12.2 Arithmetic Sequences

Objectives

1. Find the common difference of an arithmetic sequence.
2. Find the general term of an arithmetic sequence.
3. Use an arithmetic sequence in an application.
4. Find any specified term or the number of terms of an arithmetic sequence.
5. Find the sum of a specified number of terms of an arithmetic sequence.

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Find the common difference of an arithmetic sequence.

Arithmetic Sequence

An **arithmetic sequence**, or **arithmetic progression,** is a sequence in which each term after the first is found by adding a constant number to the preceding term.

CLASSROOM EXAMPLE 1 **Finding the Common Difference**

Find d for the arithmetic sequence

$$1, \frac{4}{3}, \frac{5}{3}, 2, \frac{7}{3}, \frac{8}{3}, 3, \ldots$$

Solution:

You should find the difference for all pairs of adjacent terms to determine if the sequence is arithmetic. In this case, we are given that the sequence is arithmetic, so d is the difference between any two adjacent terms. Choose the terms $\frac{5}{3}$ and $\frac{4}{3}$.

$$d = \frac{5}{3} - \frac{4}{3} = \frac{1}{3}$$

CLASSROOM EXAMPLE 2 **Writing the Terms of a Sequence from the First Term and the Common Difference**

Write the first five terms of the arithmetic sequence with first term 5 and common difference ½ .

Solution:

Given $a_1 = 5$ and $d = ½$,

$$a_2 = a_1 + d = 5 + ½ = 5\ ½$$
$$a_3 = a_2 + d = 5\ ½ + ½ = 6$$
$$a_4 = a_3 + d = 6 + ½ = 6\ ½$$
$$a_5 = a_4 + d = 6\ ½ + ½ = 7$$

The first five terms of the sequence are 5, 5 ½ , 6, 6 ½ , 7.

Find the general term of an arithmetic sequence.

General Term of an Arithmetic Sequence

The general term of an arithmetic sequence with first term a_1 and common difference d is

$$a_n = a_1 + (n - 1)d.$$

CLASSROOM EXAMPLE 3 **Finding the General Term of an Arithmetic Sequence**

Find the general term of the arithmetic sequence 4, 2, 0, –2, …

Solution:

To find d, subtract any two adjacent terms.

$$d = -2 - 0 = -2$$

The first term is $a_1 = 4$.

Now find a_n.

$$\begin{aligned} a_n &= a_1 + (n - 1)d \\ &= 4 + (n - 1)(-2) \\ &= 4 - 2n + 2 \\ &= -2n + 6 \end{aligned}$$

Thus $a_{20} = -2(20) + 6 = -40 + 6 = -34$.

CLASSROOM EXAMPLE 4 **Applying an Arithmetic Sequence**

How much will be in an account if an initial deposit of $5000 is followed by a $250 contribution each month for 36 months?

Solution:

After 1 month, the account will have
$5000 + 1 • $250 = $5250.

After 2 months, the account will have
$5000 + 2 • $250 = $5500.

In general, after n months the account will have
$5000 + n • $250.

Thus, after 36 months, the account will have
$5000 + 36 • $250 = $14,000.

Objective 4

Find any specified term or the number of terms of an arithmetic sequence.

CLASSROOM EXAMPLE 5 **Finding Specified Terms in Sequence**

Find the indicated term for the arithmetic sequence.

Given $a_1 = -15$ and $d = -4$, find a_{12}.

Solution:

$$a_n = a_1 + (n-1)d$$

$$a_{12} = a_1 + (12-1)d$$

$$= -15 + 11(-4)$$

$$= -59$$

CLASSROOM EXAMPLE 5 **Finding Specified Terms in Sequence (cont'd)**

Find the indicated term for the arithmetic sequence.

Given $a_3 = 2$ and $a_{10} = 23$, find a_{15}.

Solution:

Use $a_n = a_1 + (n-1)d$ to write a system of equations.

$$a_3 = a_1 + (3-1)d$$

$$2 = a_1 + 2d$$

$$a_{10} = a_1 + (10-1)d$$

$$23 = a_1 + 9d$$

To eliminate a_1, multiply (1) by –1 and add the result to (2).

$$\begin{array}{rcl} -2 &=& -a_1 - 2d \\ 23 &=& a_1 + 9d \\ \hline 21 &=& 7d \end{array}$$

$$3 = d$$

CLASSROOM EXAMPLE 5 **Finding Specified Terms in Sequence (cont'd)**

From (1), $2 = a_1 + 2(3)$, so $a_1 = -4$. Now find a_{15}.

$$a_{15} = a_1 + (15-1)d$$

$$= -4 + 14(3)$$

$$= 38$$

There are 5 differences from a_{10} to a_{15}, so

$$a_{15} = a_{10} + 5d$$

$$= 23 + 5(3)$$

$$= 38$$

CLASSROOM EXAMPLE 6 **Finding the Number of Terms in a Sequence**

Find the number of terms in the arithmetic sequence 8, 5, 2, –1, …, –46.

Solution:

$a_n = a_1 + (n-1)d$	**Formula for a_n**
$-46 = 8 + (n-1)(-3)$	**$d = 5 - 8 = -3$**
$-46 = 8 - 3n + 3$	**Distributive property**
$-57 = -3n$	**Simplify**
$19 = n$	**Divide by –3.**

The sequence has 19 terms.

Objective 5

Find the sum of a specified number of terms of an arithmetic sequence.

CLASSROOM EXAMPLE 7 **Finding the Sum of the First *n* Terms of an Arithmetic Sequence**

Find the sum of the first nine terms of the arithmetic sequence in which $a_n = 5 + 2n$.

Solution:

Since we want the sum of the first nine terms, we'll find a_1 and a_9 using $a_n = 5 + 2n$.

$$a_1 = 5 + 2(1) = 7$$

$$a_9 = 5 + 2(9) = 23$$

CLASSROOM EXAMPLE 7 **Finding the Sum of the First *n* Terms of an Arithmetic Sequence (cont'd)**

Now use the formula for the sum of the first *n* terms of an arithmetic sequence.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{9}{2}(a_1 + a_9)$$

$$= \frac{9}{2}(7 + 23)$$

$$= \frac{9}{2}(30)$$

$$= 135$$

Find the sum of specified number of terms of an arithmetic sequence.

Sum of the First *n* Terms of an Arithmetic Sequence

The sum of the first n terms of the arithmetic sequence with the first term a_1, nth term a_n, and common difference d is given by either formula

$$S_n = \frac{n}{2}(a_1 + a_n) \quad \text{or} \quad S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right].$$

CLASSROOM EXAMPLE 8 **Finding the Sum of the First n Terms of an Arithmetic Sequence**

Find the sum of the first 10 terms of the arithmetic sequence having first term –7 and common difference 3.

Solution:

We are given $a_1 = -7$, $d = 3$, and $n = 10$. Use the second formula for the sum of the arithmetic sequence.

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(-7) + (10-1)3\right]$$

$$= 5\left[-14 + (9)3\right]$$

$$= 5(-14 + 27)$$

$$= 65$$

CLASSROOM EXAMPLE 9 **Using S_n to Evaluate a Summation**

Evaluate $\sum_{i=1}^{20}(4i+1)$.

Solution:

To find the first and last (20th) terms, let $n = 1$ and $n = 20$ and $a_n = 4n + 1$.

$$a_1 = 4(1) + 1 = 5$$

$$a_{20} = 4(20) + 1 = 81$$

Now find S_{20}.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{20} = \frac{20}{2}(5 + 81)$$

$$= 10(86)$$

$$= 860$$