## 2.1 Linear Equations in One Variable

**Objectives**

1. Distinguish between expressions and equations.
2. Identify linear equations, and decide whether a number is a solution of a linear equation.
3. Solve linear equations by using the addition and multiplication properties of equality.
4. Solve linear equations by using the distributive property.
5. Solve linear equations with fractions or decimals.
6. Identify conditional equations, contradictions, and identities.

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### Distinguish between expressions and equations.

Equations and inequalities compare algebraic expressions.

An ***equation*** is a statement that two algebraic expressions are equal.

***An equation always contains an equals symbol, while an expression does not.***

$$\underbrace{3x - 7}_{\text{Left side}} = \underbrace{2}_{\text{Right side}} \qquad\qquad 3x - 7$$

**Equation**
**(to solve)**

**Expression**
**(to simplify or evaluate)**




***CLASSROOM EXAMPLE 1*** **Distinguishing between Expressions and Equations**

Decide whether each of the following is an ***equation*** or an ***expression***.

| | **Solution:** |
|---|---|
| $9x + 10 = 0$ | **equation** |
| $9x + 10$ | **expression** |




## Objective 2

## Identify linear equations, and decide whether a number is a solution of a linear equation.




### Identify linear equations, and decide whether a number is a solution of a linear equation.

**Linear Equation in One Variable**

A **linear equation in one variable** can be written in the form

$$Ax + B = C,$$

where $A$, $B$, and $C$ are real numbers, with $A \neq 0$.

A linear equation is a **first-degree equation,** since the greatest power on the variable is 1.




### Identify linear equations, and decide whether a number is a solution of a linear equation.

If the variable in an equation can be replaced by a real number that makes the statement true, then that number is a **solution** of the equation.

An equation is ***solved*** by finding its **solution set,** the set of all solutions.

**Equivalent equations** are related equations that have the same solution set.

## Objective 3

**Solve linear equations by using the addition and multiplication properties of equality.**





**Solve linear equations by using the addition and multiplication properties of equality.**

**Addition and Multiplication Properties of Equality**

**Addition Property of Equality**

For all real numbers *A*, *B*, and *C*, the equations

$A = B$ and $A + C = B + C$

are equivalent.

That is, ***the same number may be added to each side of an equation without changing the solution set.***

**Multiplication Property of Equality**

For all real numbers *A*, and *B*, and for $C \neq 0$, the equations

$A = B$ and $AC = BC$

are equivalent.

That is, ***each side of the equation may be multiplied by the same nonzero number without changing the solution set.***





***CLASSROOM EXAMPLE 2*** **Using the Properties of Equality to Solve a Linear Equation**

Solve.

$4x + 8x = -9 + 17x - 1$

**Solution:**

**The goal is to isolate *x* on one side of the equation.**

| | |
|---|---|
| $12x = -10 + 17x$ | **Combine like terms.** |
| $12x - 17x = -10 + 17x - 17x$ | **Subtract 17*x* from each side.** |
| $\frac{-5x}{-5} = \frac{-10}{-5}$ | **Divide each side by –5.** |
| $x = 2$ | |

**Check $x = 2$ in the original equation.**





***CLASSROOM EXAMPLE 2*** **Using the Properties of Equality to Solve a Linear Equation (cont'd)**

Check $x = 2$ in the original equation.

$4x + 8x = -9 + 17x - 1$

$4(2) + 8(2) = -9 + 17(2) - 1$

$8 + 16 = -9 + 34 - 1$

$24 = 24$

**Use parentheses around substituted values to avoid errors.**

**This is NOT the solution.**

**The true statement indicates that {2} is the solution set.**





**Solve linear equations by using the addition and multiplication properties of equality.**

**Solving a Linear Equation in One Variable**

***Step 1*** **Clear fractions or decimals.** Eliminate fractions by multiplying each side by the least common denominator. Eliminate decimals by multiplying by a power of 10.

***Step 2*** **Simplify each side separately.** Use the distributive property to clear parentheses and combine like terms as needed.

***Step 3*** **Isolate the variable terms on one side.** Use the addition property to get all terms with variables on one side of the equation and all numbers on the other.

***Step 4*** **Isolate the variable.** Use the multiplication property to get an equation with just the variable (with coefficient 1) on one side.

***Step 5*** **Check.** Substitute the proposed solution into the original equation.





## Objective 4

**Solve linear equations by using the distributive property.**

***CLASSROOM EXAMPLE 3*** **Using the Distributive Property to Solve a Linear Equation**

Solve.

$6 - (4 + x) = 8x - 2(3x + 5)$

**Solution:**

***Step 1*** Since there are no fractions in the equation, ***Step 1*** does not apply.

***Step 2*** Use the distributive property to simplify and combine like terms on the left and right.

$6 - (1)4 - (1)x = 8x - 2(3x) + (-2)(5)$ **Distributive property.**

$6 - 4 - x = 8x - 6x - 10$ **Multiply.**

$2 - x = 2x - 10$ **Combine like terms.**




***CLASSROOM EXAMPLE 3*** **Using the Distributive Property to Solve a Linear Equation (cont'd)**

***Step 3*** Next, use the addition property of equality.

$2 - 2 - x = 2x - 10 - 2$ **Subtract 2.**

$-x = 2x - 12$ **Combine like terms.**

$-x - 2x = 2x - 2x - 12$ **Subtract 2*x***

$-3x = -12$ **Combine like terms.**

***Step 4*** Use the multiplication property of equality to isolate *x* on the left side.

$$\frac{-3x}{-3} = \frac{-12}{-3}$$

$x = 4$




***CLASSROOM EXAMPLE 3*** **Using the Distributive Property to Solve a Linear Equation (cont'd)**

***Step 5*** Check. $6 - (4 + x) = 8x - 2(3x + 5)$

$6 - (4 + 4) = 8(4) - 2(3(4) + 5)$

$6 - 8 = 32 - 2(12 + 5)$

$-2 = 32 - 2(17)$

$-2 = 32 - 34$

$-2 = -2$ **True**

**The solution checks, so {4} is the solution set.**




***CLASSROOM EXAMPLE 4*** **Solving a Linear Equation with Fractions**

Solve.

