2.4 Further Applications of Linear Equations

Objectives

1. Solve problems about different denominations of money.
2. Solve problems about uniform motion.
3. Solve problems about angles.

PEARSON

Objective 1

Solve problems about different denominations of money.

Solve problems about different denominations of money.

PROBLEM-SOLVING HINT

In problems involving money, use the basic fact that

Number of monetary units of the same kind **× denomination =** **total monetary value**

For example, 30 dimes have a monetary value of 30($0.10) = $3.00.
Fifteen 5-dollar bills have a value of 15($5) = $75.

CLASSROOM EXAMPLE 1 **Solving a Money Denomination Problem**

Mohammed has a box of coins containing only dimes and half-dollars. There are 26 coins, and the total value is $8.60. How many of each denomination of coin does he have?

Solution:

Step 1 **Read** the problem. What is being asked?

To find the number of each denomination of coin.

What is given?

The total number of coins and the total value.

Step 2 **Assign a variable.** Then, organize a table.

Let x = the number of dimes.

Let $26 - x$ = number of half-dollars.

CLASSROOM EXAMPLE 1 **Solving a Money Denomination Problem (cont'd)**

Number of Coins	*Denominations*	*Value*
x	0.10	$0.10x$
$26 - x$	0.50	$0.50(26 - x)$
XXXXXXX	Total	8.60

Multiply the number of coins by the denominations, and add the results to get 8.60

Step 3 **Write an equation**.

$$0.10x + 0.50(26 - x) = 8.60$$

CLASSROOM EXAMPLE 1 **Solving a Money Denomination Problem (cont'd)**

Step 4 **Solve.**

$$0.10x + 0.50(26 - x) = 8.60$$

Move decimal point 1 place to the right

$$1x + 5(26 - x) = 86$$ **Multiply by 10.**

$$1x + 130 - 5x = 86$$ **Distributive property.**

$$-4x = -44$$

$$x = 11$$

Step 5 **State the answer**.

He has 11 dimes and 26 – 11 = 15 half-dollars.

Step 6 **Check**.

He has 11 + 15 = 26 coins, and the value is $0.10(11) + $0.50(15) = $8.60.

CAUTION ***Be sure that your answer is reasonable*** when you are working with problems like this. Because you are working with coins, the correct answer can be neither negative nor a fraction.

Solve problems about uniform motion.

PROBLEM-SOLVING HINT

Uniform motion problems use the distance formula, $d = rt$. ***When rate (or speed) is given in miles per hour, time must be given in hours. Draw a sketch*** to illustrate what is happening. ***Make a table*** to summarize the given information.

CLASSROOM EXAMPLE 2 **Solving a Motion Problem (Motion in Opposite Directions)**

Two cars leave the same town at the same time. One travels north at 60 mph and the other south at 45 mph. In how many hours will they be 420 mi apart?

Solution:

Step 1 **Read** the problem. What is to be found?

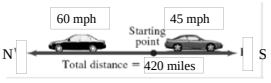
The time for the cars to be 420 miles apart.

What information is given?

Both their speeds and the distance between them.

Step 2 **Assign a variable.** Make a sketch to illustrate the situation.

Let x = the amount of time needed for the cars to be 420 mi apart.



CLASSROOM EXAMPLE 2 **Solving a Motion Problem (Motion in Opposite Directions) (cont'd)**

	Rate	***Time***	***Distance***
Northbound Car	60	x	$60x$
Southbound Car	45	x	$45x$
XXXXXXX	XXXXX	XXXXX	420

Total

Step 3 **Write an equation**.

$60x + 45x = 420$

CLASSROOM EXAMPLE 2 **Solving a Motion Problem (Motion in Opposite Directions) (cont'd)**

Step 4 **Solve**.

$$60x + 45x = 420$$

$$105x = 420$$

$$x = \frac{420}{105} = 4$$

Step 5 **State the answer**.

The cars will be 420 mi apart in 4 hr.

Step 6 **Check**.

$$60(4) + 45(4) = 420$$

$$240 + 180 = 420$$

$$420 = 420$$

It is a common error to write 420 as the distance traveled by each car. However, 420 is ***total*** distance traveled by both cars.

CLASSROOM EXAMPLE 3 **Solving a Motion Problem (Motion in the Same Direction)**

When Chris drives his car to work, the trip takes ½ hr. When he rides the bus, it takes ¾ hr. The average rate of the bus is 12 mph less than his rate when driving. Find the distance he travels to work.

Solution:

Step 1 **Read** the problem. What is to be found?

The distance Chris travels to his workplace.

What is given?

The time it takes Chris to drive, the time it takes the bus to arrive and the average rate for the bus relative to driving.

Step 2 **Assign a variable.**

Let x = the average rate of the car.

Then, $x - 12$ = average rate of the bus.

CLASSROOM EXAMPLE 3 **Solving a Motion Problem (Motion in the Same Direction) (cont'd)**

	Rate	***Time***	***Distance***
Car	x	½	½ x
Bus	$x - 12$	¾	¾ $(x - 12)$

Same

Step 3 **Write an equation**.

$$\tfrac{1}{2}x = \tfrac{3}{4}(x - 12)$$

Step 4 **Solve**.

$$\frac{1}{2}x = \frac{3}{4}(x-12)$$

$$2x = 3(x-12) \qquad \textbf{Multiply by 4.}$$

$$2x = 3x - 36$$

$$36 = x$$

CLASSROOM EXAMPLE 3 Solving a Motion Problem (Motion in the Same Direction) (cont'd)

Step 5 **State the answer**.

The required distance is

$$d = \frac{1}{2}x = \frac{1}{2}(36) = 18 \text{ miles.}$$

Step 6 **Check**.

$$d = \frac{3}{4}(x - 12)$$
$$d = \frac{3}{4}(36 - 12)$$
$$d = \frac{3}{4}(24)$$
$$d = 18 \text{ miles}$$

Same result

Solve problems about uniform motion.

PROBLEM-SOLVING HINT

As in **Example 3,** sometimes it is easier to let the variable represent a quantity other than the one that we are asked to find. It takes practice to learn when this approach works best.

Objective 3

Solve problems about angles.

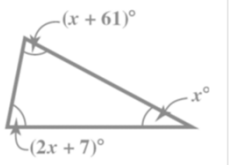
Solve problems about angles.

An important result of Euclidean geometry is that ***the sum of the angle measures of any triangle is 180°.***

CLASSROOM EXAMPLE 4 Finding Angle Measures

Find the value of x, and determine the measure of each angle.

Solution:



Step 1 **Read** the problem. What is to be found?
The measure of each angle.
What is given?
The expression for each angle relative to one another and the knowledge that the sum of all three angles combined is 180.

Step 2 **Assign a variable.**

Let x = the measure of one angle.

CLASSROOM EXAMPLE 4 Finding Angle Measures (cont'd)

Step 3 **Write an equation**. The sum of the three measures shown in the figure must be 180°.

$$x + (x + 61) + (2x + 7) = 180$$

Step 4 **Solve**.

$$4x + 68 = 180$$
$$4x = 112$$
$$x = 28$$

Step 5 **State the answer**.

The angles measure 28°, 28 + 61 = 89°, and 2(28) + 7 = 63°.

Step 6 **Check**.

28° + 89° + 63° = 180°