1.4 Properties of Real Numbers

Objectives

1 Use the distributive property.

2 Use the identity properties.

3 Use the inverse properties.

4 Use the commutative and associative properties.

5 Use the multiplication property of 0.

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Use the distributive property.

The Distributive Property

For any real numbers, *a, b,* and *c*, the following are true.

$a(b + c) = ab + ac$ and $(b + c)a = ba + ca.$

The distributive property can be extended to more than two numbers and provides a way to rewrite a product as a sum.

$a(b + c + d) = ab + ac + ad$

When we rewrite $a(b + c)$ as $ab + ac$, we sometimes refer to the process as "removing" or "clearing" parentheses.

CLASSROOM EXAMPLE 1

Using the Distributive Property

Use the distributive property to rewrite each expression.

Solution:

$-4(p - 5)$

$= -4p - (-4)(5)$

$= -4p + 20$

$-6m + 2m$

$= (-6 + 2)m$

$= -6m + 2m$

$= -4m$

CLASSROOM EXAMPLE 1

Using the Distributive Property (cont'd)

$2r + 3s$

Solution:

Because there is no common number or variable here, we cannot use the distributive property to rewrite the expression.

$5(4p - 2q + r)$

$= 5(4p) - 5(2q) + 5r$

$= 20p - 10q + 5r$

Use the identity properties.

The number 0 is the only number that can be added to any number and leaves the number unchanged. Thus, zero is called the **identity element for addition,** or the **additive identity.**

Similarly, the number 1 is the only number that can be multiplied with another number and leaves the number unchanged. Thus, one is called the **identity element for multiplication** or the **multiplicative identity.**

Identity Properties

For any real number *a*, the following are true.

$a + 0 = 0 + a = a$

$a \cdot 1 = 1 \cdot a = a$

CLASSROOM EXAMPLE 2

Using the Identity Property $1 \cdot a = a$

Simplify each expression.

$x - 3x$ **Solution:**

$= 1x - 3x$

$= 1x - 3x$ **Identity property.**

$= (1 - 3)x$ **Distributive property.**

$= -2x$ **Subtract inside parentheses.**

$-(3 + 4p)$

$= -1(3 + 4p)$

$= -1(3) + (-1)(4p)$ **Identity property.**

$= -3 - 4p$ **Multiply.**

Use the inverse properties.

The ***additive inverse*** (or ***opposite***) of a number a is $-a$. Additive inverses have a sum of 0.

The ***multiplicative inverse*** (or ***reciprocal***) of a number a is $\frac{1}{a}$. Multiplicative inverses have a product of 1.

Inverse Properties

For any real number a, the following are true.

$a + (-a) = 0$ and $-a + a = 0$

$a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1 \quad (a \neq 0).$

The inverse properties "undo" addition or multiplication.

Use the inverse properties.

A **term** is a number or the product of a number and one or more variables raised to powers.

The numerical factor in a term is called the **numerical coefficient,** or just the **coefficient.**

Terms with exactly the same variables raised to exactly the same powers are called **like terms.**

$5y$ and $-21y$	$-6x^2$ and $9x^2$	**Like terms**
$3m$ and $16x$	$7y^3$ and $-3y^2$	**Unlike terms**

Remember that only like terms may be combined.

Use the commutative and associative properties.

Commutative and Associative Properties

For any real numbers a, b, and c, the following are true.

$a + b = b + a$
$ab = ba$
Commutative properties

The order of the two terms or factors changes.

$a + (b + c) = (a + b) + c$
$a(bc) = (ab)c$
Associative properties

The grouping among the terms or factors changes, but the order stays the same.

CLASSROOM EXAMPLE 3 **Using the Commutative and Associative Properties**

Simplify.

Solution:

$12b - 9 + 4b - 7b + 1$

$= (12b + 4b) - 9 - 7b + 1$

$= (12 + 4)b - 9 - 7b + 1$

$= 16b - 9 - 7b + 1$

$= (16b - 7b) - 9 + 1$

$= (16 - 7)b - 9 + 1$

$= 9b - 8$

CLASSROOM EXAMPLE 4 **Using the Properties of Real Numbers**

Simplify each expression.

Solution:

$12b - 9b + 5b - 7b$

$= (12 - 9 + 5 - 7)b$ **Distributive property.**

$= b$ **Combine like terms.**

$6 - (2x + 7) - 3$

$= 6 - 2x - 7 - 3$ **Distributive property.**

$= -2x + 6 - 7 - 3$ **Commutative property.**

$= -2x - 4$ **Combine like terms.**

CLASSROOM EXAMPLE 4 **Using the Properties of Real Numbers (cont'd)**

$4m(2n)$

Solution:

$= (4)(2)mn$

$= 8mn$