$$\frac{x+1}{2} + \frac{x+3}{4} = \frac{1}{2}$$

**Solution:**

***Step 1*** Start by eliminating the fractions. Multiply both sides by the LCD.

$$4\left(\frac{x+1}{2} + \frac{x+3}{4}\right) = 4\left(\frac{1}{2}\right)$$

***Step 2***

$$4\left(\frac{x+1}{2}\right) + 4\left(\frac{x+3}{4}\right) = 4\left(\frac{1}{2}\right)$$ **Distributive property.**

$$\frac{4(x+1)}{2} + \frac{4(x+3)}{4} = 2$$ **Multiply; 4.**




***CLASSROOM EXAMPLE 4*** **Solving a Linear Equation with Fractions (cont'd)**

$$\frac{4(x+1)}{2} + \frac{4(x+3)}{4} = 2$$

$$2(x+1) + x + 3 = 2$$

$2(x) + 2(1) + x + 3 = 2$ **Distributive property.**

$2x + 2 + x + 3 = 2$ **Multiply.**

$3x + 5 = 2$ **Combine like terms.**

***Step 3*** $3x + 5 - 5 = 2 - 5$ **Subtract 5.**

$3x = -3$ **Combine like terms.**

***Step 4*** $\frac{3x}{3} = \frac{-3}{3}$ **Divide by 3.**

$x = -1$




***CLASSROOM EXAMPLE 4*** **Solving a Linear Equation with Fractions (cont'd)**

***Step 5*** Check.

$$\frac{(x+1)}{2} + \frac{(x+3)}{4} = \frac{1}{2}$$

$$\frac{(x+1)}{2} + \frac{(x+3)}{4} = \frac{1}{2}$$

$$\frac{(-1+1)}{2} + \frac{(-1+3)}{4} = \frac{1}{2}$$

$$\frac{0}{2} + \frac{2}{4} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

**The solution checks, so the solution set is {–1}.**

**CLASSROOM EXAMPLE 5** — Solving a Linear Equation with Decimals

Solve.

$0.02(60) + 0.04x = 0.03(50 + x)$

**Solution:**

$$2(60) + 4x = 3(50 + x)$$

$$120 + 4x = 150 + 3x$$

$$120 - 120 + 4x = 150 - 120 + 3x$$

$$4x = 30 + 3x$$

$$4x - 3x = 30 + 3x - 3x$$

$$x = 30$$

**Since each decimal number is given in hundredths, multiply both sides of the equation by 100.**




## Objective 6

## Identify conditional equations, contradictions, and identities.




### Identify conditional equations, contradictions, and identities.

| *Type of Linear Equation* | *Number of Solutions* | *Indication when Solving* |
|---|---|---|
| **Conditional** | One | Final line is $x = a$ number. |
| **Identity** | Infinite; solution set {all real numbers} | Final line is true, such as 0 = 0. |
| **Contradiction** | None; solution set ∅ | Final line is false, such as –15 = –20 . |




**CLASSROOM EXAMPLE 6** — Recognizing Conditional Equations, Identities, and Contradictions

Solve each equation. Decide whether it is a ***conditional equation,*** an ***identity,*** or a ***contradiction***.

$5(x + 2) - 2(x + 1) = 3x + 1$

**Solution:**

$$5x + 10 - 2x - 2 = 3x + 1$$

$$3x + 8 = 3x + 1$$

$$3x - 3x + 8 = 3x - 3x + 1$$

$$8 = 1 \qquad \textbf{False}$$

**The result is false, the equation has no solution. The equation is a contradiction. The solution set is ∅.**




**CLASSROOM EXAMPLE 6** — Recognizing Conditional Equations, Identities, and Contradictions (cont'd)

$$\frac{x+1}{3} + \frac{2x}{3} = x + \frac{1}{3}$$

**Solution:**

$$3\left(\frac{x+1}{3}\right) + 3\left(\frac{2x}{3}\right) = 3\left(x + \frac{1}{3}\right)$$

$$x + 1 + 2x = 3x + 1$$

$$3x + 1 = 3x + 1$$

**This is an identity. Any real number will make the equation true. The solution set is {all real numbers}.**




**CLASSROOM EXAMPLE 6** — Recognizing Conditional Equations, Identities, and Contradictions (cont'd)

$5(3x + 1) = x + 5$

**Solution:**

$$15x + 5 = x + 5$$

$$15x - x + 5 = x - x + 5$$

$$14x + 5 = 5$$

$$14x + 5 - 5 = 5 - 5$$

$$14x = 0$$

$$x = 0$$

**This is a conditional equation. The solution set is {0}.**

## 2.2 Formulas and Percent

**Objectives**

1. Solve a formula for a specified variable.
2. Solve applied problems by using formulas.
3. Solve percent problems.
4. Solve problems involving percent increase or decrease.



## Formulas and Percent

A **mathematical model** is an equation or inequality that describes a real situation. Models for many applied problems, called ***formulas,*** already exist. A **formula** is an equation in which variables are used to describe a relationship.

A few commonly used formulas are:

| $d = rt$, | $I = prt$, | $A = \frac{1}{2}bh$, | and | $P = 2L + 2W$ |
|---|---|---|---|---|
| **Distance Formula** | **Interest Formula** | **Area of a Triangle Formula** | | **Perimeter of a Rectangle Formula** |




## Objective 1

## Solve a formula for a specified variable.




## Solve a formula for a specified variable.

***When solving for a specified variable, the key is to treat that variable as if it were the only one. Treat all other variables like numbers (constants).***

**Solving for a Specified Variable**

***Step 1*** If the equation contains fractions, multiply both sides by the LCD to clear the fractions.

***Step 2*** Transform so that all terms containing the specified variable are on one side of the equation and all terms without that variable are on the other side.

***Step 3*** Divide each side by the factor that is the coefficient of the specified variable.




***CLASSROOM EXAMPLE 1*** **Solving for a Specified Variable**

Solve the formula *d* = *rt* for *r*.

**Solution:**

**Solve the formula by isolating the *r* on one side of the equals sign.**

$$\frac{d}{t} = \frac{rt}{t}$$ **Divide by 2.**

$$r = \frac{d}{t}$$




***CLASSROOM EXAMPLE 2*** **Solving for a Specified Variable**

Solve the formula for *L*.

