2.3 Applications of Linear Equations

Objectives

1. Translate from words to mathematical expressions.
2. Write equations from given information.
3. Distinguish between simplifying expressions and solving equations.
4. Use the six steps in solving an applied problem.
5. Solve percent problems.
6. Solve investment problems.
7. Solve mixture problems.

Translate from words to mathematical expressions.

PROBLEM-SOLVING HINT

There are usually key words and phrases in a verbal problem that translate into mathematical expressions involving addition, subtraction, multiplication and division. Translations of some commonly used expressions follow.

Translate from words to mathematical expressions.

Verbal Expression	*Mathematical Expression (where x and y are numbers)*
Addition	
The **sum** of a number and 7	$x + 7$
6 **more than** a number	$x + 6$
3 **plus** a number	$3 + x$
24 **added to** a number	$x + 24$
A number **increased by** 5	$x + 5$
The **sum** of two numbers	$x + y$

Translate from words to mathematical expressions.

Verbal Expression	*Mathematical Expression (where x and y are numbers)*
Subtraction	
2 **less than** a number	$x – 2$
2 **less** a number	$2 – x$
12 **minus** a number	$12 – x$
A number **decreased by** 12	$x – 12$
A number **subtracted from** 10	$10 – x$
From a number, **subtract** 10	$x – 10$
The **difference between** two numbers	$x – y$

Translate from words to mathematical expressions.

Verbal Expression	*Mathematical Expression (where x and y are numbers)*
Multiplication	
16 **times** a number	$16x$
A number **multiplied by** 6	$6x$
2/3 **of** a number (used with fractions and percent)	$\frac{2}{3}x$
¾ **as much as** a number	$\frac{3}{4}x$
Twice (2 times) a number	$2x$
The **product** of two numbers	xy

Translate from words to mathematical expressions.

Verbal Expression	*Mathematical Expression (where x and y are numbers)*
Division	
The quotient of 8 and a number	$\frac{8}{x}\ (x \neq 0)$
A number **divided by** 13	$\frac{x}{13}$
The **ratio** of two numbers or the **quotient** of two numbers	$\frac{x}{y}\ (y \neq 0)$

CLASSROOM EXAMPLE 1 **Translating Words into Equations**

Translate each verbal sentence into an equation, using *x* as the variable.

Solution:

The sum of a number and 6 is 28.

$$x + 6 = 28$$

The product of a number and 7 is twice the number plus 12.

$$7x = 2x + 12$$

The quotient of a number and 6, added to twice the number is 7.

$$2x + \frac{x}{6} = 7$$

Objective 3

Distinguish between simplifying expressions and solving equations.

Distinguish between simplifying expressions and solving equations.

An ***expression*** translates as a phrase.

An ***equation*** includes the = symbol, with expressions on both sides, and translates as a sentence.

CLASSROOM EXAMPLE 2 **Distinguishing between Simplifying Expressions and Solving Equations**

Decide whether each is an ***expression*** or an ***equation***. Simplify any expressions, and solve any equations.

Solution:

$5x – 3(x + 2) = 7$

This is an equation. There is an equals symbol with an expression on either side of it.

$$5x - 3x - 6 = 7 \qquad 2x = 13$$

$$2x - 6 = 7 \qquad x = \frac{13}{2}$$

$5x – 3(x + 2)$

This is an expression. There is no equals symbol.

$$5x - 3x - 6$$

$$2x - 6$$

Use the six steps in solving an applied problem.

Solving an Applied Problem

Step 1 **Read** the problem, several times if necessary. What information is given? What is to be found?

Step 2 **Assign a variable** to represent the unknown value. Use a sketch, diagram, or table, as needed. Write down what the variable represents. If necessary, express any other unknown values in terms of the variable.

Step 3 **Write an equation** using the variable expression(s).

Step 4 **Solve** the equation.

Step 5 **State the answer.** Label it appropriately. Does it seem reasonable?

Step 6 **Check** the answer in the words of the ***original*** problem.

CLASSROOM EXAMPLE 3 **Solving a Perimeter Problem**

The length of a rectangle is 5 cm more than its width. The perimeter is five times the width. What are the dimensions of the rectangle?

Solution:

Step 1 **Read** the problem. What must be found?

The length and width of the rectangle.

