1.1 Basic Concepts

Objectives

1 Write sets using set notation.

2 Use number lines.

3 Know the common sets of numbers.

4 Find the additive inverses.

5 Use absolute value.

6 Use inequality symbols.

7 Graph sets of real numbers.

Write sets using set notation.

A **set** is a collection of objects called the **elements** or **members** of the set.

In algebra, the elements of a set are usually numbers. Set braces, { }, are used to enclose the elements.

Since we can count the number of elements in the set {1, 2, 3}, it is a **finite set.**

The set ***N*** **= {1, 2, 3, 4, 5, 6…}** is called the **natural numbers,** or **counting numbers**.

The three dots (***ellipsis*** **points**) show that the list continues in the same pattern indefinitely.

We cannot list all the elements of the set of natural numbers, so it is an **infinite set.**

Write sets using set notation.

When 0 is included with the set of natural numbers, we have the set of **whole numbers**, written

W **= {0, 1, 2, 3, 4, 5, 6…}.**

The set containing no elements, such as the set of whole numbers less than 0, is called the **empty set,** or **null set**, usually written ∅ or { }.

To write the fact that 2 is an element of the set {1, 2, 3}, we use the symbol ∈ (read "is an element of").

$2 \in \{1, 2, 3\}$

CAUTION Do not write {∅} for the empty set. {∅} is a set with one element: ∅. Use the notation ∅ or { } for the empty set.

Write sets using set notation.

Two sets are equal if they contain exactly the same elements. For example, {1, 2} = {2, 1} (Order does not matter.)

{0, 1, 2} ≠ {1, 2} (≠ means "is not equal to"), since one set contains the element 0 while the other does not.

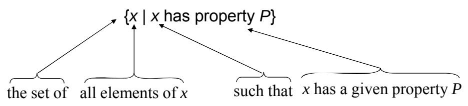
Letters called **variables** are often used to represent numbers or to define sets of numbers. For example,

{*x* | *x* is a natural number between 3 and 15}

(read "the set of all elements *x* such that *x* is a natural number between 3 and 15") defines the set {4, 5, 6, 7, …14}.

Write sets using set notation.

The notation {*x* | *x* is a natural number between 3 and 15} is an example of **set-builder notation.**



CLASSROOM EXAMPLE 1

Listing the Elements in Sets

List the elements in { *x* | *x* is a natural number greater than 12 }.

Solution:

{13, 14, 15, …}

CLASSROOM EXAMPLE 2

Using Set-Builder Notation to Describe Sets

Use set builder notation to describe the set.

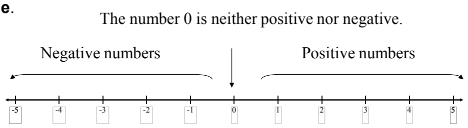
{ 0, 1, 2, 3, 4, 5 }

Solution:

{ *x* | *x* is a whole number less than 6 }

Use number lines.

A good way to get a picture of a set of numbers is to use a **number line**.

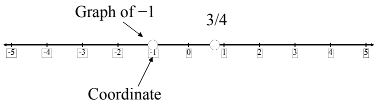


The set of numbers identified on the number line above, including positive and negative numbers and 0, is part of the set of **integers**, written

***I* = {…, –3, –2, –1, 0, 1, 2, 3…}.**

Use number lines.

Each number on a number line is called the **coordinate** of the point that it labels while the point is the **graph** of the number.



The fraction ¾ graphed on the number line is an example of a ***rational number.*** A **rational number** can be expressed as the quotient of two integers, with denominator not 0. The set of all rational numbers is written

$$\left\{\frac{p}{q} \mid p \text{ and } q \text{ are integers, } q \neq 0\right\}.$$

Use number lines.

The set of rational numbers includes the natural numbers, whole numbers, and integers, since these numbers can be written as fractions.

For example, $20 = \frac{20}{1}.$

A rational number written as a fraction, such as ½ or 1/8, can also be expressed as a decimal by dividing the numerator by the denominator.

