11.5 Second-Degree Inequalities and Systems of Inequalities

Objectives

1 Graph second-degree inequalities.

2 Graph the solution set of a system of inequalities.

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Graph second-degree inequalities.

A **second-degree inequality** is an inequality with at least one variable of degree 2 and no variable with degree greater than 2.

CLASSROOM EXAMPLE 1 **Graphing a Second-Degree Inequality**

Graph $x^2 + y^2 \geq 25$.

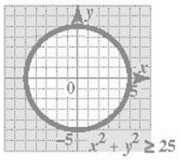
Solution:

The boundary of the inequality is the graph of the equation $x^2 + y^2 = 25$, a circle with radius 5 and center at the origin.

The inequality $x^2 + y^2 \geq 25$ will include either the points outside the circle or the points inside the circle, as well as the boundary. To determine which region to shade, we use a test point not on the circle.

Use test point (0, 0) from inside the circle.

$$x^2 + y^2 \geq 25$$
$$0 + 0 \geq 25$$
$$0 \geq 25 \quad \text{False}$$



False, shade the region that does not contains (0, 0).

CLASSROOM EXAMPLE 2 **Graphing a Second-Degree Inequality**

Graph $y \geq (x+1)^2 - 5$.

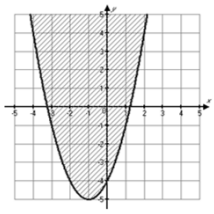
Solution:

The boundary is a parabola that opens upward with vertex at (–1, – 5).

It is a solid curve.

Use (0, 0) as a test point.

$$y \geq (x+1)^2 - 5$$
$$0 \geq (0+1)^2 - 5$$
$$0 \geq 1 - 5$$
$$0 \geq -4$$



True, shade the region that contains (0, 0).

CLASSROOM EXAMPLE 3 **Graphing a Second-Degree Inequality**

Graph $x^2 + 4y^2 > 36$.

Solution:

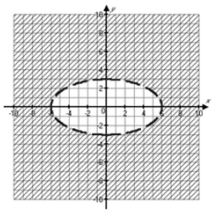
The boundary is an ellipse.

The line is dashed.

Intercepts (6, 0), (–6, 0), (0, 3), and (0, –3).

Use (0, 0) as a test point.

$$\frac{x^2}{36} + \frac{y^2}{9} > 1$$
$$x^2 + 4y^2 > 36.$$
$$0 + 0 > 36$$



False, shade the region that does not contain (0, 0).

Graph the solution set of a system of inequalities.

If two or more inequalities are considered at the same time, we have a **system of inequalities.** To find the solution set of the system, we find the intersection of the graphs (solution sets) of the inequalities in the system.

CLASSROOM EXAMPLE 4 **Graphing a System of Two Inequalities**

Graph the solution set of the system $y \ge x^2 - 3$

$y \le -x^2 + 3.$

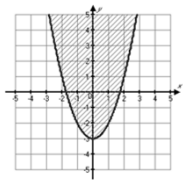
Solution:

For the first equation, the boundary is an upward opening parabola with vertical axis $x = 0$ and vertex $(0, -3)$.

It is a solid curve.

Use (0, 0) as a test point.

$$y \ge x^2 - 3$$
$$0 \ge 0^2 - 3$$
$$0 \ge -3$$



True, shade the region that contains (0, 0).

CLASSROOM EXAMPLE 4 **Graphing a System of Two Inequalities (cont'd)**

Graph the solution set of the system $y \ge x^2 - 3$

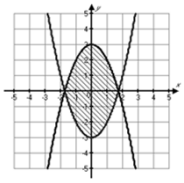
$y \le -x^2 + 3.$

For the second equation, the boundary is a downward opening parabola with vertical axis $x = 0$ and vertex $(0, 3)$.

It is a solid curve.

Use (0, 0) as a test point.

$$y \le -x^2 + 3$$
$$0 \le -0^2 + 3$$
$$0 \le 3$$



True, shade the region that contains (0, 0).

CLASSROOM EXAMPLE 5 **Graphing a Linear System of Three Inequalities**

Graph the solution set of the system $2x - 5y \ge 10$

$x + 3y \ge 6$

$y \le 2.$

Solution:

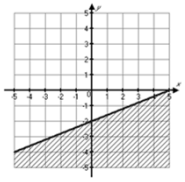
$2x - 5y$ is a solid line with intercepts (5, 0) and $(0, -2)$.

Test (0, 0).

$2(0) - 5(0) \ge 10$

$0 \ge 10$ False

Shade the side of the line that does not contain (0, 0).



CLASSROOM EXAMPLE 5 **Graphing a Linear System of Three Inequalities (cont'd)**

$2x - 5y \ge 10$

$x + 3y \ge 6$

$y \le 2.$

$x + 3y$ is a solid line with intercepts (6, 0) and (0, 2).

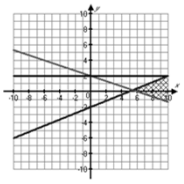
Test (0, 0).

$(0) + 5(0) \ge 6$

$0 \ge 6$ False

Shade the side of the line that does not contain (0, 0).

$y \le 2$ is a solid horizontal line through (0, 2).



Shade the region below the line.

CLASSROOM EXAMPLE 6 **Graphing a System of Three Inequalities**

Graph the solution set of the system $y \ge x^2 + 1$

$\frac{x^2}{9} + \frac{y^2}{4} \ge 1$

$y \le 5$

Solution:

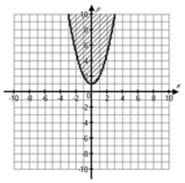
$x^2 + 1$ is a parabola open upward with vertex (0, 1).

Test (0, 0).

$0 \ge 0 + 1$

$0 \ge 1$ False

Shade the side of the parabola that does not contain (0, 0).



CLASSROOM EXAMPLE 6 **Graphing a System of Three Inequalities (cont'd)**

$y \ge x^2 + 1$

$\frac{x^2}{9} + \frac{y^2}{4} \ge 1$

$y \le 5$

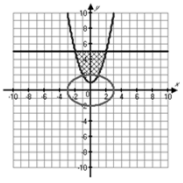
$\frac{x^2}{9} + \frac{y^2}{4} \ge 1$ is an ellipse with intercepts (3, 0), (–3, 0), (0, 2), and (0, –2).

Test (0, 0).

$\frac{0^2}{9} + \frac{0^2}{4} \ge 1$

$0 \ge 1$ False

Shade the region that does not contain (0, 0).



$y \le 5$ is a solid horizontal line through (0, 5).

Shade the region below the line.