2.1 Linear Equations in One Variable

Objectives

1 Distinguish between expressions and equations.

2 Identify linear equations, and decide whether a number is a solution of a linear equation.

3 Solve linear equations by using the addition and multiplication properties of equality.

4 Solve linear equations by using the distributive property.

5 Solve linear equations with fractions or decimals.

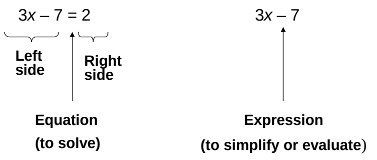
6 Identify conditional equations, contradictions, and identities.

Distinguish between expressions and equations.

Equations and inequalities compare algebraic expressions.

An ***equation*** is a statement that two algebraic expressions are equal.

An equation always contains an equals symbol, while an expression does not.



$3x - 7 = 2$

Left side

Right side

Equation

(to solve)

$3x - 7$

Expression

(to simplify or evaluate)

CLASSROOM EXAMPLE 1 **Distinguishing between Expressions and Equations**

Decide whether each of the following is an ***equation*** or an ***expression***.

	Solution:
$9x + 10 = 0$	**equation**
$9x + 10$	**expression**

Objective 2

Identify linear equations, and decide whether a number is a solution of a linear equation.

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Linear Equation in One Variable

A **linear equation in one variable** can be written in the form

$$Ax + B = C,$$

where A, B, and C are real numbers, with $A \neq 0$.

A linear equation is a **first-degree equation,** since the greatest power on the variable is 1.

Identify linear equations, and decide whether a number is a solution of a linear equation.

If the variable in an equation can be replaced by a real number that makes the statement true, then that number is a **solution** of the equation.

An equation is ***solved*** by finding its **solution set,** the set of all solutions.

Equivalent equations are related equations that have the same solution set.

Objective 3

Solve linear equations by using the addition and multiplication properties of equality.

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Addition and Multiplication Properties of Equality

Addition Property of Equality

For all real numbers *A, B*, and *C*, the equations

$\boldsymbol{A = B}$ and $\boldsymbol{A + C = B + C}$

are equivalent.

That is, ***the same number may be added to each side of an equation without changing the solution set.***

Multiplication Property of Equality

For all real numbers *A,* and *B*, and for $C \neq 0$, the equations

$\boldsymbol{A = B}$ and $\boldsymbol{AC = BC}$

are equivalent.

That is, ***each side of the equation may be multiplied by the same nonzero number without changing the solution set.***

CLASSROOM EXAMPLE 2 **Using the Properties of Equality to Solve a Linear Equation**

Solve.

$4x + 8x = -9 + 17x - 1$

Solution:

The goal is to isolate *x* on one side of the equation.

$12x = -10 + 17x$	**Combine like terms.**
$12x - 17x = -10 + 17x - 17x$	**Subtract 17*x* from each side.**
$\dfrac{-5x}{-5} = \dfrac{-10}{-5}$	**Divide each side by –5.**
$x = 2$	

Check *x* = 2 in the original equation.

CLASSROOM EXAMPLE 2 **Using the Properties of Equality to Solve a Linear Equation (cont'd)**

Check *x* = 2 in the original equation.

$$4x + 8x = -9 + 17x - 1$$

$$4(2) + 8(2) = -9 + 17(2) - 1$$

Use parentheses around substituted values to avoid errors.

$$8 + 16 = -9 + 34 - 1$$

$$24 = 24$$

This is NOT the solution.

The true statement indicates that {2} is the solution set.

Solve linear equations by using the addition and multiplication properties of equality.

Solving a Linear Equation in One Variable

Step 1 **Clear fractions or decimals.** Eliminate fractions by multiplying each side by the least common denominator. Eliminate decimals by multiplying by a power of 10.

Step 2 **Simplify each side separately.** Use the distributive property to clear parentheses and combine like terms as needed.

Step 3 **Isolate the variable terms on one side.** Use the addition property to get all terms with variables on one side of the equation and all numbers on the other.

Step 4 **Isolate the variable.** Use the multiplication property to get an equation with just the variable (with coefficient 1) on one side.

Step 5 **Check.** Substitute the proposed solution into the original equation.

Objective 4

Solve linear equations by using the distributive property.

CLASSROOM EXAMPLE 3

Using the Distributive Property to Solve a Linear Equation

Solve.

$6 – (4 + x) = 8x – 2(3x + 5)$

Solution:

Step 1 Since there are no fractions in the equation, ***Step 1*** does not apply.

Step 2 Use the distributive property to simplify and combine like terms on the left and right.

$6 – (1)4 – (1)x = 8x – 2(3x) + (–2)(5)$ **Distributive property.**

$6 – 4 – x = 8x – 6x – 10$ **Multiply.**

$2 – x = 2x – 10$ **Combine like terms.**

CLASSROOM EXAMPLE 3

Using the Distributive Property to Solve a Linear Equation (cont'd)

Step 3 Next, use the addition property of equality.