*P* = 2*L* + 2*W*

**Solution:**

$$P - 2W = 2L + 2W - 2W$$ **Subtract 2*W* from both sides.**

$$P - 2W = 2L$$ **Combine like terms.**

$$\frac{P - 2W}{2} = \frac{2L}{2}$$ **Divide both sides by 2 to isolate *L*.**

$$L = \frac{P - 2W}{2}$$

*CLASSROOM EXAMPLE 3* **Solving a Formula Involving Parentheses**

Solve the equation for *x*.

$$y = \frac{1}{2}(x+3)$$

**Solution:**

$$y = \frac{x}{2} + \frac{3}{2}$$ **Use distributive property on the right side to eliminate the parentheses.**

$$2y = x + 3$$ **Multiply both sides by 2 to eliminate fractions.**

$$2y - 3 = x \quad \text{or} \quad x = 2y - 3$$ **Subtract 3 from both sides.**




*CLASSROOM EXAMPLE 4* **Solving an Equation for One of the Variables**

Solve the equation for *y*.

$$2x + 7y = 5$$

**Solution:**

$$2x + 7y - 2x = 5 - 2x$$ **Subtract 2*x* from both sides.**

$$7y = 5 - 2x$$ **Combine like terms.**

$$y = \frac{5-2x}{7}$$ **Divide both sides by 7.**




## Objective 2

## Solve applied problems by using formulas.




*CLASSROOM EXAMPLE 5* **Finding Average Rate**

It takes James Harmon one third of an hour to travel 15 miles. What is his average rate?

**Solution:**

**Find the rate by using the formula *d = rt* and solving for *r*.**

$$15 = r \cdot \frac{1}{3}$$ **Multiply both sides by 3.**

$$45 = r$$

**Average rate of speed is 45 mph.**




**Solve percent problems.**

An important everyday use of mathematics involves the concept of percent. Percent is written with the symbol %. The word **percent** means “per one hundred”.

1% = 0.01 or 1% = $\frac{1}{100}$

**Solving a Percent Problem**

Let *a* represent a partial amount of *b*, the base, or whole amount. Then the following equation can be used to solve a percent problem.

$$\frac{\text{partial amount } a}{\text{base } b} = \text{percent (represented as a decimal)}$$




*CLASSROOM EXAMPLE 6* **Solving Percent Problems**

Solve each problem.

A mixture of gasoline oil contains 20 oz, of which 1 oz is oil. What percent of the mixture is oil?

**Solution:**

**The whole amount of mixture is 20 oz. The part that is oil is 1 oz.**

$$x = \frac{1}{20}$$ **partial amount** (numerator) / **whole amount** (denominator)

$$x = 0.05, \quad \text{or} \quad 5\%.$$

**Thus, 5% of the mixture is oil.**

**CLASSROOM EXAMPLE 6** — **Solving Percent Problems (cont'd)**

An automobile salesman earns an 8% commission on every car he sells. How much does he earn on a car that sells for $12,000?

**Solution:**

**Let *x* represent the amount of commission earned.**

**8% = 8 · 0.01 = 0.08**

$$\frac{x}{12,000} = 0.08 \qquad \frac{\text{partial}}{\text{whole}} = \text{percent}$$

$$x = 0.08(12,000) \qquad \textbf{Multiply by 12,000.}$$

$$x = 960$$

**The salesman earns $960.**




**CLASSROOM EXAMPLE 7** — **Interpreting Percents from a Graph**

In 2007, Americans spent about $41.2 billion on their pets. Use the graph to determine how much was spent on pet supplies/medicine? Round your answer to the nearest tenth of a billion dollars.

**Solution:**

**Let *x* represent the amount spent on pet supplies/medicine.**

$$\frac{x}{41.2} = 0.238$$

$$x = 0.238(41.2)$$

$$x = 9.8056$$


**Therefore, about $9.8 billion was spent on pet supplies/medicine.**




## Objective 4

## Solve problems involving percent increase or decrease.




### Solve problems involving percent increase or decrease.

Percent is often used to express a change in some quantity. To solve problems of this type, we use the following form of the percent equation.

$$\text{percent change} = \frac{\text{amount of change}}{\text{base}}$$

CAUTION When calculating percent increase or decrease, be sure that you use the original number (***before*** the change) as the base. A common error is to use the final number (***after*** the change) in the denominator of the fraction.




**CLASSROOM EXAMPLE 8** — **Solving Problems about Percent Increase or Decrease**

A cost-of-living salary increase resulted in Keith's monthly salary to go from $1300 to $1352. What percent increase was this?

**Solution:**

**Let *x* represent the percent increase in salary.**

$$\text{percent increase} = \frac{\text{amount of change}}{\text{base}}$$

$$x = \frac{1352 - 1300}{1300}$$

$$x = \frac{52}{1300}$$

$$x = 0.04$$

**The increase in salary was 4%.**




**CLASSROOM EXAMPLE 8** — **Solving Problems about Percent Increase or Decrease (cont'd)**

The price of a concert ticket was changed from $54.00 to $51.30. What percent decrease was this?

**Solution:**

**Let *x* represent the percent decrease in ticket price.**

$$\text{percent decrease} = \frac{\text{amount of change}}{\text{base}}$$

$$x = \frac{54.00 - 51.30}{54.00}$$

$$x = \frac{2.70}{54.00}$$

$$x = 0.05$$

**The decrease in ticket price was 5%.**

## 2.3 Applications of Linear Equations

**Objectives**

1. Translate from words to mathematical expressions.
2. Write equations from given information.
3. Distinguish between simplifying expressions and solving equations.
4. Use the six steps in solving an applied problem.
5. Solve percent problems.
6. Solve investment problems.
7. Solve mixture problems.



### Translate from words to mathematical expressions.

**PROBLEM-SOLVING HINT**

There are usually key words and phrases in a verbal problem that translate into mathematical expressions involving addition, subtraction, multiplication and division. Translations of some commonly used expressions follow.




