1.2 Operations on Real Numbers

Objectives

1. Add real numbers.
2. Subtract real numbers.
3. Find the distance between two points on a number line.
4. Multiply real numbers.
5. Find reciprocals and divide real numbers.

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Add real numbers.

Recall that the answer to an addition problem is called the **sum.**

Adding Real Numbers

Same Sign To add two numbers with the ***same*** sign, add their absolute values. The sum has the same sign as the given numbers.

Different Signs To add two numbers with ***different*** signs, find the absolute values of the numbers, and subtract the smaller absolute value from the larger. The sum has the same sign as the number with the larger absolute value.

CLASSROOM EXAMPLE 1 Adding Two Negative Real Numbers

Find each sum.

$-6 + (-15)$

Solution:
Find the absolute values.

$|-6| = 6 \quad |-15| = 15$

Because they have the same sign, add their absolute values.

$-6 + (-15) = -(6 + 15)$ **Add the absolute values.**

$= -(21)$
$= -21$

Both numbers are negative, so the answer will be negative.

CLASSROOM EXAMPLE 1 Adding Two Negative Real Numbers (cont'd)

$-1.1 + (-1.2)$

Solution:
$= -(1.1 + 1.2)$
$= -2.3$

$-\frac{3}{4} + \left(-\frac{5}{8}\right) = -\left(\frac{3}{4} + \frac{5}{8}\right)$ **Add the absolute values. Both numbers are negative, so the answer will be negative.**

$= -\left(\frac{6}{8} + \frac{5}{8}\right)$ **The least common denominator is 8.**

$= -\frac{11}{8}$ **Add numerators; keep the same denominator.**

CLASSROOM EXAMPLE 2 Adding Real Numbers with Different Signs

Find each sum.

$3 + (-7)$

Solution:
Find the absolute values.
$|3| = 3 \quad |-7| = 7$

Because they have *different* signs, subtract their absolute values. The number –7 has the larger absolute value, so the answer is negative.

$3 + (-7) = -4$

The sum is negative because $|-7| > |3|$

CLASSROOM EXAMPLE 2 Adding Real Numbers with Different Signs (cont'd)

$-3 + 7 = 7 - 3$ **Solution:**
$= 4$

$-3.8 + 4.6 = 4.6 - 3.8$
$= 0.8$

$-\frac{3}{8} + \frac{1}{4} = -\frac{3}{8} + \frac{2}{8}$

$= -\left(\frac{3}{8} - \frac{2}{8}\right)$ **Subtract the absolute values. −3/8 has the larger absolute value, so the answer will be negative.**

$= -\frac{1}{8}$ **Subtract numerators; keep the same denominator.**

Subtract real numbers.

Recall that the answer to a subtraction problem is called the **difference.**

Subtraction

For all real numbers a and b,

$$a - b = a + (-b).$$

That is, to subtract b from a, add the additive inverse (or opposite) of b to a.

CLASSROOM EXAMPLE 3 **Subtracting Real Numbers**

Find each difference.

Solution:

$12 - (-5)$

$= 12 + 5$ Change to addition. The additive inverse of -5 is 5.

$= 17$

$-11.5 - (-6.3)$

$= -15.5 + 6.3$ Change to addition. The additive inverse of -6.3 is 6.3.

$= -5.2$

CLASSROOM EXAMPLE 3 **Subtracting Real Numbers (cont'd)**

$$\frac{3}{4} - \left(-\frac{2}{3}\right) = \frac{3}{4} + \frac{2}{3}$$ To subtract, add the additive inverse (opposite).

$$= \frac{3 \cdot 3}{4 \cdot 3} + \frac{2 \cdot 4}{3 \cdot 4}$$ Write each fraction with the least common denominator, 12.

$$= \frac{9}{12} + \frac{8}{12}$$ Add numerators; keep the same denominator.

$$= \frac{17}{12}$$

CLASSROOM EXAMPLE 4 **Adding and Subtracting Real Numbers**

Perform the indicated operations.

$-6 - (-2) - 8 - 1$ **Work from left to right.**

Solution:

$= (-6 + 2) - 8 - 1$

$= -4 - 8 - 1$

$= -12 - 1$

$= -13$

$-3 - [(-7) + 15] - 6$ **Work inside brackets.**

$= -3 - [8] - 6$

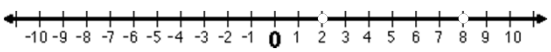
$= -11 - 6$

$= -17$

Find the distance between two points on a number line.

To find the distance between the points 2 and 8, we subtract $8 - 2 = 6$. Since distance is always positive, we must be careful to subtract in such a way that the answer is positive.

Or, to avoid this problem altogether, we can find the absolute value of the difference. Then the distance is either $|8 - 2| = 6$ or $|2 - 8| = 6$.



CLASSROOM EXAMPLE 5 **Finding Distance Between Points on the Number Line**

Find the distance between the points -12 and -1.

Solution:

Find the absolute value of the difference of the numbers, taken in either order.

$|-12 - (-1)| = 11$

or

$|-1 - (-12)| = 11$

Multiply real numbers.

Recall that the answer to a multiplication problem is called the **product.**

Multiplying Real Numbers

Same Sign The product of two numbers with the *same* sign is positive.

Different Signs The product of two numbers with *different* signs is negative.

CLASSROOM EXAMPLE 6 **Multiplying Real Numbers**

Find each product.

	Solution:	
$7(-2)$	$= -14$	**Different signs; product is negative.**
$-0.9(-15)$	$= 13.5$	**Same signs; product is positive.**
$-\frac{5}{8}(16)$	$= -10$	**Different signs; product is negative.**

Find reciprocals and divide real numbers.

The definition of division depends on the idea of a **multiplicative inverse,** or ***reciprocal.***

Reciprocal

The reciprocal of a nonzero number a is $\frac{1}{a}$.

A number and its additive inverse have opposite signs. However, a number and its reciprocal always have the same sign.

Find reciprocals and divide real numbers.

Reciprocals have a product of 1.

Number	Reciprocal
$-\frac{2}{5}$	$-\frac{5}{2}$
-6	$-\frac{1}{6}$
$\frac{7}{11}$	$\frac{11}{7}$
0.05	20
0	None

Division by 0 is undefined, whereas dividing 0 by a nonzero number gives the quotient 0.

Find reciprocals and divide real numbers.

The result of dividing one number by another is called the **quotient.**

Division

For all real numbers a and b (where $b \neq 0$),

$$a \div b = \frac{a}{b} = a \cdot \frac{1}{b}.$$

That is, multiply the first number (the **dividend**) by the reciprocal of the second number (the **divisor**).

Dividing Real Numbers

Same Sign The quotient of two nonzero real numbers with the *same* sign is positive.

Different Signs The product of two nonzero real numbers with *different* signs is negative.

CLASSROOM EXAMPLE 7 **Dividing Real Numbers**

Find each quotient.

Solution:

$$\frac{-15}{-3} = -15 \cdot \frac{1}{-3} = 5$$ **Same signs; quotient is positive.**

$$\frac{-\frac{3}{8}}{\frac{11}{16}} = -\frac{3}{8} \cdot \left(\frac{16}{11}\right) = -\frac{6}{11}$$ **The reciprocal of 11/16 is 16/11.**

$$\frac{3}{4} \div \left(-\frac{7}{16}\right) = \frac{3}{4} \cdot -\frac{16}{7} = -\frac{48}{28}$$ **Multiply by the reciprocal.**

$$= -\frac{4 \cdot 2 \cdot 6}{4 \cdot 7} = -\frac{12}{7}$$