$2 – 2 – x = 2x – 10 – 2$ **Subtract 2.**

$–x = 2x – 12$ **Combine like terms.**

$–x – 2x = 2x – 2x – 12$ **Subtract 2*x***

$–3x = –12$ **Combine like terms.**

Step 4 Use the multiplication property of equality to isolate *x* on the left side.

$$\frac{–3x}{–3} = \frac{–12}{–3}$$

$$x = 4$$

CLASSROOM EXAMPLE 3

Using the Distributive Property to Solve a Linear Equation (cont'd)

Step 5 Check. $6 – (4 + x) = 8x– 2(3x + 5)$

$6 – (4 + 4) = 8(4) – 2(3(4) + 5)$

$6 – 8 = 32 – 2(12 + 5)$

$–2 = 32 – 2(17)$

$–2 = 32 – 34$

$–2 = –2$ **True**

The solution checks, so {4} is the solution set.

CLASSROOM EXAMPLE 4

Solving a Linear Equation with Fractions

Solve.

$$\frac{x+1}{2}+\frac{x+3}{4}=\frac{1}{2}$$

Solution:

Step 1 Start by eliminating the fractions. Multiply both sides by the LCD.

$$4\left(\frac{x+1}{2}+\frac{x+3}{4}\right)=4\left(\frac{1}{2}\right)$$

Step 2

$$4\left(\frac{x+1}{2}\right)+4\left(\frac{x+3}{4}\right)=4\left(\frac{1}{2}\right)$$ **Distributive property.**

$$\frac{4(x+1)}{2}+\frac{4(x+3)}{4}=2$$ **Multiply; 4.**

CLASSROOM EXAMPLE 4

Solving a Linear Equation with Fractions (cont'd)

$$\frac{4(x+1)}{2}+\frac{4(x+3)}{4}=2$$

$$2(x+1)+x+3=2$$

$$2(x)+2(1)+x+3=2$$ **Distributive property.**

$$2x+2+x+3=2$$ **Multiply.**

$$3x+5=2$$ **Combine like terms.**

Step 3 $$3x+5-5=2-5$$ **Subtract 5.**

$$3x=-3$$ **Combine like terms.**

Step 4 $$\frac{3x}{3}=\frac{-3}{3}$$ **Divide by 3.**

$$x=-1$$

CLASSROOM EXAMPLE 4

Solving a Linear Equation with Fractions (cont'd)

Step 5 Check.

$$\frac{(x+1)}{2}+\frac{(x+3)}{4}=\frac{1}{2}$$

$$\frac{(x+1)}{2}+\frac{(x+3)}{4}=\frac{1}{2}$$

$$\frac{(-1+1)}{2}+\frac{(-1+3)}{4}=\frac{1}{2}$$

$$\frac{0}{2}+\frac{2}{4}=\frac{1}{2}$$

$$\frac{1}{2}=\frac{1}{2}$$

The solution checks, so the solution set is {–1}.

CLASSROOM EXAMPLE 5

Solving a Linear Equation with Decimals

Solve.

$0.02(60) + 0.04x = 0.03(50 + x)$

Solution:

$$2(60) + 4x = 3(50 + x)$$

$$120 + 4x = 150 + 3x$$

$$120 - 120 + 4x = 150 - 120 + 3x$$

$$4x = 30 + 3x$$

$$4x - 3x = 30 + 3x - 3x$$

$$x = 30$$

Since each decimal number is given in hundredths, multiply both sides of the equation by 100.

Objective 6

Identify conditional equations, contradictions, and identities.

Identify conditional equations, contradictions, and identities.

Type of Linear Equation	*Number of Solutions*	*Indication when Solving*
Conditional	One	Final line is $x = a$ number.
Identity	Infinite; solution set {all real numbers}	Final line is true, such as 0 = 0.
Contradiction	None; solution set ∅	Final line is false, such as –15 = –20 .

CLASSROOM EXAMPLE 6

Recognizing Conditional Equations, Identities, and Contradictions

Solve each equation. Decide whether it is a ***conditional equation,*** an ***identity,*** or a ***contradiction*.**

$5(x + 2) - 2(x + 1) = 3x + 1$

Solution:

$$5x + 10 - 2x - 2 = 3x + 1$$

$$3x + 8 = 3x + 1$$

$$3x - 3x + 8 = 3x - 3x + 1$$

$$8 = 1 \qquad \textbf{False}$$

The result is false, the equation has no solution. The equation is a contradiction. The solution set is ∅.

CLASSROOM EXAMPLE 6

Recognizing Conditional Equations, Identities, and Contradictions (cont'd)

$$\frac{x+1}{3} + \frac{2x}{3} = x + \frac{1}{3}$$

Solution:

$$3\left(\frac{x+1}{3}\right) + 3\left(\frac{2x}{3}\right) = 3\left(x + \frac{1}{3}\right)$$

$$x + 1 + 2x = 3x + 1$$

$$3x + 1 = 3x + 1$$

This is an identity. Any real number will make the equation true. The solution set is {all real numbers}.

CLASSROOM EXAMPLE 6

Recognizing Conditional Equations, Identities, and Contradictions (cont'd)

$5(3x + 1) = x + 5$

Solution:

$$15x + 5 = x + 5$$

$$15x - x + 5 = x - x + 5$$

$$14x + 5 = 5$$

$$14x + 5 - 5 = 5 - 5$$

$$14x = 0$$

$$x = 0$$

This is a conditional equation. The solution set is {0}.