11.4 **Nonlinear Systems of Equations**

Objectives

1 Solve a nonlinear system by substitution.

2 Solve a nonlinear system by elimination.

3 Solve a nonlinear system that requires a combination of methods.

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Nonlinear Systems of Equations

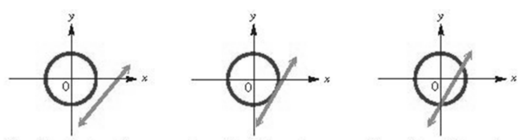
An equation in which some terms have more than one variable or a variable of degree 2 or greater is called a **nonlinear equation.**

A **nonlinear system of equations** includes at least one nonlinear equation.

Nonlinear Systems of Equations

When solving a nonlinear system, it helps to visualize the types of graphs of the equations of the system to determine the possible number of points of intersection.

For example, if a system includes two equations where the graph of one is a circle and the graph of the other is a line, then there may be zero, one, or two points of intersection.



No points of intersection One point of intersection Two points of intersection

CLASSROOM EXAMPLE 1 **Solving a Nonlinear System by Substitution**

Solve the system $x^2 - 2y^2 = 8$

$x + y = 6.$

Solution:

Solve equation (2) for *y.*

y = 6 – *x*

Substitute 6 – *x* for *y* in equation (1).

$$x^2 - 2y^2 = 8$$
$$x^2 - 2(6-x)^2 = 8$$
$$x^2 - 2(x^2 - 12x + 36) = 8$$
$$x^2 - 2x^2 + 24x - 72 = 8$$
$$-x^2 + 24x - 80 = 0$$
$$x^2 - 24x + 80 = 0$$

$$(x-4)(x-20) = 0$$
$$x - 4 = 0 \qquad x - 20 = 0$$
$$x = 4 \qquad x = 20$$

CLASSROOM EXAMPLE 1 **Solving a Nonlinear System by Substitution (cont'd)**

$$x^2 - 2y^2 = 8$$
$$x + y = 6$$

Using the equation *y* = 6 – *x,* find *y*.

x = 4	*x* = 20
y = 6 – 4	*y* = 6 – 20
y = 2	*y* = –14
(4, 2)	(20, –14)

Check each ordered pair in the equations.

The solution set is {(4, 2) and (20, –14)}.

CLASSROOM EXAMPLE 2 **Solving a Nonlinear System by Substitution**

Solve the system $xy + 10 = 0$

$4x + 9y = -2.$

Solution:

Solve equation (1) for *y.* $y = -\dfrac{10}{x}$

Substitute for *y* in equation (2).

$$4x + 9y = -2$$
$$4x + 9\left(\frac{-10}{x}\right) = -2$$
$$4x - \frac{90}{x} = -2$$

$$4x^2 - 90 = -2x$$
$$4x^2 + 2x - 90 = 0$$
$$2x^2 + x - 45 = 0$$
$$(2x-9)(x+5) = 0$$
$$x = \frac{9}{2} \qquad x = -5$$

CLASSROOM EXAMPLE 2 **Solving a Nonlinear System by Substitution (cont'd)**

Using the equation $y = -\frac{10}{x}$, find *y*.

$$xy + 10 = 0$$
$$4x + 9y = -2$$

x = 9/2

$$y = -\frac{10}{x}$$
$$y = -\frac{10}{\frac{9}{2}}$$
$$y = -\frac{20}{9}$$
$$\left(\frac{9}{2}, \frac{-20}{9}\right)$$

x = –5

$$y = -\frac{10}{x}$$
$$y = -\frac{10}{(-5)}$$
$$y = 2$$
$$(-5, 2)$$

Check each ordered pair in the equations.

The solution set is

$$\left\{\left(\frac{9}{2}, \frac{-20}{9}\right), (-5, 2)\right\}.$$

CLASSROOM EXAMPLE 3 **Solving a Nonlinear System by Elimination**

Solve the system $x^2 - 5y^2 = 4$

$$x^2 - 3y^2 = 6.$$

Solution:

Multiply equation (1) by –1 and add the result to equation (2).

$$x^2 - 5y^2 = 4 \longrightarrow -x^2 + 5y^2 = -4$$
$$x^2 - 3y^2 = 6 \qquad\qquad \underline{x^2 - 3y^2 = 6}$$
$$2y^2 = 2$$
$$y^2 = 1$$
$$y = 1 \qquad y = -1$$

CLASSROOM EXAMPLE 3 **Solving a Nonlinear System by Elimination (cont'd)**

Substitute 1 for y^2 in equation (1).

$$x^2 - 5y^2 = 4$$
$$x^2 - 3y^2 = 6.$$

$$x^2 - 5y^2 = 4$$
$$x^2 - 5(1) = 4$$
$$x^2 = 9$$
$$x = 3 \qquad x = -3$$

Check: If $x = \pm 3$, $x^2 = 9$, and if $y = \pm 1$, $y^2 = 1$.

Thus in any case, we get 9 – 5 = 4 in (1) and 9 – 3 = 6 in (2).
All *four* ordered pairs check.

The solution set is {(3, 1), (–3, 1), (–3, –1), (3, –1)}.

Objective 3

Solve a nonlinear system that requires a combination of methods.

CLASSROOM EXAMPLE 4 **Solving a Nonlinear System by a Combination of Methods**

Solve the system $x^2 + 7xy - 2y^2 = -8$

$$-2x^2 + 4y^2 = 16.$$

Solution:

Multiply equation (1) by 2 and add the result to equation (2).

$$x^2 + 7xy - 2y^2 = -8 \longrightarrow 2x^2 + 14xy - 4y^2 = -16$$
$$-2x^2 + 4y^2 = 16 \qquad\qquad \underline{-2x^2 \qquad + 4y^2 = 16}$$
$$14xy = 0$$
$$xy = 0$$

If *xy* = 0, then either *x* = 0 or *y* = 0.

CLASSROOM EXAMPLE 4 **Solving a Nonlinear System by a Combination of Methods (cont'd)**

If *x* = 0, then substitute 0 for *x* in equation (1).

$$x^2 + 7xy - 2y^2 = -8$$
$$-2x^2 + 4y^2 = 16$$

$$x^2 + 7xy - 2y^2 = -8$$
$$0 + 0 - 2y^2 = -8$$
$$y^2 = 4$$
$$y = \pm 2$$

If *y* = 0, then substitute 0 for *y* in equation (1).

$$x^2 + 7xy - 2y^2 = -8$$
$$x^2 + 0 - 0 = -8$$
$$x^2 = -8$$
$$x = \pm 2i\sqrt{2}$$

The solution set is {(0, 2), (0, –2),

$$\left(2i\sqrt{2}, 0\right), \left(-2i\sqrt{2}, 0\right)\}.$$