Decimal numbers that neither terminate nor repeat are ***not*** rational numbers and thus are called **irrational numbers.**

For example,

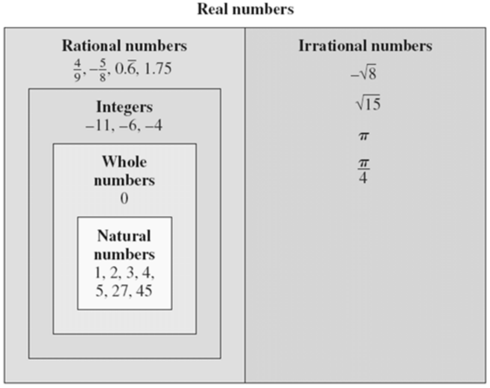
$$\sqrt{2} = 1.414213562\ldots \text{ and } -\sqrt{7} = -2.6457513\ldots$$

Know the common sets of numbers.

Sets of Numbers

Natural numbers, or counting numbers	{1, 2, 3, 4, 5, 6, …}
Whole numbers	{0, 1, 2, 3, 4, 5, 6, …}
Integers	{…, –3, –2, –1, 0, 1, 2, 3, …}
Rational numbers	$\left\{\frac{p}{q} \mid p \text{ and } q \text{ are integers, } q \neq 0\right\}.$
Irrational numbers	{*x* | *x* is a real number that is not rational}
Real numbers	{*x* | *x* is a rational number or an irrational number}

Use number lines.



CLASSROOM EXAMPLE 3

Identifying Examples of Number Sets

Select all the sets from the following list that apply to each number.

–7 3.14 $\sqrt{4}$

Solution:

–7

whole number
[✓ integer]
[✓ rational number]
irrational number
[✓ real number]

CLASSROOM EXAMPLE 3

Identifying Examples of Number Sets (cont'd)

Select all the sets from the following list that apply to each number.

–7 3.14 $\sqrt{4}$

Solution:

3.14

whole number
integer
[✓ rational number]
irrational number
[✓ real number]

CLASSROOM EXAMPLE 3

Identifying Examples of Number Sets (cont'd)

Select all the sets from the following list that apply to each number.

–7 3.14 $\sqrt{4}$

Solution:

$\sqrt{4} = 2$

[✓ whole number]
[✓ integer]
[✓ rational number]
irrational number
[✓ real number]

CLASSROOM EXAMPLE 4

Determining Relationships Between Sets of Numbers

Decide whether the statement is ***true*** or ***false.*** If it is false, tell why.

Solution:

Some integers are whole numbers.

true

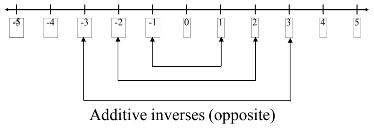
Every real number is irrational.

false; some real numbers are irrational, but others are rational numbers.

Find the additive inverses.

Additive Inverse

For any real number *a*, the number –*a* is the **additive inverse** of *a*.



Change the sign of a number to get its additive inverse. The sum of a number and it additive inverse is always 0.

Find the additive inverses.

Uses of the Symbol –

The symbol "–" is used to indicate any of the following:

1. **a negative number,** such as –9 or –15;
2. **the additive inverse of a number,** as in "– 4 is the additive inverse of 4";
3. **subtraction,** as in 12 – 3.

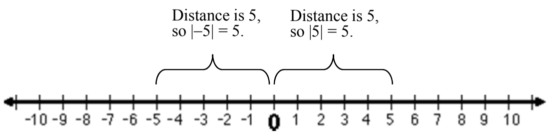
– (–*a*)

For any real number *a*, – (–*a*) = *a*.

Use absolute value.

The **absolute value** of a number *a*, written |*a*|, is the distance on a number line from 0 to *a*.

For example, the absolute value of 5 is the same as the absolute value of –5 because each number lies 5 units from 0.



Use absolute value.

Absolute Value

For any real number *a*, | *a* | = $\begin{cases} a & \text{if } a \text{ is positive or 0} \\ -a & \text{if } a \text{ is negative.} \end{cases}$

Because absolute value represents distance, and distance is never negative, the absolute value of a number is always positive or 0.

CLASSROOM EXAMPLE 5 **Finding Absolute Value**

Simplify by finding each absolute value.