### Translate from words to mathematical expressions.

| ***Verbal Expression*** | ***Mathematical Expression (where x and y are numbers)*** |
|---|---|
| **Addition** | |
| The **sum** of a number and 7 | $x + 7$ |
| 6 **more than** a number | $x + 6$ |
| 3 **plus** a number | $3 + x$ |
| 24 **added to** a number | $x + 24$ |
| A number **increased by** 5 | $x + 5$ |
| The **sum** of two numbers | $x + y$ |




### Translate from words to mathematical expressions.

| ***Verbal Expression*** | ***Mathematical Expression (where x and y are numbers)*** |
|---|---|
| **Subtraction** | |
| 2 **less than** a number | $x - 2$ |
| 2 **less** a number | $2 - x$ |
| 12 **minus** a number | $12 - x$ |
| A number **decreased by** 12 | $x - 12$ |
| A number **subtracted from** 10 | $10 - x$ |
| **From** a number, **subtract** 10 | $x - 10$ |
| The **difference between** two numbers | $x - y$ |




### Translate from words to mathematical expressions.

| ***Verbal Expression*** | ***Mathematical Expression (where x and y are numbers)*** |
|---|---|
| **Multiplication** | |
| 16 **times** a number | $16x$ |
| A number **multiplied by** 6 | $6x$ |
| 2/3 **of** a number (used with fractions and percent) | $\frac{2}{3}x$ |
| ¾ **as much as** a number | $\frac{3}{4}x$ |
| **Twice** (2 times) a number | $2x$ |
| The **product** of two numbers | $xy$ |




### Translate from words to mathematical expressions.

| ***Verbal Expression*** | ***Mathematical Expression (where x and y are numbers)*** |
|---|---|
| **Division** | |
| The quotient of 8 and a number | $\frac{8}{x}\ (x \neq 0)$ |
| A number **divided by** 13 | $\frac{x}{13}$ |
| The **ratio** of two numbers or the **quotient** of two numbers | $\frac{x}{y}\ (y \neq 0)$ |

**CLASSROOM EXAMPLE 1** — **Translating Words into Equations**

Translate each verbal sentence into an equation, using $x$ as the variable.

**Solution:**

The sum of a number and 6 is 28.

$$x + 6 = 28$$

The product of a number and 7 is twice the number plus 12.

$$7x = 2x + 12$$

The quotient of a number and 6, added to twice the number is 7.

$$2x + \frac{x}{6} = 7$$




## Objective 3

## Distinguish between simplifying expressions and solving equations.




### Distinguish between simplifying expressions and solving equations.

An ***expression*** translates as a phrase.

An ***equation*** includes the = symbol, with expressions on both sides, and translates as a sentence.




**CLASSROOM EXAMPLE 2** — **Distinguishing between Simplifying Expressions and Solving Equations**

Decide whether each is an ***expression*** or an ***equation***. Simplify any expressions, and solve any equations.

**Solution:**

$5x – 3(x + 2) = 7$

**This is an equation. There is an equals symbol with an expression on either side of it.**

$$5x - 3x - 6 = 7 \qquad 2x = 13$$

$$2x - 6 = 7 \qquad x = \frac{13}{2}$$

$5x – 3(x + 2)$

**This is an expression. There is no equals symbol.**

$$5x - 3x - 6$$

$$2x - 6$$




### Use the six steps in solving an applied problem.

**Solving an Applied Problem**

***Step 1*** **Read** the problem, several times if necessary. What information is given? What is to be found?

***Step 2*** **Assign a variable** to represent the unknown value. Use a sketch, diagram, or table, as needed. Write down what the variable represents. If necessary, express any other unknown values in terms of the variable.

***Step 3*** **Write an equation** using the variable expression(s).

***Step 4*** **Solve** the equation.

***Step 5*** **State the answer.** Label it appropriately. Does it seem reasonable?

***Step 6*** **Check** the answer in the words of the ***original*** problem.




**CLASSROOM EXAMPLE 3** — **Solving a Perimeter Problem**

The length of a rectangle is 5 cm more than its width. The perimeter is five times the width. What are the dimensions of the rectangle?

**Solution:**

***Step 1*** **Read** the problem. What must be found?

*The length and width of the rectangle.*

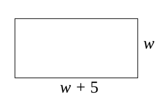
What is given?

*The length is 5 cm more than its width; the perimeter is 5 times the width*.

***Step 2*** **Assign a variable.**

Let $W$ = the width; then $W + 5$ = the length.

Make a sketch.



***CLASSROOM EXAMPLE 3*** **Solving a Perimeter Problem (cont'd)**

***Step 3*** **Write an equation.**

Use the formula for the perimeter of a rectangle.

$P = 2L + 2W$

$5W = 2(W + 5) + 2(W)$

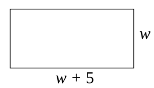
***Step 4*** **Solve** the equation.

$5W = 2W + 10 + 2W$

$5W = 4W + 10$

$5W - 4W = 4W - 4W + 10$

$W = 10$







***CLASSROOM EXAMPLE 3*** **Solving a Perimeter Problem (cont'd)**

***Step 5*** **State the answer.**

The width of the rectangle is 10 cm and the length is 10 + 5 = 15 cm.

***Step 6*** **Check.**

The perimeter is 5 times the width.

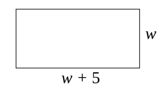
$P = 2L + 2W$

$5W = 2(15) + 2(10)$

$50 = 30 + 20$

$50 = 50$

**The solution checks.**







***CLASSROOM EXAMPLE 4*** **Finding Unknown Numerical Quantities**

For the 2009 regular season, the MLB leaders in number of hits were Ichiro Suzuki and Derek Jeter. These two players had a total of 437 hits. Suzuki had 13 more hits than Jeter. How many hits did each player have? (*Source:* www.mlb.com)

**Solution:**

***Step 1*** **Read** the problem. What are we asked to find?

The number of hits for each player.

What is given?

Total hits for both players was 437.

Suzuki had 13 more hits than Jeter.




***CLASSROOM EXAMPLE 4*** **Finding Unknown Numerical Quantities (cont'd)**

***Step 2*** **Assign a variable.**

Let $x$ represents the number of hits for Jeter.

Let $x + 13$ represent the number of hits for Suzuki.

***Step 3*** **Write an equation**.

The sum of the hits is 437.

$x + x + 13 = 437$

***Step 4*** **Solve** the equation.

$x + x + 13 = 437$

$2x + 13 = 437$

$2x + 13 - 13 = 437 - 13$

$2x = 424$

$x = 212$




***CLASSROOM EXAMPLE 4*** **Finding Unknown Numerical Quantities (cont'd)**

***Step 5*** **State the answer.**

We let $x$ represent the number of hits for Jeter, so Jeter had 212 hits.

Then Suzuki has *212* + 13, or 225 hits.

***Step 6*** **Check.**

225 is 13 more than 212, and 212 + 225 = 437.

The conditions of the problem are satisfied, and our answer checks.




***CLASSROOM EXAMPLE 5*** **Solving a Percent Problem**

In 2002, there were 301 long-distance area codes in the United States, an increase of 250% over the number when the area code plan originated in 1947. How many area codes were there in 1947? (*Source:* SBC Telephone Directory.)

