12.3 Geometric Sequences

Objectives

1 Find the common ratio of a geometric sequence.

2 Find the general term of a geometric sequence.

3 Find any specified term of a geometric sequence.

4 Find the sum of a specified number of terms of a geometric sequence.

5 Apply the formula for the future value of an ordinary annuity.

6 Find the sum of an infinite number of terms of a certain geometric sequences.

PEARSON

Geometric Sequences

Geometric Sequence

A **geometric sequence**, or **geometric progression**, is a sequence in which each term after the first is found by multiplying the preceding term by a nonzero constant.

Find the common ratio of a geometric sequence.

We find the constant multiplier, called the **common ratio,** by dividing any term after the first by the preceding term. That is, the common ratio is

$$r = \frac{a_{n+1}}{a_n}.$$

CLASSROOM EXAMPLE 1 **Finding the Common Ratio**

Determine *r* for the geometric sequence $1, -2, 4, -8, 16, \ldots$

Solution:

You should find the ratio for all pairs of adjacent terms to determine if the sequence is geometric. In this case, we are ***given*** that the sequence is geometric, so to find *r*, choose any two adjacent terms and divide the second one by the first one. Choose the terms 1 and –2.

$$r = \frac{-2}{1} = -2$$

Notice that any other two adjacent terms could have been used with the same result.

The common ration is $r = -2$.

Find the general term of a geometric sequence.

General Term of a Geometric Sequence

The general term of the geometric sequence with first term a_1 and common ratio *r* is

$$a_n = a_1 r^{n-1}.$$

CAUTION In finding $a_1 r^{n-1}$, be careful to use the correct order of operations. The value of r^{n-1} must be found first. Then multiply the result by a_1.

CLASSROOM EXAMPLE 2 **Finding the General Term of a Geometric Sequence**

Find the general term of the sequence –2, –8, –32, ….

Solution:

The common ratio is $r = \frac{-8}{-2} = 4.$

Substitute into the formula for a_n.

$$a_n = a_1 r^{n-1}$$
$$= -2(4)^{n-1}$$

CLASSROOM EXAMPLE 3 **Finding Specified Terms in Sequences**

Find the indicated term for each geometric sequence.

Solution:

$a_1 = -2$, $r = 5$; a_6

$a_n = a_1 r^{n-1}$

$a_6 = -2(5)^{6-1}$

$= -2(5)^5$

$= -6250$

4, 28, 196, 1372, …; a_8

The common ratio is $r = \frac{28}{4} = 7.$

Substitute into the formula for a_n.

$a_n = a_1 r^{n-1}$

$a_8 = 4(7)^{8-1}$

$= 3{,}294{,}172$

CLASSROOM EXAMPLE 4 **Writing the Terms of a Sequence**

Write the first five terms of the geometric sequence whose first term is 3 and whose common ratio is –2.

Solution:

$a_1 = 3$

$a_2 = a_1 r = 3(-2) = -6$

$a_3 = a_2 r = -6(-2) = 12$

$a_4 = a_3 r = 12(-2) = -24$

$a_5 = a_4 r = -24(-2) = 48$

Objective 4

Find the sum of a specified number of terms of a geometric sequence.

Find the sum of a specified number of terms of a geometric sequence.

Sum of the First *n* Terms of a Geometric Sequence

The sum of the first *n* terms of the geometric sequence with the first term a_1 and common ratio *r* is

$$S_n = \frac{a_1(r^n - 1)}{r - 1} \quad (r \neq 1).$$

CLASSROOM EXAMPLE 5 **Finding the Sum of the First *n* Terms of a Geometric Sequence**

Evaluate the sum of the first six terms of the geometric sequence with first term 6 and common ratio 3.

Solution:

$$S_n = \frac{a_1(r^n - 1)}{r - 1}$$

$$S_6 = \frac{6(3^6 - 1)}{3 - 1}$$

$$= \frac{6(729 - 1)}{2}$$

$$= 3(728)$$

$$= 2184$$

CLASSROOM EXAMPLE 6 **Using the Formula for S_n to Find a Summation**

Evaluate $\sum_{i=1}^{6} 3\left(\frac{1}{4}\right)^i$.