Solution:

|–3| **= 3**

−|3| **= – 3**

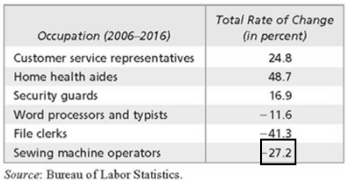
−|–3| **= – 3**

| 8 | + |–1| **= 8 + 1** **= 9**

| 8 – 1| **= |7|** **= 7**

CLASSROOM EXAMPLE 6 **Comparing Rates of Change in Industries**

Of the customer service representatives and sewing machine operators, which will show the greater change (without regard to sign)?



Occupation (2006–2016)	Total Rate of Change (in percent)
Customer service representatives	24.8
Home health aides	48.7
Security guards	16.9
Word processors and typists	−11.6
File clerks	−41.3
Sewing machine operators	$\boxed{-27.2}$

Source: Bureau of Labor Statistics.

Solution:

Look for the number with the largest absolute value.

sewing machine operators

Use inequality symbols.

The statement

4 + 2 = 6

is an **equation** — a statement that two quantities are equal.

The statement

4 ≠ 6 (read “4 is not equal to 6”)

is an **inequality** — a statement that two quantities are ***not*** equal.

The symbol **<** means “is less than.”

8 < 9, –7 < 16, –8 < –2, and 0 < 4/3

The symbol **>** means “is greater than.”

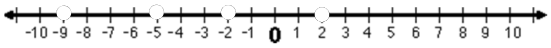
13 > 8, 8 > –2, –3 > –7 and 5/3 > 0

Use inequality symbols.

Inequalities on a Number Line

On a number line,
a* < *b if *a* is to the left of *b*; ***a* > *b*** if *a* is to the right of *b*.

You can use a number line to determine order.



–9 < 2 –2 > –5 or –5 < –2

Be careful when ordering negative numbers.

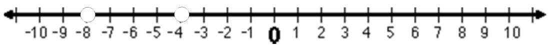
CLASSROOM EXAMPLE 7

Determining Order on a Number Line

Use a number line to determine whether each statement is ***true*** or ***false.***

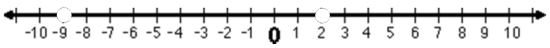
Solution:

$-8 > -4$



False

$-9 < 2$



True

Use inequality symbols.

In addition to the symbols ≠, **<**, and **>**, the symbols ≤ and ≥ are often used.

Symbol	Meaning	Example
≠	is not equal to	3 ≠ 7
<	is less than	–4 < –1
>	is greater than	3 > –2
≤	is less than or equal to	6 ≤ 6
≥	is greater than or equal to	–8 ≥ –10

CLASSROOM EXAMPLE 8

Using Inequality Symbols

Determine whether each statement is ***true*** or ***false.***

	Solution:
$-2 \leq -3$	**false**
$-1 \geq -9$	**true**
$8 \leq 8$	**true**
$3(4) > 2(6)$	**false**

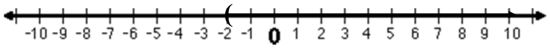
Graph sets of real numbers.

Inequality symbols and variables are used to write sets of real numbers. For example, the set

$$\{x \mid x > -2\}$$

consists of all the real numbers greater than –2.

On a number line, we graph the elements of this set by drawing an arrow from –2 to the right. We use a parenthesis at –2 to indicate that –2 is ***not*** an element of the given set.



The set of numbers greater than –2 is an example of an **interval** on the number line.

Graph sets of real numbers.

To write intervals, use **interval notation**.

The interval of all numbers greater than –2, would be $(-2, \infty)$.

The **infinity symbol (∞)** does not indicate a number; it shows that the interval includes all real numbers greater than –2.

The left parenthesis indicated that –2 is not included.
A parenthesis is always used next to the infinity symbol.

The set of real numbers is written in interval notation as $(-\infty, \infty)$.

CLASSROOM EXAMPLE 9

Graphing an Inequality Written in Interval Notation

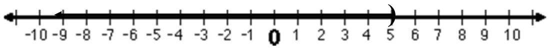
Write in interval notation and graph.

$\{x \mid x < 5\}$

Solution:

The interval is the set of all real numbers less than 5.

$(-\infty, 5)$



CLASSROOM EXAMPLE 10

Graphing an Inequality Written in Interval Notation

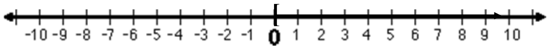
Write in interval notation and graph.