**Solution:**

***Step 1*** **Read** the problem. What are we being asked to find?

The number of area codes in 1947.

What are we given?

The number of area codes in 2002 and the percent increase from 1947 to 2002.

***Step 2*** **Assign a variable.**

Let $x$ = the number of area codes in 1947.

Then, 2.5x represents the number of codes in 2002.

***Step 3*** **Write an equation** from the given information.

$x + 2.5x = 301$

**CLASSROOM EXAMPLE 5** Solving a Percent Problem (cont'd)

***Step 4*** **Solve** the equation.

$x + 2.5x = 301$

$3.5x = 301$

$$\frac{3.5x}{3.5} = \frac{301}{3.5}$$

$x = 86$

***Step 5*** **State the answer.**

The number of area codes in 1947 was 86 and the increase in the number of area codes between 1947 and 2002 was 215 (2.5 X 86).

***Step 6*** **Check.**

86 area codes in 1947 + an increase of 215 codes = 301 total area codes by 2002




## Objective 6

## Solve investment problems.




**CLASSROOM EXAMPLE 6** Solving an Investment Problem

A man has $34,000 to invest. He invests some of the money at 5% and the balance at 4%. His total annual interest income is $1545. Find the amount invested at each rate.

**Solution:**

***Step 1*** **Read** the problem. What is to be found?

We must find the two amounts; the amount invested at 5% and the amount invested at 4%.

What information is given?

The total to invest and the interest earned.

***Step 2*** **Assign a variable.**

Let $x$ = the amount to invest at 5%

$34,000 – x$ = the amount to invest at 4%




**CLASSROOM EXAMPLE 6** Solving an Investment Problem (cont'd)

***Step 2*** **Assign a variable.** Use a table to organize the given information.

| ***Principal*** | ***Rate (as a decimal)*** | ***Interest*** |
|---|---|---|
| $x$ | 0.05 | $0.05x$ |
| $34,000 – x$ | 0.04 | $0.04(34,000 – x)$ |
| 34,000 | XXXXXXXXX | 1545 |

***Step 3*** **Write an equation.**

The formula for simple interest is $I = prt$.

Here the time is 1 yr.

$0.05x + 0.04(34,000 – x) = 1545$

interest at 5% + interest at 4% = total interest




**CLASSROOM EXAMPLE 6** Solving an Investment Problem (cont'd)

***Step 4*** **Solve** the equation.

$0.05x + 0.04(34,000 – x) = 1545$

$0.05x + 1360 – 0.04x = 1545$

$0.01x + 1360 = 1545$

$0.01x = 185$

$x = 18,500$

***Step 5*** **State the answer.**

$18,500 was invested at 5%; 15,500 at 4%.

***Step 6*** **Check** by finding the annual interest at each rate.

0.05(18,500) = $925 0.04(15,500) = $620

$925 + $620 = $1545




### Solve investment problems.

### PROBLEM-SOLVING HINT

In the **Example 6,** we chose to let the variable $x$ represent the amount invested at 5%. It would have also been acceptable to let $x$ represent the amount invested at 4% instead. The equation to solve would have been different, but in the end the answers would be the same.

## Objective 7

## Solve mixture problems.




*CLASSROOM EXAMPLE 7* **Solving a Mixture Problem**

How many pounds of candy worth \$8 per lb should be mixed with 100 lb of candy worth \$4 per lb to get a mixture that can be sold for \$7 per lb?

**Solution:**

***Step 1*** **Read** the problem. What is to be found?

How much candy worth \$8 per lb is to be used.

What is given?

The amount used at \$4 per lb and the selling price per pound.

***Step 2*** **Assign a variable.**

Let $x$ = the amount of \$8 per lb candy.

| *Number of pounds* | *\$ Amount* | *Pounds of Candy worth \$7* |
|---|---|---|
| 100 | \$4 | 100(4) = 400 |
| $x$ | \$8 | $8x$ |
| 100 + $x$ | \$7 | 7(100 + $x$) |




*CLASSROOM EXAMPLE 7* **Solving a Mixture Problem (cont'd)**

***Step 3*** **Write an equation.**

$400 + 8x = 7(100 + x)$

***Step 4*** **Solve.**

$400 + 8x = 700 + 7x$ **Subtract 400 and 7*x*.**

$x = 300$

***Step 5*** **State the answer.**

300 ibs of candy worth \$8 per pound should be used.

***Step 6*** **Check.**

300 lb worth \$8 + 100 lb worth \$4 = \$7(100 + 300)

\$2400 + \$400 = \$7(400)

\$2800 = \$2800




**Solve mixture problems.**

**PROBLEM-SOLVING HINT**

In the **Example 8,** remember that when pure water is added to a solution, water is 0% of the chemical (acid, alcohol, etc.). Similarly, pure chemical is 100% chemical.




*CLASSROOM EXAMPLE 8* **Solving a Mixture Problem When One Ingredient is Pure**

How much water must be added to 20 L of 50% antifreeze solution to reduce it to 40% antifreeze?

**Solution:**

***Step 1*** **Read** the problem. What is to be found?

What amount of pure water is to be added.

What is given?

The amount of antifreeze and its purity percentage.

***Step 2*** **Assign a variable.** Use a table.

Let $x$ = the number of liters of pure water.

| *Number of liters* | *Percent (as a decimal)* | *Liters of Pure Antifreeze* |
|---|---|---|
| $x$ | 0 | 0 |
| 20 | 0.5 | 20(0.5) |
| $x$ + 20 | 0.4 | 0.4($x$ + 20) |




*CLASSROOM EXAMPLE 8* **Solving a Mixture Problem When One Ingredient is Pure (cont'd)**

***Step 3*** **Write an equation.**

$0 + 20(0.5) = 0.4(x + 20)$

***Step 4*** **Solve.**

$10 = 0.4x + 8$

$2 = 0.4x$

$x = 5$

***Step 5*** **State the answer.**

5 L of water are needed.

***Step 6*** **Check.**

$20(0.5) = 0.4(5 + 20)$

$10 = 0.4(25)$

$10 = 10$

## 2.4 Further Applications of Linear Equations

**Objectives**

1 Solve problems about different denominations of money.

2 Solve problems about uniform motion.

3 Solve problems about angles.

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## Objective 1

## Solve problems about different denominations of money.





### Solve problems about different denominations of money.

**PROBLEM-SOLVING HINT**

In problems involving money, use the basic fact that

**Number of monetary units of the same kind × denomination = total monetary value**

For example, 30 dimes have a monetary value of 30($0.10) = $3.00. Fifteen 5-dollar bills have a value of 15($5) = $75.





