1.3 Exponents, Roots, and Order of Operations

Objectives

1 Use exponents.

2 Find square roots.

3 Use the order of operations.

4 Evaluate algebraic expressions for given values of variables.

PEARSON

Use exponents.

In algebra we use ***exponents*** as a way of writing products of repeated factors.

$$\underbrace{2\cdot 2\cdot 2\cdot 2\cdot 2}_{\text{5 factors of 2}} = 2^5$$

The number 5 shows that 2 is used as a factor 5 times.

The number 5 is the ***exponent*****,** and 2 is the *base*.

2^5 ← Exponent; 2 ← Base

Use exponents.

Exponential Expression

If *a* is a real number and *n* is a natural number, then

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a},$$

where *n* is the **exponent,** *a* is the **base,** and a^n is an **exponential expression**. Exponents are also called **powers.**

CLASSROOM EXAMPLE 1 **Using Exponential Notation**

Write using exponents.

$$\frac{2}{7}\cdot\frac{2}{7}\cdot\frac{2}{7}\cdot\frac{2}{7}$$

Solution:

Here, $\frac{2}{7}$ **is used as a factor 4 times.**

$$\underbrace{\frac{2}{7}\cdot\frac{2}{7}\cdot\frac{2}{7}\cdot\frac{2}{7}}_{\text{4 factors of } \frac{2}{7}.} = \left(\frac{2}{7}\right)^4$$

Read as "$\frac{2}{7}$ **to the fourth power."**

CLASSROOM EXAMPLE 1 **Using Exponential Notation (cont'd)**

$(-10)(-10)(-10)$

Solution:

$(-10)^3$

Read as "–10 cubed."

$y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$

y^8

CLASSROOM EXAMPLE 2 **Evaluating Exponential Expressions**

Evaluate.

3^4

Solution:

$3\cdot 3\cdot 3\cdot 3 = 81$ **3 is used as a factor 4 times.**

$(-3)^2$

$(-3)(-3) = 9$ **The base is –3.**

-3^2

$-(3\cdot 3) = -9$ **There are no parentheses. The exponent 2 applies *only* to the number 3, not to –3.**

Use exponents.

Sign of an Exponential Expression

The product of an ***odd*** number of negative factors is negative.

The product of an ***even*** number of negative factors is positive.

CAUTION It is important to distinguish between $-a^n$ and $(-a)^n$.

$$-a^n = -1\underbrace{(a \cdot a \cdot a \cdot \ldots \cdot a)}_{n \text{ factors of } a} \quad \text{The base is } a.$$

$$(-a)^n = \underbrace{(-a)(-a) \cdot \ldots \cdot (-a)}_{n \text{ factors of } -a} \quad \text{The base is } -a.$$

Be careful when evaluating an exponential expression with a negative sign.

Find square roots.

The opposite (inverse) of squaring a number is called taking its **square root.**

The square root of 49 is 7.

Another square root of 49 is –7, since $(-7)^2 = 49$.

Thus 49 has two square roots: 7 and –7.

We write the **positive** or **principal square root** of a number with the symbol $\sqrt{\ }$, called a **radical symbol.**

Find square roots.

The **negative square root** of 49 is written

$$-\sqrt{49} = -7.$$

Since the square of any nonzero real number is positive, the square root of a negative number, such as $\sqrt{-49}$ is not a real number.

CAUTION The symbol $\sqrt{\ }$ is used only for the *positive* square root, except that $\sqrt{0} = 0$. The symbol $-\sqrt{\ }$ is used for the negative square root.

CLASSROOM EXAMPLE 3 **Finding Square Roots**

Find each square root that is a real number.

Solution:

$$-\sqrt{\frac{121}{81}} = -\frac{11}{9}$$

$$\sqrt{49} = 7$$

$$-\sqrt{49} = -7$$

$\sqrt{-49}$ **Not a real number, because the negative sign is inside the radical sign. No *real number* squared equals –49.**

Use the order of operations.