$\{x \mid x \geq 0\}$

Solution:

The interval is the set of all real numbers greater than or equal to 0. We use a square bracket [since 0 is part of the set.

$[0, \infty)$



Graph sets of real numbers.

Sometimes we graph sets of numbers that are ***between*** two given numbers.

For example: $\{x \mid 2 < x < 8\}$

This is called a **three-part inequality**, is read "2 is less than *x* and *x* is less than 8" or "*x* is between 2 and 8."

CLASSROOM EXAMPLE 11

Graphing a Three-Part Inequality

Write in interval notation and graph.

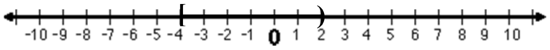
$\{x \mid x - 4 \leq x < 2\}$

Solution:

Use a square bracket at –4.

Use a parenthesis at 2.

$[-4, 2)$



1.2 Operations on Real Numbers

Objectives

1 Add real numbers.

2 Subtract real numbers.

3 Find the distance between two points on a number line.

4 Multiply real numbers.

5 Find reciprocals and divide real numbers.

Add real numbers.

Recall that the answer to an addition problem is called the **sum.**

Adding Real Numbers

Same Sign To add two numbers with the ***same*** sign, add their absolute values. The sum has the same sign as the given numbers.

Different Signs To add two numbers with ***different*** signs, find the absolute values of the numbers, and subtract the smaller absolute value from the larger. The sum has the same sign as the number with the larger absolute value.

CLASSROOM EXAMPLE 1

Adding Two Negative Real Numbers

Find each sum.

–6 + (–15)

Solution:

Find the absolute values.

| –6| = 6| –15| = 15

Because they have the same sign, add their absolute values.

–6 + (–15) = –(6 + 15) **Add the absolute values.**

= –(21)

= –21

Both numbers are negative, so the answer will be negative.

CLASSROOM EXAMPLE 1

Adding Two Negative Real Numbers (cont'd)

–1.1 + (–1.2)

Solution:

= –(1.1 +1.2)

= –2.3

$-\frac{3}{4}+\left(-\frac{5}{8}\right) \quad =-\left(\frac{3}{4}+\frac{5}{8}\right)$ **Add the absolute values. Both numbers are negative, so the answer will be negative.**

$=-\left(\frac{6}{8}+\frac{5}{8}\right)$ **The least common denominator is 8.**

$=-\frac{11}{8}$ **Add numerators; keep the same denominator.**

CLASSROOM EXAMPLE 2

Adding Real Numbers with Different Signs

Find each sum.

3 + (–7)

Solution:

Find the absolute values.

| 3| = 3 | –7| = 7

Because they have *different* signs, subtract their absolute values.

The number –7 has the larger absolute value, so the answer is negative.

3 + (–7) = –4

The sum is negative because |–7| > |3|

CLASSROOM EXAMPLE 2

Adding Real Numbers with Different Signs (cont'd)

–3 + 7 = 7 – 3 **Solution:**

= 4

–3.8 + 4.6 = 4.6 – 3.8

= 0.8

$-\frac{3}{8}+\frac{1}{4} \quad =-\frac{3}{8}+\frac{2}{8}$

$=-\left(\frac{3}{8}-\frac{2}{8}\right)$ **Subtract the absolute values. −3/8 has the larger absolute value, so the answer will be negative.**

$=-\frac{1}{8}$ **Subtract numerators; keep the same denominator.**

Subtract real numbers.

Recall that the answer to a subtraction problem is called the **difference.**

Subtraction

For all real numbers *a* and *b*,

$a - b = a + (-b)$.

That is, to subtract *b* from *a*, add the additive inverse (or opposite) of *b* to *a*.

CLASSROOM EXAMPLE 3

Subtracting Real Numbers

Find each difference.

Solution:

12 – (–5) $= 12 + 5$ $= 17$

Change to addition.

The additive inverse of –5 is 5.

–11.5 – (–6.3) $= -15.5 + 6.3$ $= -5.2$

Change to addition.

The additive inverse of –6.3 is 6.3.

CLASSROOM EXAMPLE 3

Subtracting Real Numbers (cont'd)

$\frac{3}{4} - \left(-\frac{2}{3}\right)$

$= \frac{3}{4} + \frac{2}{3}$ To subtract, add the additive inverse (opposite).