***CLASSROOM EXAMPLE 1*** **Solving a Money Denomination Problem**

Mohammed has a box of coins containing only dimes and half-dollars. There are 26 coins, and the total value is $8.60. How many of each denomination of coin does he have?

**Solution:**

***Step 1*** **Read** the problem. What is being asked?

To find the number of each denomination of coin.

What is given?

The total number of coins and the total value.

***Step 2*** **Assign a variable.** Then, organize a table.

Let $x$ = the number of dimes.

Let $26 - x$ = number of half-dollars.





***CLASSROOM EXAMPLE 1*** **Solving a Money Denomination Problem (cont'd)**

| *Number of Coins* | *Denominations* | *Value* |
|---|---|---|
| $x$ | 0.10 | $0.10x$ |
| $26 - x$ | 0.50 | $0.50(26 - x)$ |
| XXXXXXX | Total | 8.60 |

**Multiply the number of coins by the denominations, and add the results to get 8.60**

***Step 3*** **Write an equation**.

$$0.10x + 0.50(26 - x) = 8.60$$





***CLASSROOM EXAMPLE 1*** **Solving a Money Denomination Problem (cont'd)**

***Step 4*** **Solve.**

$$0.10x + 0.50(26 - x) = 8.60$$

$$1x + 5(26 - x) = 86 \quad \textbf{Multiply by 10.}$$

$$1x + 130 - 5x = 86 \quad \textbf{Distributive property.}$$

$$-4x = -44$$

$$x = 11$$

**Move decimal point 1 place to the right**

***Step 5*** **State the answer**.

He has 11 dimes and 26 – 11 = 15 half-dollars.

***Step 6*** **Check**.

He has 11 + 15 = 26 coins, and the value is $0.10(11) + $0.50(15) = $8.60.

CAUTION ***Be sure that your answer is reasonable*** when you are working with problems like this. Because you are working with coins, the correct answer can be neither negative nor a fraction.

## Solve problems about uniform motion.

### PROBLEM-SOLVING HINT

Uniform motion problems use the distance formula, $d = rt$. ***When rate (or speed) is given in miles per hour, time must be given in hours. Draw a sketch*** to illustrate what is happening. ***Make a table*** to summarize the given information.




***CLASSROOM EXAMPLE 2*** **Solving a Motion Problem (Motion in Opposite Directions)**

Two cars leave the same town at the same time. One travels north at 60 mph and the other south at 45 mph. In how many hours will they be 420 mi apart?

**Solution:**

***Step 1*** **Read** the problem. What is to be found?

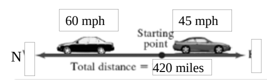
The time for the cars to be 420 miles apart.

What information is given?

Both their speeds and the distance between them.

***Step 2*** **Assign a variable.** Make a sketch to illustrate the situation.

Let $x$ = the amount of time needed for the cars to be 420 mi apart.







***CLASSROOM EXAMPLE 2*** **Solving a Motion Problem (Motion in Opposite Directions) (cont'd)**

| | ***Rate*** | ***Time*** | ***Distance*** |
|---|---|---|---|
| *Northbound Car* | 60 | $x$ | $60x$ |
| *Southbound Car* | 45 | $x$ | $45x$ |
| XXXXXXX | XXXXX | XXXXX | 420 |

**Total**

***Step 3*** **Write an equation**.

$60x + 45x = 420$




***CLASSROOM EXAMPLE 2*** **Solving a Motion Problem (Motion in Opposite Directions) (cont'd)**

***Step 4*** **Solve**.

$$60x + 45x = 420$$
$$105x = 420$$
$$x = \frac{420}{105} = 4$$

***Step 5*** **State the answer**.

The cars will be 420 mi apart in 4 hr.

***Step 6*** **Check**.

$$60(4) + 45(4) = 420$$
$$240 + 180 = 420$$
$$420 = 420$$

CAUTION It is a common error to write 420 as the distance traveled by each car. However, 420 is ***total*** distance traveled by both cars.




***CLASSROOM EXAMPLE 3*** **Solving a Motion Problem (Motion in the Same Direction)**

When Chris drives his car to work, the trip takes ½ hr. When he rides the bus, it takes ¾ hr. The average rate of the bus is 12 mph less than his rate when driving. Find the distance he travels to work.

**Solution:**

***Step 1*** **Read** the problem. What is to be found?

The distance Chris travels to his workplace.

What is given?

The time it takes Chris to drive, the time it takes the bus to arrive and the average rate for the bus relative to driving.

***Step 2*** **Assign a variable.**

Let $x$ = the average rate of the car.

Then, $x - 12$ = average rate of the bus.




***CLASSROOM EXAMPLE 3*** **Solving a Motion Problem (Motion in the Same Direction) (cont'd)**

| | ***Rate*** | ***Time*** | ***Distance*** |
|---|---|---|---|
| *Car* | $x$ | ½ | ½ $x$ |
| *Bus* | $x - 12$ | ¾ | ¾ $(x - 12)$ |

**Same**

***Step 3*** **Write an equation**.

$$\tfrac{1}{2}x = \tfrac{3}{4}(x - 12)$$

***Step 4*** **Solve**.

$$\frac{1}{2}x = \frac{3}{4}(x-12)$$
$$2x = 3(x-12) \quad \textbf{Multiply by 4.}$$
$$2x = 3x - 36$$
$$36 = x$$

**CLASSROOM EXAMPLE 3** Solving a Motion Problem (Motion in the Same Direction) (cont'd)

***Step 5*** **State the answer**.

The required distance is

$$d = \frac{1}{2}x = \frac{1}{2}(36) = 18 \text{ miles.}$$

***Step 6*** **Check**.

$$d = \frac{3}{4}(x - 12)$$
$$d = \frac{3}{4}(36 - 12)$$
$$d = \frac{3}{4}(24)$$
$$d = 18 \text{ miles}$$

**Same result**




**Solve problems about uniform motion.**

**PROBLEM-SOLVING HINT**

As in **Example 3,** sometimes it is easier to let the variable represent a quantity other than the one that we are asked to find. It takes practice to learn when this approach works best.