Order of Operations

1. Work separately above and below any **fraction bar**.

2. If **grouping symbols** such as **parentheses ( ), brackets [ ],** or **absolute value bars | |** are present, start with the innermost set and work outward.

3. Evaluate all **powers, roots,** and **absolute values.**

4. Multiply or **divide** in order from left to right.

5. Add or **subtract** in order from left to right.

Use the order of operations.

Some students like to use the mnemonic “Please Excuse My Dear Aunt Sally” to help remember the rules for order of operations.

Please	Excuse	My	Dear	Aunt	Sally
↑	↑	↑	↑	↑	↑
Parentheses	Exponents	Multiply	Divide	Add	Subtract

Be sure to multiply or divide in order from left to right. Then add or subtract in order from left to right.

CLASSROOM EXAMPLE 4 **Using Order of Operations**

Simplify.

$5 \cdot 9 + 2 \cdot 4$

Solution:

$= 45 + 2 \cdot 4$	**Multiply.**
$= 45 + 8$	**Multiply.**
$= 53$	**Add.**

$4 - 12 \div 4 \cdot 2$

$= 4 - 3 \cdot 2$	**Divide.**
$= 4 - 6$	**Multiply.**
$= -2$	**Subtract.**

CLASSROOM EXAMPLE 5 **Using Order of Operations**

Simplify.

$(4 + 2) - 3^2 - (8 - 3)$

$3^2 = 3 \cdot 3$ not $3 \cdot 2$

Solution:

$= 6 - 3^2 - 5$	**Perform operations inside parentheses.**
$= 6 - 9 - 5$	**Evaluate the power.**
$= -3 - 5$	**Subtract 6 – 9.**
$= -8$	**Subtract.**

CLASSROOM EXAMPLE 6 **Using the Order of Operations**

Simplify.

$$\frac{\frac{1}{2}\cdot 10 - 6 + \sqrt{9}}{\frac{5}{6}\cdot 12 - 3(2)^2}$$

Work separately above and below the fraction bar.

Solution:

$$\frac{\frac{1}{2}\cdot 10 - 6 + \sqrt{9}}{\frac{5}{6}\cdot 12 - 3(2)^2} = \frac{\frac{1}{2}\cdot 10 - 6 + 3}{\frac{5}{6}\cdot 12 - 3(4)}$$

Evaluate the root and the power.

$$= \frac{5 - 6 + 3}{10 - 3(4)}$$

Multiply the fraction and whole number.

$$= \frac{2}{10 - 12} \qquad = \frac{2}{-2} = -1$$

Subtract and add in the numerator.

Multiply 3(4).

Evaluate algebraic expressions for given values of variables.

Any sequence of numbers, variables, operation symbols, and/or grouping symbols formed in accordance with the rules of algebra is called an **algebraic expression.**

$6ab$, $5m - 9n$, and $-2(x^2 + 4y)$

We evaluate algebraic expressions by ***substituting*** given values for the variables.

CLASSROOM EXAMPLE 7 **Evaluating Algebraic Expressions**

Evaluate the expression if $w = 4$, $x = -12$, $y = 64$ and $z = -3$.

$$\frac{5x + z\sqrt{y}}{x - 1}$$

Solution:

$$= \frac{5(-12) + (-3)\sqrt{64}}{-12 - 1}$$

Substitute *x* = – 12, *y* = 64 and *z* = – 3.

$$= \frac{-60 + (-3)(8)}{-13}$$

Work separately above and below the fraction bar.

$$= \frac{-60 - 24}{-13} \qquad = \frac{-84}{-13} = \frac{84}{13}$$

CLASSROOM EXAMPLE 7 **Evaluating Algebraic Expressions (cont'd)**

Evaluate the expression $w^2 + 2z^3$ if $w = 4$, $x = -12$, $y = 64$ and $z = -3$.

Solution:

$= (4)^2 + 2(-3)^3$	**Substitute *w* = 4 and *z* = – 3.**
$= 16 + 2(-27)$	**Evaluate the powers.**
$= 16 - 54$	**Multiply.**
$= -38$	**Subtract.**