$= \frac{3 \cdot 3}{4 \cdot 3} + \frac{2 \cdot 4}{3 \cdot 4}$ Write each fraction with the least common denominator, 12.

$= \frac{9}{12} + \frac{8}{12}$ Add numerators; keep the same denominator.

$= \frac{17}{12}$

CLASSROOM EXAMPLE 4

Adding and Subtracting Real Numbers

Perform the indicated operations.

–6 – (–2) – 8 – 1 **Work from left to right.**

Solution:

= (–6 + 2) – 8 – 1
= –4 – 8 – 1
= –12 – 1
= –13

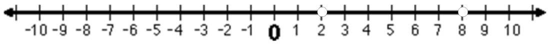
–3 – [(–7) + 15] – 6 **Work inside brackets.**

= –3 – [8] – 6
= –11 – 6
= –17

Find the distance between two points on a number line.

To find the distance between the points 2 and 8, we subtract 8 – 2 = 6. Since distance is always positive, we must be careful to subtract in such a way that the answer is positive.

Or, to avoid this problem altogether, we can find the absolute value of the difference. Then the distance is either |8 – 2| = 6 or |2 – 8| = 6.



CLASSROOM EXAMPLE 5

Finding Distance Between Points on the Number Line

Find the distance between the points –12 and –1.

Solution:

Find the absolute value of the difference of the numbers, taken in either order.

| –12 – (–1)| = 11

or

| –1 – (–12)| = 11

Multiply real numbers.

Recall that the answer to a multiplication problem is called the **product.**

Multiplying Real Numbers

Same Sign The product of two numbers with the *same* sign is positive.

Different Signs The product of two numbers with *different* signs is negative.

CLASSROOM EXAMPLE 6 **Multiplying Real Numbers**

Find each product.

	Solution:	
7(–2)	$= -14$	**Different signs; product is negative.**
–0.9(–15)	$= 13.5$	**Same signs; product is positive.**
$-\frac{5}{8}(16)$	$= -10$	**Different signs; product is negative.**

Find reciprocals and divide real numbers.

The definition of division depends on the idea of a **multiplicative inverse,** or ***reciprocal.***

Reciprocal

The reciprocal of a nonzero number a is $\frac{1}{a}$.

A number and its additive inverse have opposite signs. However, a number and its reciprocal always have the same sign.

Find reciprocals and divide real numbers.

Reciprocals have a product of 1.

Number	Reciprocal
$-\frac{2}{5}$	$-\frac{5}{2}$
–6	$-\frac{1}{6}$
$\frac{7}{11}$	$\frac{11}{7}$
0.05	20
0	None

Division by 0 is undefined, whereas dividing 0 by a nonzero number gives the quotient 0.

Find reciprocals and divide real numbers.

The result of dividing one number by another is called the **quotient.**

Division

For all real numbers *a* and *b* (where *b* ≠ 0),

$$a \div b = \frac{a}{b} = a \cdot \frac{1}{b}.$$

That is, multiply the first number (the **dividend**) by the reciprocal of the second number (the **divisor**).

Dividing Real Numbers

Same Sign The quotient of two nonzero real numbers with the *same* sign is positive.

Different Signs The product of two nonzero real numbers with *different* signs is negative.

CLASSROOM EXAMPLE 7 **Dividing Real Numbers**

Find each quotient.

	Solution:		
$\frac{-15}{-3}$	$= -15 \cdot \frac{1}{-3}$	$= 5$	**Same signs; quotient is positive.**
$\frac{-\frac{3}{8}}{\frac{11}{16}}$	$= -\frac{3}{8} \cdot \left(\frac{16}{11}\right)$	$= -\frac{6}{11}$	**The reciprocal of 11/16 is 16/11.**
$\frac{3}{4} \div \left(-\frac{7}{16}\right)$	$= \frac{3}{4} \cdot -\frac{16}{7}$	$= -\frac{48}{28}$	**Multiply by the reciprocal.**

$$= -\frac{4 \cdot 2 \cdot 6}{4 \cdot 7} \qquad = -\frac{12}{7}$$

1.3 Exponents, Roots, and Order of Operations

Objectives

1 Use exponents.

2 Find square roots.

3 Use the order of operations.

4 Evaluate algebraic expressions for given values of variables.

Use exponents.

