer Function**

Graph $f(x)=[\![x+1]\!]$. Give the domain and range.

Solution:

The domain is $(-\infty,\infty)$.



The range is the set of integers {…, –2, –1, 0, 1, 2, …}.

CLASSROOM EXAMPLE 6

Applying a Greatest Integer Function

Assume that the post office charges 80 cents per oz (or fraction of an ounce) to mail a letter to Europe. Graph the ordered pairs (ounces, cost) in the interval (0, 4].

Solution:

This function is similar to the greatest integer function, but in this case, we use the integer that is ***greater than*** or equal to the number.

Interval	Ounces Charged for	Cost
(0, 1]	1	$0.80
(1, 2]	2	$1.60
(2, 3]	3	$2.40
(3, 4]	4	$3.20

CLASSROOM EXAMPLE 6

Applying a Greatest Integer Function (cont'd)

Interval	Ounces Charged for	Cost
(0, 1]	1	$0.80
(1, 2]	2	$1.60
(2, 3]	3	$2.40
(3, 4]	4	$3.20