11.2 The Circle and the Ellipse

Objectives

1 Find an equation of a circle given the center and radius.

2 Determine the center and radius of a circle given its equation.

3 Recognize an equation of an ellipse.

4 Graph ellipses.

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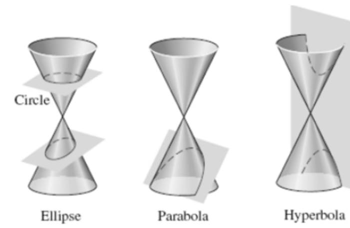
The Circle and the Ellipse

When an infinite cone is intersected by a plane, the resulting figure is called a **conic section**. The parabola is one example of a conic section; circles, ellipses, and hyperbolas may also result.



Find an equation of a circle given the center and radius.

A **circle** is the set of all points in a plane that lie a fixed distance from a fixed point.

The fixed point is called the **center**, and the fixed distance is called the **radius.**

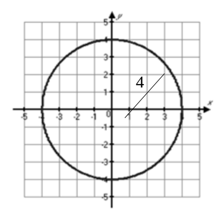
CLASSROOM EXAMPLE 1 **Finding an Equation of a Circle and Graphing It**

Find an equation of the circle with radius 4 and center at (0, 0), and graph it.

Solution:

If the point (x, y) is on the circle, the distance from (x, y) to the center (0, 0) is 4. Use the distance formula.

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=d$$
$$\sqrt{(x-0)^2+(y-0)^2}=4$$
$$\sqrt{x^2+y^2}=4$$
$$x^2+y^2=16$$



The equation of this circle is $x^2 + y^2 = 16$.

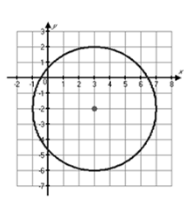
CLASSROOM EXAMPLE 2 **Finding an Equation of a Circle and Graphing It**

Find an equation of the circle with center at (3, –2) and radius 4, and graph it.

Solution:

If the point (x, y) is on the circle, the distance from (x, y) to the center (0, 0) is 4. Use the distance formula.

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=d$$
$$\sqrt{(x-3)^2+[(y-(-2)]^2}=4$$
$$\sqrt{(x-3)^2+(y+2)^2}=4$$
$$(x-3)^2+(y+2)^2=16$$



Find an equation of a circle given the center and radius.

Equation of a Circle (Center-Radius Form)

An equation of a circle with radius r and center at (h, k) is

$$(x-h)^2+(y-k)^2=r^2.$$

CLASSROOM EXAMPLE 3 — **Using the Center-Radius Form of the Equation of a Circle**

Find an equation of the circle with center at (2, –1) and radius $\sqrt{10}$.

Solution:

Substitute h = 2, k = –1, and r = 3.

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-2)^2+[(y-(-1)]^2=\sqrt{10}^2$$

$$(x-2)^2+(y+1)^2=10$$

CLASSROOM EXAMPLE 4 — **Completing the Square to Find the Center and Radius**

Find the center and radius of the circle $x^2 + y^2 + 6x - 4y - 51 = 0$.

Solution:

Complete the squares on x and y.

$$(x^2+6x)+(y^2-4y)=51$$

$$(x^2+6x+9)+(y^2-4y+4)=51+9+4$$

$$(x+3)^2+(y-2)^2=64$$

$$[x-(-3)]^2+(y-2)^2=8^2$$

The circle has center at (–3, 2) and radius 8.

Recognize an equation of an ellipse.

An **ellipse** is the set of all points in a plane the ***sum*** of whose distances from two fixed points is constant. These fixed points are called **foci** (singular: ***focus***).

Equation of an Ellipse

The ellipse whose x-intercepts are (a, 0) and (–a, 0) and whose y-intercepts are (0, b) and (0, –b) has an equation of the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

Graphs of circles and ellipses are not graphs of functions. The only conic section whose graph represents a function is the vertical parabola with equation $f(x)=ax^2+bx+c.$

CLASSROOM EXAMPLE 5 — **Graphing Ellipses**

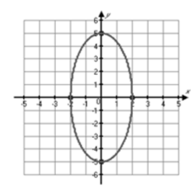
Graph $\frac{x^2}{4}+\frac{y^2}{25}=1.$

Solution:

a^2 = 4, so a = 2 and the x-intercepts are (2, 0) and (–2, 0).

b^2 = 25, so b = 5 and the y-intercepts are (0, 5) and (0, –5).

Plot the intercepts, and draw the ellipse through them.



CLASSROOM EXAMPLE 6 — **Graphing an Ellipse Shifted Horizontally and Vertically**

Graph $\frac{(x+4)^2}{16}+\frac{(y-1)^2}{36}=1.$

Solution:

$$\frac{[x-(-4)]^2}{4^2}+\frac{(y-1)^2}{6^2}=1$$

The center is (–4, 1).

The ellipse passes through the four points:

(–4 – 4, 1) = (–8, 1)
(–4 + 4, 1) = (0, 1)
(–4, 1 + 6) = (–4, 7)
(–4, 1 – 6) = (–4, –5)

CLASSROOM EXAMPLE 6 — **Graphing an Ellipse Shifted Horizontally and Vertically (cont'd)**

The center is (–4, 1).

The ellipse passes through the four points:

(–8, 1)
(0, 1)
(–4, 7)
(–4, –5)