In algebra we use ***exponents*** as a way of writing products of repeated factors.

$$\underbrace{2\cdot 2\cdot 2\cdot 2\cdot 2}_{\text{5 factors of 2}} = 2^5$$

The number 5 shows that 2 is used as a factor 5 times.

The number 5 is the ***exponent*****,** and 2 is the *base*.

2^5 ← Exponent

Base

Use exponents.

Exponential Expression

If a is a real number and n is a natural number, then

$$a^n = \underbrace{a\cdot a\cdot a\cdot \ldots \cdot a}_{n\text{ factors of }a},$$

where n is the **exponent,** a is the **base,** and a^n is an **exponential expression**. Exponents are also called **powers.**

CLASSROOM EXAMPLE 1 **Using Exponential Notation**

Write using exponents.

$$\frac{2}{7}\cdot\frac{2}{7}\cdot\frac{2}{7}\cdot\frac{2}{7}$$

Solution:

Here, $\frac{2}{7}$ **is used as a factor 4 times.**

$$\underbrace{\frac{2}{7}\cdot\frac{2}{7}\cdot\frac{2}{7}\cdot\frac{2}{7}}_{\text{4 factors of }\frac{2}{7}.} = \left(\frac{2}{7}\right)^4$$

Read as " $\frac{2}{7}$ **to the fourth power."**

CLASSROOM EXAMPLE 1 **Using Exponential Notation (cont'd)**

$(-10)(-10)(-10)$

Solution:

$(-10)^3$

Read as "–10 cubed."

$y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y$

y^8

CLASSROOM EXAMPLE 2 **Evaluating Exponential Expressions**

Evaluate.

3^4

Solution:

$3\cdot 3\cdot 3\cdot 3 = 81$ **3 is used as a factor 4 times.**

$(-3)^2$

$(-3)(-3) = 9$ **The base is –3.**

-3^2

$-(3\cdot 3) = -9$ **There are no parentheses. The exponent 2 applies *only* to the number 3, not to –3.**

Use exponents.

Sign of an Exponential Expression

The product of an ***odd*** number of negative factors is negative.

The product of an ***even*** number of negative factors is positive.

CAUTION

It is important to distinguish between $-a^n$ and $(-a)^n$.

$$-a^n = -1\underbrace{(a \cdot a \cdot a \cdot \ldots \cdot a)}_{n \text{ factors of } a} \qquad \text{The base is } a.$$

$$(-a)^n = \underbrace{(-a)(-a) \cdot \ldots \cdot (-a)}_{n \text{ factors of } -a} \qquad \text{The base is } -a.$$

Be careful when evaluating an exponential expression with a negative sign.

Find square roots.

The opposite (inverse) of squaring a number is called taking its **square root.**

The square root of 49 is 7.

Another square root of 49 is –7, since $(-7)^2 = 49$.

Thus 49 has two square roots: 7 and –7.

We write the **positive** or **principal square root** of a number with the symbol $\sqrt{}$, called a **radical symbol.**

Find square roots.

The **negative square root** of 49 is written

$$-\sqrt{49} = -7.$$

Since the square of any nonzero real number is positive, the square root of a negative number, such as $\sqrt{-49}$ is not a real number.

CAUTION

The symbol $\sqrt{}$ is used only for the *positive* square root, except that $\sqrt{0} = 0$. The symbol $-\sqrt{}$ is used for the negative square root.

CLASSROOM EXAMPLE 3

Finding Square Roots

Find each square root that is a real number.

Solution:

$-\sqrt{\dfrac{121}{81}} \quad = -\dfrac{11}{9}$

$\sqrt{49} \quad = 7$

$-\sqrt{49} \quad = -7$

$\sqrt{-49}$ **Not a real number, because the negative sign is inside the radical sign. No *real number* squared equals –49.**

Use the order of operations.

Order of Operations

1. Work separately above and below any **fraction bar**.

2. If **grouping symbols** such as **parentheses ( ), brackets [ ],** or **absolute value bars | |** are present, start with the innermost set and work outward.