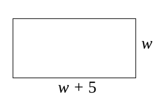
What is given?

The length is 5 cm more than its width; the perimeter is 5 times the width.

Step 2 **Assign a variable.**

Let W = the width; then $W + 5$ = the length.

Make a sketch.



CLASSROOM EXAMPLE 3 — **Solving a Perimeter Problem (cont'd)**

Step 3 **Write an equation.**

Use the formula for the perimeter of a rectangle.

$P = 2L + 2W$

$5W = 2(W + 5) + 2(W)$

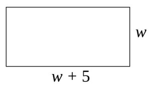
Step 4 **Solve** the equation.

$5W = 2W + 10 + 2W$

$5W = 4W + 10$

$5W - 4W = 4W - 4W + 10$

$W = 10$



CLASSROOM EXAMPLE 3 — **Solving a Perimeter Problem (cont'd)**

Step 5 **State the answer.**

The width of the rectangle is 10 cm and the length is 10 + 5 = 15 cm.

Step 6 **Check.**

The perimeter is 5 times the width.

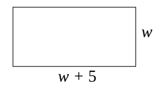
$P = 2L + 2W$

$5W = 2(15) + 2(10)$

$50 = 30 + 20$

$50 = 50$

The solution checks.



CLASSROOM EXAMPLE 4 — **Finding Unknown Numerical Quantities**

For the 2009 regular season, the MLB leaders in number of hits were Ichiro Suzuki and Derek Jeter. These two players had a total of 437 hits. Suzuki had 13 more hits than Jeter. How many hits did each player have? (*Source:* www.mlb.com)

Solution:

Step 1 **Read** the problem. What are we asked to find?

The number of hits for each player.

What is given?

Total hits for both players was 437.

Suzuki had 13 more hits than Jeter.

CLASSROOM EXAMPLE 4 — **Finding Unknown Numerical Quantities (cont'd)**

Step 2 **Assign a variable.**

Let x represents the number of hits for Jeter.

Let $x + 13$ represent the number of hits for Suzuki.

Step 3 **Write an equation**.

The sum of the hits is 437.

$x + x + 13 = 437$

Step 4 **Solve** the equation.

$x + x + 13 = 437$

$2x + 13 = 437$

$2x + 13 - 13 = 437 - 13$

$2x = 424$

$x = 212$

CLASSROOM EXAMPLE 4 — **Finding Unknown Numerical Quantities (cont'd)**

Step 5 **State the answer.**

We let x represent the number of hits for Jeter, so Jeter had 212 hits.

Then Suzuki has *212* + 13, or 225 hits.

Step 6 **Check.**

225 is 13 more than 212, and 212 + 225 = 437.

The conditions of the problem are satisfied, and our answer checks.

CLASSROOM EXAMPLE 5 — **Solving a Percent Problem**

In 2002, there were 301 long-distance area codes in the United States, an increase of 250% over the number when the area code plan originated in 1947. How many area codes were there in 1947? (*Source:* SBC Telephone Directory.)

Solution:

Step 1 **Read** the problem. What are we being asked to find?

The number of area codes in 1947.

What are we given?

The number of area codes in 2002 and the percent increase from 1947 to 2002.

Step 2 **Assign a variable.**

Let x = the number of area codes in 1947.

Then, 2.5x represents the number of codes in 2002.

Step 3 **Write an equation** from the given information.

$x + 2.5x = 301$

CLASSROOM EXAMPLE 5 Solving a Percent Problem (cont'd)

Step 4 **Solve** the equation.

$x + 2.5x = 301$

$3.5x = 301$

$$\frac{3.5x}{3.5} = \frac{301}{3.5}$$

$x = 86$

Step 5 **State the answer.**

The number of area codes in 1947 was 86 and the increase in the number of area codes between 1947 and 2002 was 215 (2.5 X 86).

Step 6 **Check.**

86 area codes in 1947 + an increase of 215 codes = 301 total area codes by 2002

Objective 6

Solve investment problems.

CLASSROOM EXAMPLE 6 Solving an Investment Problem

A man has $34,000 to invest. He invests some of the money at 5% and the balance at 4%. His total annual interest income is $1545. Find the amount invested at each rate.

Solution:

Step 1 **Read** the problem. What is to be found?