## Objective 3

# Solve problems about angles.



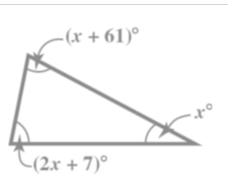

**Solve problems about angles.**

An important result of Euclidean geometry is that ***the sum of the angle measures of any triangle is 180°.***




**CLASSROOM EXAMPLE 4** **Finding Angle Measures**

Find the value of *x*, and determine the measure of each angle.




**Solution:**

***Step 1*** **Read** the problem. What is to be found?

The measure of each angle.

What is given?

The expression for each angle relative to one another and the knowledge that the sum of all three angles combined is 180.

***Step 2*** **Assign a variable.**

Let *x* = the measure of one angle.




**CLASSROOM EXAMPLE 4** **Finding Angle Measures (cont'd)**

***Step 3*** **Write an equation**. The sum of the three measures shown in the figure must be 180º.

$$x + (x + 61) + (2x + 7) = 180$$

***Step 4*** **Solve**.

$$4x + 68 = 180$$
$$4x = 112$$
$$x = 28$$

***Step 5*** **State the answer**.

The angles measure 28º, 28 + 61 = 89º, and 2(28) + 7 = 63º.

***Step 6*** **Check**.

28º + 89º + 63º = 180º

## 2.5 Linear Inequalities in One Variable

**Objectives**

1 Solve linear inequalities by using the addition property.

2 Solve linear inequalities by using the multiplication property.

3 Solve linear inequalities with three parts.

4 Solve applied problems by using linear inequalities.

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## Linear Inequalities in One Variable

| *Type of Interval* | *Set-Builder Notation* | *Interval Notation* | *Graph* |
|---|---|---|---|
| Open interval | $\{x \mid a < x < b\}$ | $(a, b)$ | a b |
| Closed interval | $\{x \mid a \le x \le b\}$ | $[a, b]$ | a b |
| Half-open (or half-closed) interval | $\{x \mid a \le x < b\}$ | $[a, b)$ | a b |
| | $\{x \mid a < x \le b\}$ | $(a, b]$ | a b |
| Disjoint interval* | $\{x \mid x < a \text{ or } x > b\}$ | $(-\infty, a) \cup (b, \infty)$ | a b |
| Infinite interval | $\{x \mid x > a\}$ | $(a, \infty)$ | a |
| | $\{x \mid x \ge a\}$ | $[a, \infty)$ | a |
| | $\{x \mid x < a\}$ | $(-\infty, a)$ | a |
| | $\{x \mid x \le a\}$ | $(-\infty, a]$ | a |
| | $\{x \mid x \text{ is a real number}\}$ | $(-\infty, \infty)$ | 0 |

Notice that a parenthesis is ***always*** used next to an infinity symbol, regardless whether it is negative or positive.




## Linear Inequalities in One Variable

An **inequality** says that two expressions are ***not*** equal. Solving inequalities is similar to solving equations.

### Linear Inequality in One Variable

A **linear inequality in one variable** can be written in the form

$$Ax + B < C,\ Ax + B \le C,\ Ax + B > C, \text{ or } Ax + B \ge C,$$

where $A$, $B$, and $C$ are real numbers, with $A \neq 0$.




## Solve linear inequalities by using the addition property.

We solve an inequality by finding all numbers that make the inequality true. Usually, an inequality has an infinite number of solutions. These solutions are found by producing a series of simpler related inequalities. **Equivalent inequalities** are inequalities with the same solution set.

### Addition Property of Inequality

For all real numbers $A$, $B$, and $C$, the inequalities

$$A < B \quad \text{and} \quad A + C < B + C$$

are equivalent.

That is, adding the same number to each side of an inequality does not change the solution set.




***CLASSROOM EXAMPLE 1***

### Using the Addition Property of Inequality

Solve $k - 5 > 1$, and graph the solution set.

**Solution:**

$k - 5 > 1$

$k - 5 + 5 > 1 + 5$ **Add 5.**

$k > 6$

**Check:**

Substitute 6 for $k$ in the equation $k - 5 = 1$

$k - 5 = 1$

$6 - 5 = 1$

$1 = 1$ **True**

This shows that 6 is a boundary point. Now test a number on each side of the 6 to verify that numbers ***greater than*** 6 make the inequality true.



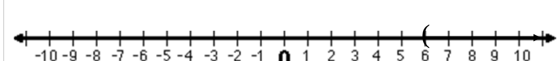

***CLASSROOM EXAMPLE 1***

### Using the Addition Property of Inequality (cont'd)

$k - 5 > 1$

| | |
|---|---|
| Let $k = 4$ | Let $k = 7$ |
| $4 - 5 > 1$ | $7 - 5 > 1$ |
| $-1 > 1$ **False** | $2 > 1$ **True** |
| $-1$ is not in the solution set | 7 is in the solution set |

**The check confirms that $(6, \infty)$, is the correct solution.**



**CLASSROOM EXAMPLE 2**

## Using the Addition Property of Inequality

Solve $5x + 3 \geq 4x - 1$, and graph the solution set.

**Solution:**

$$5x + 3 - 3 \geq 4x - 1 - 3$$
$$5x \geq 4x - 4$$
$$5x - 4x \geq 4x - 4x - 4$$
$$x \geq -4$$

**Check:**

$$5x + 3 = 4x - 1$$
$$5(-4) + 3 = 4(-4) - 1$$
$$-20 + 3 = -16 - 1$$
$$-17 = -17 \qquad \textbf{True}$$

This shows that –4 is a boundary point.



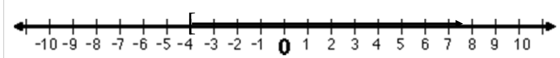

**CLASSROOM EXAMPLE 2**

## Using the Addition Property of Inequality

$$5x + 3 \geq 4x - 1$$

| Let $x = -5$ | Let $x = 0$ |
|---|---|
| $5(-5) + 3 \geq 4(-5) - 1$ | $5(0) + 3 \geq 4(0) - 1$ |
| $-25 + 3 \geq -20 - 1$ | $0 + 3 \geq 0 - 1$ |
| $-22 \geq -21$ False | $3 \geq -1$ **True** |
| –5 is not in the solution set | 0 is in the solution set |

**The check confirms that [–4, ∞), is the correct solution.**







## Solve linear inequalities by using the multiplication property.

### Multiplication Property of Inequality

For all real numbers *A, B,* and *C*, with $C \neq 0$,

**a.** the inequalities

***A < B*** and ***AC < BC*** are equivalent **if *C* > 0**;

**b.** the inequalities

***A < B*** and ***AC > BC*** are equivalent **if *C* < 0**.

That is, each side of an inequality may be multiplied (or divided) by a ***positive*** number without changing the direction of the inequality symbol. ***Multiplying (or dividing) by a negative number requires that we reverse the inequality symbol.***




**CLASSROOM EXAMPLE 3**

## Using the Multiplication Property of Inequality

Solve each inequality, and graph the solution set.