3. Evaluate all **powers, roots,** and **absolute values.**

4. Multiply or **divide** in order from left to right.

5. Add or **subtract** in order from left to right.

Use the order of operations.

Some students like to use the mnemonic “Please Excuse My Dear Aunt Sally” to help remember the rules for order of operations.

Please	Excuse	My	Dear	Aunt	Sally
↑	↑	↑	↑	↑	↑
Parentheses	Exponents	Multiply	Divide	Add	Subtract

Be sure to multiply or divide in order from left to right. Then add or subtract in order from left to right.

CLASSROOM EXAMPLE 4

Using Order of Operations

Simplify.

$5 \cdot 9 + 2 \cdot 4$

Solution:

$= 45 + 2 \cdot 4$ **Multiply.**

$= 45 + 8$ **Multiply.**

$= 53$ **Add.**

$4 - 12 \div 4 \cdot 2$

$= 4 - 3 \cdot 2$ **Divide.**

$= 4 - 6$ **Multiply.**

$= -2$ **Subtract.**

CLASSROOM EXAMPLE 5

Using Order of Operations

Simplify.

$(4 + 2) - 3^2 - (8 - 3)$

$3^2 = 3 \cdot 3$ not $3 \cdot 2$

Solution:

$= 6 - 3^2 - 5$ **Perform operations inside parentheses.**

$= 6 - 9 - 5$ **Evaluate the power.**

$= -3 - 5$ **Subtract 6 – 9.**

$= -8$ **Subtract.**

CLASSROOM EXAMPLE 6

Using the Order of Operations

Simplify.

$$\frac{\frac{1}{2}\cdot 10-6+\sqrt{9}}{\frac{5}{6}\cdot 12-3(2)^2}$$

Work separately above and below the fraction bar.

Solution:

$$\frac{\frac{1}{2}\cdot 10-6+\sqrt{9}}{\frac{5}{6}\cdot 12-3(2)^2}=\frac{\frac{1}{2}\cdot 10-6+3}{\frac{5}{6}\cdot 12-3(4)}$$

Evaluate the root and the power.

$$=\frac{5-6+3}{10-3(4)}$$

Multiply the fraction and whole number.

$$=\frac{2}{10-12} \qquad =\frac{2}{-2}=-1$$

Subtract and add in the numerator.

Multiply 3(4).

Evaluate algebraic expressions for given values of variables.

Any sequence of numbers, variables, operation symbols, and/or grouping symbols formed in accordance with the rules of algebra is called an **algebraic expression.**

$6ab$, $5m - 9n$, and $-2(x^2 + 4y)$

We evaluate algebraic expressions by ***substituting*** given values for the variables.

CLASSROOM EXAMPLE 7

Evaluating Algebraic Expressions

Evaluate the expression if $w = 4$, $x = -12$, $y = 64$ and $z = -3$.

$$\frac{5x+z\sqrt{y}}{x-1}$$

Solution:

$$=\frac{5(-12)+(-3)\sqrt{64}}{-12-1}$$

Substitute $x = -12$, $y = 64$ and $z = -3$.

$$=\frac{-60+(-3)(8)}{-13}$$

Work separately above and below the fraction bar.

$$=\frac{-60-24}{-13} \qquad =\frac{-84}{-13}=\frac{84}{13}$$

CLASSROOM EXAMPLE 7

Evaluating Algebraic Expressions (cont’d)

Evaluate the expression $w^2 + 2z^3$ if $w = 4$, $x = -12$, $y = 64$ and $z = -3$.

Solution:

$= (4)^2 + 2(-3)^3$ **Substitute $w = 4$ and $z = -3$.**

$= 16 + 2(-27)$ **Evaluate the powers.**

$= 16 - 54$ **Multiply.**

$= -38$ **Subtract.**

1.4 Properties of Real Numbers

Objectives

1 Use the distributive property.

2 Use the identity properties.

3 Use the inverse properties.

4 Use the commutative and associative properties.

5 Use the multiplication property of 0.

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Use the distributive property.

The Distributive Property

For any real numbers, *a, b,* and *c*, the following are true.

$a(b + c) = ab + ac$ and $(b + c)a = ba + ca.$

The distributive property can be extended to more than two numbers and provides a way to rewrite a product as a sum.

