2.2 Formulas and Percent

Objectives

1. Solve a formula for a specified variable.
2. Solve applied problems by using formulas.
3. Solve percent problems.
4. Solve problems involving percent increase or decrease.

PEARSON

Formulas and Percent

A **mathematical model** is an equation or inequality that describes a real situation. Models for many applied problems, called ***formulas,*** already exist. A **formula** is an equation in which variables are used to describe a relationship.

A few commonly used formulas are:

$d = rt$, $I = prt$, $A = \frac{1}{2}bh$, and $P = 2L + 2W$

Distance Formula | **Interest Formula** | **Area of a Triangle Formula** | **Perimeter of a Rectangle Formula**

Objective 1

Solve a formula for a specified variable.

Solve a formula for a specified variable.

When solving for a specified variable, the key is to treat that variable as if it were the only one. Treat all other variables like numbers (constants).

Solving for a Specified Variable

Step 1 If the equation contains fractions, multiply both sides by the LCD to clear the fractions.

Step 2 Transform so that all terms containing the specified variable are on one side of the equation and all terms without that variable are on the other side.

Step 3 Divide each side by the factor that is the coefficient of the specified variable.

CLASSROOM EXAMPLE 1 **Solving for a Specified Variable**

Solve the formula $d = rt$ for r.

Solution:

Solve the formula by isolating the r on one side of the equals sign.

$$\frac{d}{t} = \frac{rt}{t}$$ **Divide by 2.**

$$r = \frac{d}{t}$$

CLASSROOM EXAMPLE 2 **Solving for a Specified Variable**

Solve the formula for L.

$P = 2L + 2W$

Solution:

$P - 2W = 2L + 2W - 2W$ **Subtract $2W$ from both sides.**

$P - 2W = 2L$ **Combine like terms.**

$\frac{P - 2W}{2} = \frac{2L}{2}$ **Divide both sides by 2 to isolate L.**

$$L = \frac{P - 2W}{2}$$

CLASSROOM EXAMPLE 3 **Solving a Formula Involving Parentheses**

Solve the equation for x.

$$y = \frac{1}{2}(x+3)$$

Solution:

$$y = \frac{x}{2} + \frac{3}{2}$$ **Use distributive property on the right side to eliminate the parentheses.**

$$2y = x + 3$$ **Multiply both sides by 2 to eliminate fractions.**

$$2y - 3 = x \quad \text{or} \quad x = 2y - 3$$ **Subtract 3 from both sides.**

CLASSROOM EXAMPLE 4 **Solving an Equation for One of the Variables**

Solve the equation for y.

$$2x + 7y = 5$$

Solution:

$$2x + 7y - 2x = 5 - 2x$$ **Subtract 2*x* from both sides.**

$$7y = 5 - 2x$$ **Combine like terms.**

$$y = \frac{5 - 2x}{7}$$ **Divide both sides by 7.**

Objective 2

Solve applied problems by using formulas.

CLASSROOM EXAMPLE 5 **Finding Average Rate**

It takes James Harmon one third of an hour to travel 15 miles. What is his average rate?

Solution:

Find the rate by using the formula *d* = *rt* and solving for *r*.

$$15 = r \cdot \frac{1}{3}$$ **Multiply both sides by 3.**

$$45 = r$$

Average rate of speed is 45 mph.

Solve percent problems.

An important everyday use of mathematics involves the concept of percent. Percent is written with the symbol %. The word **percent** means "per one hundred".

1% = 0.01 or $1\% = \frac{1}{100}$

Solving a Percent Problem

Let a represent a partial amount of b, the base, or whole amount. Then the following equation can be used to solve a percent problem.

$$\frac{\text{partial amount } a}{\text{base } b} = \text{percent (represented as a decimal)}$$

CLASSROOM EXAMPLE 6 **Solving Percent Problems**

Solve each problem.

A mixture of gasoline oil contains 20 oz, of which 1 oz is oil. What percent of the mixture is oil?

Solution:

The whole amount of mixture is 20 oz. The part that is oil is 1 oz.

$$x = \frac{1}{20}$$ ← **partial amount** ← **whole amount**

$$x = 0.05, \quad \text{or} \quad 5\%.$$

Thus, 5% of the mixture is oil.

CLASSROOM EXAMPLE 6 **Solving Percent Problems (cont'd)**

An automobile salesman earns an 8% commission on every car he sells. How much does he earn on a car that sells for $12,000?

Solution:

Let *x* represent the amount of commission earned.
8% = 8 · 0.01 = 0.08

$$\frac{x}{12,000} = 0.08 \qquad \frac{\text{partial}}{\text{whole}} = \text{percent}$$

$$x = 0.08(12,000) \quad \textbf{Multiply by 12,000.}$$

$$x = 960$$

The salesman earns $960.

CLASSROOM EXAMPLE 7 **Interpreting Percents from a Graph**

In 2007, Americans spent about $41.2 billion on their pets. Use the graph to determine how much was spent on pet supplies/medicine? Round your answer to the nearest tenth of a billion dollars.

Solution:

Let *x* represent the amount spent on pet supplies/medicine.

$$\frac{x}{41.2} = 0.238$$

$$x = 0.238(41.2)$$

$$x = 9.8056$$

Therefore, about $9.8 billion was spent on pet supplies/medicine.

Objective 4

Solve problems involving percent increase or decrease.

Solve problems involving percent increase or decrease.

Percent is often used to express a change in some quantity. To solve problems of this type, we use the following form of the percent equation.

$$\text{percent change} = \frac{\text{amount of change}}{\text{base}}$$

When calculating percent increase or decrease, be sure that you use the original number (***before*** the change) as the base. A common error is to use the final number (***after*** the change) in the denominator of the fraction.

CLASSROOM EXAMPLE 8 **Solving Problems about Percent Increase or Decrease**

A cost-of-living salary increase resulted in Keith's monthly salary to go from $1300 to $1352. What percent increase was this?

Solution:

Let *x* represent the percent increase in salary.

$$\text{percent increase} = \frac{\text{amount of change}}{\text{base}}$$

$$x = \frac{1352 - 1300}{1300}$$

$$x = \frac{52}{1300}$$

$$x = 0.04$$

The increase in salary was 4%.

CLASSROOM EXAMPLE 8 **Solving Problems about Percent Increase or Decrease (cont'd)**

The price of a concert ticket was changed from $54.00 to $51.30. What percent decrease was this?

Solution:

Let *x* represent the percent decrease in ticket price.

$$\text{percent decrease} = \frac{\text{amount of change}}{\text{base}}$$

$$x = \frac{54.00 - 51.30}{54.00}$$

$$x = \frac{2.70}{54.00}$$

$$x = 0.05$$

The decrease in ticket price was 5%.