Solution:

For $\sum_{i=1}^{6} 3\left(\frac{1}{4}\right)^i$, $a_1 = \frac{3}{4}$ and $r = \frac{1}{4}$. Find S_6.

$$S_n = \frac{a_1(r^n - 1)}{r - 1}$$

$$S_6 = \frac{\frac{3}{4}\left[\left(\frac{1}{4}\right)^6 - 1\right]}{\frac{1}{4} - 1}$$

$$= \frac{\frac{3}{4}\left[\frac{1}{4096} - 1\right]}{-\frac{3}{4}}$$

$$= -1\left(\frac{1}{4096} - \frac{4096}{4096}\right)$$

$$= -1\left(\frac{-4095}{4096}\right)$$

$$= \frac{4095}{4096} \text{ or } \approx 0.9998$$

Apply the formula for the future value of an ordinary annuity.

Future Value of an Ordinary Annuity

The future value for an ordinary annuity is

$$S = R\left[\frac{(1+i)^n - 1}{i}\right],$$

where S is the future value,
R is the payment at the end of each period,
i is the interest rate per period, and
n is the number of periods.

CLASSROOM EXAMPLE 7 Applying the Formula for the Future Value of an Annuity

Sonny's Building Specialties deposits $2500 at the end of each year into an account paying 4% per yr, compounded annually. Find the total amount on deposit after 10 yr.

Solution:

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$S = 2500\left[\frac{(1+0.04)^{10} - 1}{0.04}\right]$$

$$= 30,015.268$$

The future value of the annuity is $30,015.27.

CLASSROOM EXAMPLE 7 Applying the Formula for the Future Value of an Annuity (cont'd)

Igor Kalugin is an athlete who believes that his playing career will last 10 yr. How much will be in Igor's account after 10 yr if he deposits $2000 at the end of each month at 6% interest compounded monthly?

Solution:

Now use r = $2000, $i = \frac{0.06}{12} = 0.005,$ and n = 12(10) = 120.

$$S = \$2000\left[\frac{(1+0.005)^{120} - 1}{0.005}\right]$$

$$= \$327,758.69$$

Find the sum of an infinite number of terms of certain geometric sequences.

Sum of the Terms of an Infinite Geometric Sequence

The sum S of the terms of an infinite geometric sequence with the first term a_1 and common ratio r, $|r| < 1$, is

$$S = \frac{a_1}{1-r}.$$

If $|r| \geq 1$, then the sum does not exist.

CLASSROOM EXAMPLE 8 Finding the Sum of the Terms of an Infinite Geometric Sequence

Find the sum of the terms of the infinite geometric sequence with $a_1 = -2$ and $r = -\frac{5}{8}$.

Solution:

Use the formula for the sum of the terms of an infinite geometric sequence with $a_1 = -2$ and $r = -\frac{5}{8}$.

$$S = \frac{a_1}{1-r} = \frac{-2}{1-\left(-\frac{5}{8}\right)} = \frac{-2}{\frac{13}{8}} = -\frac{16}{13}$$

CLASSROOM EXAMPLE 9 Finding the Sum of the Terms of an Infinite Geometric Series

Find $\sum_{i=1}^{\infty}\left(\frac{1}{5}\right)\left(\frac{5}{7}\right)^i$.

Solution:

For $\sum_{i=1}^{\infty}\left(\frac{1}{5}\right)\left(\frac{5}{7}\right)^i$, $a_1 = \frac{1}{7}$ and $r = \frac{5}{7}$.

$$S = \frac{a_1}{1-r} = \frac{\frac{1}{7}}{1-\frac{5}{7}} = \frac{\frac{1}{7}}{\frac{2}{7}} = \frac{1}{2}$$