$a(b + c + d) = ab + ac + ad$

When we rewrite $a(b + c)$ as $ab + ac,$ we sometimes refer to the process as "removing" or "clearing" parentheses.

CLASSROOM EXAMPLE 1 Using the Distributive Property

Use the distributive property to rewrite each expression.

Solution:

$–4(p – 5)$

$= – 4p – (–4)(5)$

$= – 4p + 20$

$–6m + 2m$

$= (–6 + 2)m$

$= – 6m + 2m$

$= – 4m$

CLASSROOM EXAMPLE 1 Using the Distributive Property (cont'd)

$2r + 3s$

Solution:

Because there is no common number or variable here, we cannot use the distributive property to rewrite the expression.

$5(4p – 2q + r)$

$= 5(4p) – 5(2q) + 5r$

$= 20p – 10q + 5r$

Use the identity properties.

The number 0 is the only number that can be added to any number and leaves the number unchanged. Thus, zero is called the **identity element for addition,** or the **additive identity.**

Similarly, the number 1 is the only number that can be multiplied with another number and leaves the number unchanged. Thus, one is called the **identity element for multiplication** or the **multiplicative identity.**

Identity Properties

For any real number *a,* the following are true.

$a + 0 = 0 + a = a$

$a \cdot 1 = 1 \cdot a = a$

CLASSROOM EXAMPLE 2 Using the Identity Property $1 \cdot a = a$

Simplify each expression.

$x – 3x$ **Solution:**

$= 1x – 3x$

$= 1x – 3x$ **Identity property.**

$= (1 – 3)x$ **Distributive property.**

$= –2x$ **Subtract inside parentheses.**

$–(3 + 4p)$

$= –1(3 + 4p)$

$= –1(3) + (–1)(4p)$ **Identity property.**

$= –3 – 4p$ **Multiply.**

Use the inverse properties.

The ***additive inverse*** (or ***opposite***) of a number *a* is − *a*. Additive inverses have a sum of 0.

The ***multiplicative inverse*** (or ***reciprocal***) of a number *a* is $\frac{1}{a}$. Multiplicative inverses have a product of 1.

Inverse Properties

For any real number *a*, the following are true.

$a + (-a) = 0$ and $-a + a = 0$

$a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1 \quad (a \neq 0).$

The inverse properties "undo" addition or multiplication.

Use the inverse properties.

A **term** is a number or the product of a number and one or more variables raised to powers.

The numerical factor in a term is called the **numerical coefficient,** or just the **coefficient.**

Terms with exactly the same variables raised to exactly the same powers are called **like terms.**

$5y$ and $-21y$	$-6x^2$ and $9x^2$	**Like terms**
$3m$ and $16x$	$7y^3$ and $-3y^2$	**Unlike terms**

Remember that only like terms may be combined.

Use the commutative and associative properties.

Commutative and Associative Properties

For any real numbers *a, b,* and *c*, the following are true.

$a + b = b + a$

$ab = ba$

Commutative properties

The order of the two terms or factors changes.

$a + (b + c) = (a + b) + c$

$a(bc) = (ab)c$

Associative properties

The grouping among the terms or factors changes, but the order stays the same.

CLASSROOM EXAMPLE 3

Using the Commutative and Associative Properties

Simplify.

Solution:

$12b - 9 + 4b - 7b + 1$

$= (12b + 4b) - 9 - 7b + 1$

$= (12 + 4)b - 9 - 7b + 1$

$= 16b - 9 - 7b + 1$

$= (16b - 7b) - 9 + 1$

$= (16 - 7)b - 9 + 1$

$= 9b - 8$

CLASSROOM EXAMPLE 4

Using the Properties of Real Numbers

Simplify each expression.

Solution:

$12b - 9b + 5b - 7b$

$= (12 - 9 + 5 - 7)b$ **Distributive property.**

$= b$ **Combine like terms.**

$6 - (2x + 7) - 3$

$= 6 - 2x - 7 - 3$ **Distributive property.**

$= -2x + 6 - 7 - 3$ **Commutative property.**

$= -2x - 4$ **Combine like terms.**

CLASSROOM EXAMPLE 4

Using the Properties of Real Numbers (cont'd)

$4m(2n)$

Solution:

$= (4)(2)mn$

$= 8mn$