We must find the two amounts; the amount invested at 5% and the amount invested at 4%.

What information is given?

The total to invest and the interest earned.

Step 2 **Assign a variable.**

Let x = the amount to invest at 5%

34,000 – x = the amount to invest at 4%

CLASSROOM EXAMPLE 6 Solving an Investment Problem (cont'd)

Step 2 **Assign a variable.** Use a table to organize the given information.

Principal	*Rate (as a decimal)*	*Interest*
x	0.05	$0.05x$
34,000 – x	0.04	0.04(34,000 – x)
34,000	XXXXXXXXX	1545

Step 3 **Write an equation.**

The formula for simple interest is $I = prt.$

Here the time is 1 yr.

$0.05x + 0.04(34{,}000 – x) = 1545$

interest at 5% + interest at 4% = total interest

CLASSROOM EXAMPLE 6 Solving an Investment Problem (cont'd)

Step 4 **Solve** the equation.

$0.05x + 0.04(34{,}000 – x) = 1545$

$0.05x + 1360 – 0.04x = 1545$

$0.01x + 1360 = 1545$

$0.01x = 185$

$x = 18{,}500$

Step 5 **State the answer.**

$18,500 was invested at 5%; 15,500 at 4%.

Step 6 **Check** by finding the annual interest at each rate.

0.05(18,500) = $925 0.04(15,500) = $620

$925 + $620 = $1545

Solve investment problems.

PROBLEM-SOLVING HINT

In the **Example 6,** we chose to let the variable x represent the amount invested at 5%. It would have also been acceptable to let x represent the amount invested at 4% instead. The equation to solve would have been different, but in the end the answers would be the same.

Objective 7

Solve mixture problems.

CLASSROOM EXAMPLE 7 **Solving a Mixture Problem**

How many pounds of candy worth $8 per lb should be mixed with 100 lb of candy worth $4 per lb to get a mixture that can be sold for $7 per lb?

Solution:

Step 1 **Read** the problem. What is to be found?

How much candy worth $8 per lb is to be used.

What is given?

The amount used at $4 per lb and the selling price per pound.

Step 2 **Assign a variable.**

Let x = the amount of $8 per lb candy.

Number of pounds	***$ Amount***	***Pounds of Candy worth $7***
100	$4	100(4) = 400
x	$8	$8x$
100 + x	$7	7(100 + x)

CLASSROOM EXAMPLE 7 **Solving a Mixture Problem (cont'd)**

Step 3 **Write an equation.**

$400 + 8x = 7(100 + x)$

Step 4 **Solve.**

$400 + 8x = 700 + 7x$ **Subtract 400 and 7*x*.**

$x = 300$

Step 5 **State the answer.**

300 ibs of candy worth $8 per pound should be used.

Step 6 **Check.**

300 lb worth $8 + 100 lb worth $4 = $7(100 + 300)

$2400 + $400 = $7(400)

$2800 = $2800

Solve mixture problems.

PROBLEM-SOLVING HINT

In the **Example 8,** remember that when pure water is added to a solution, water is 0% of the chemical (acid, alcohol, etc.). Similarly, pure chemical is 100% chemical.

CLASSROOM EXAMPLE 8 **Solving a Mixture Problem When One Ingredient is Pure**

How much water must be added to 20 L of 50% antifreeze solution to reduce it to 40% antifreeze?

Solution:

Step 1 **Read** the problem. What is to be found?

What amount of pure water is to be added.

What is given?

The amount of antifreeze and its purity percentage.

Step 2 **Assign a variable.** Use a table.

Let x = the number of liters of pure water.

Number of liters	***Percent (as a decimal)***	***Liters of Pure Antifreeze***
x	0	0
20	0.5	20(0.5)
x + 20	0.4	0.4(x + 20)

CLASSROOM EXAMPLE 8 **Solving a Mixture Problem When One Ingredient is Pure (cont'd)**

Step 3 **Write an equation.**

$0 + 20(0.5) = 0.4(x + 20)$

Step 4 **Solve.**

$10 = 0.4x + 8$

$2 = 0.4x$

$x = 5$

Step 5 **State the answer.**

5 L of water are needed.

Step 6 **Check.**

$20(0.5) = 0.4(5 + 20)$

$10 = 0.4(25)$

$10 = 10$