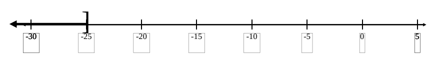
$$4m \leq -100$$

**Solution:**

Divide each side by 4. ***Since 4 > 0, do not reverse the inequality symbol.***

$$\frac{4m}{4} \leq \frac{-100}{4}$$
$$m \leq -25$$

The solution set is the interval (–∞, –25].







**CLASSROOM EXAMPLE 3**

## Using the Multiplication Property of Inequality (cont'd)

$$-9m < -81$$

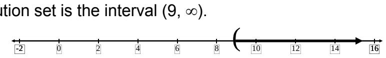
**Solution:**

Divide each side by –9. ***Since –9 < 0, reverse the inequality symbol.***

$$-9m < -81$$
$$\frac{-9m}{-9} > \frac{-81}{-9}$$
$$m > 9$$

**Reverse the inequality symbol when dividing by a *negative* number.**

The solution set is the interval (9, ∞).







## Solve linear inequalities by using the multiplication property.

### Solving a Linear Inequality

***Step 1*** **Simplify each side separately.** Clear parentheses, fractions, and decimals using the distributive property, and combine like terms.

***Step 2*** **Isolate the variable terms on one side.** Use the addition property of inequality to get all terms with variables on one side of the inequality and all numbers on the other side.

***Step 3*** **Isolate the variable.** Use the multiplication property of inequality to change the inequality to one of these forms.

$x < k, \quad x \leq k, \quad x > k, \quad$ or $x \geq k$

***CLASSROOM EXAMPLE 4*** **Solving a Linear Inequality by Using the Distributive Property**

Solve $6(x – 1) + 3x \geq –x – 3(x + 2)$, and graph the solution set.

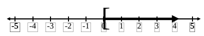
**Solution:**

***Step 1*** $$6(x-1)+3x \geq -x-3(x+2)$$
$$6x-6+3x \geq -x-3x-6$$
$$9x-6 \geq -4x-6$$

***Step 2*** $$9x-6+4x \geq -4x-6+4x$$
$$13x-6 \geq -6$$
$$13x-6+6 \geq -6+6$$

***Step 3*** $$13x \geq 0$$
$$\frac{13x}{13} \geq \frac{0}{13}$$
$$x \geq 0$$

The solution set is the interval $[0, \infty)$.







***CLASSROOM EXAMPLE 5*** **Solving a Linear Inequality with Fractions**

Solve and graph the solution set.

$$\frac{1}{4}(x+3)+2 \leq \frac{3}{4}(x+8)$$

**Solution:** $$4\left[\frac{1}{4}(x+3)+2\right] \leq 4\left[\frac{3}{4}(x+8)\right]$$ **Multiply by 4.**

$$4\left[\frac{1}{4}(x+3)\right]+4(2) \leq 4\left[\frac{3}{4}(x+8)\right]$$ **Distributive property.**

$$x+3+8 \leq 3(x+8)$$ **Multiply.**

$$x+3+8 \leq 3x+24$$ **Distributive property.**

$$x+11 \leq 3x+24$$

$$x+11-11 \leq 3x+24-11$$ **Subtract 11.**




***CLASSROOM EXAMPLE 5*** **Solving a Linear Inequality with Fractions (cont'd)**

$$x+11-11 \leq 3x+24-11$$ **Subtract 11.**

$$x+ \leq 3x+13$$

$$x-3x \leq 3x-3x+13$$ **Subtract 3*m*.**

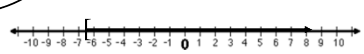
$$-2x \leq 13$$

$$\frac{-2x}{-2} \geq \frac{13}{-2}$$ **Divide –2.**

$$x \geq -\frac{13}{2}$$

**Reverse the inequality symbol when dividing by a *negative* number.**

The solution set is the interval $[-13/2, \infty)$.







## Solve linear inequalities with three parts.

For some applications, it is necessary to work with a **three-part inequality** such as

$$3 < x + 2 < 8,$$

where $x + 2$ is ***between*** 3 and 8.

CAUTION ***In three-part inequalities, the order of the parts is important. Write them so that the symbols point in the same direction and both point toward the lesser number.***



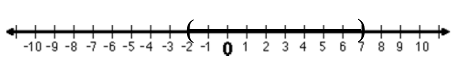

***CLASSROOM EXAMPLE 6*** **Solving a Three-Part Inequality**

Solve $−4 < x – 2 < 5$, and graph the solution set.

**Solution:**

$$−4 + 2 < x – 2 + 2 < 5 + 2$$ **Add 2 to each part.**
$$−2 < x < 7$$

The solution set is the interval $(−2, 7)$.







***CLASSROOM EXAMPLE 7*** **Solving a Three-Part Inequality**

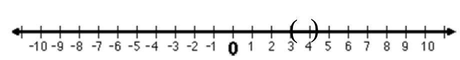
Solve $5 < 3x – 4 < 9$ and graph the solution set.

**Solution:**

$$5 + 4 < 3x – 4 + 4 < 9 + 4$$ **Add 4 to each part.**
$$9 < 3x < 13$$

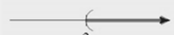
$$\frac{9}{3} < \frac{3x}{3} < \frac{13}{3}$$ **Divide by 3.**

$$3 < x < \frac{13}{3}$$



## Solve linear inequalities with three parts.

Types of solution sets for linear equations and inequalities are summarized in the table below.

| *Equation or Inequality* | *Typical Solution Set* | *Graph of Solution Set* |
|---|---|---|
| Linear equation<br>$5x + 4 = 14$ | $\{2\}$ |  2 |
| Linear inequality<br>$5x + 4 < 14$ | $(-\infty, 2)$ |  2 |
| or<br>$5x + 4 > 14$ | $(2, \infty)$ |  2 |
| Three-part inequality<br>$-1 \le 5x + 4 \le 14$ | $[-1, 2]$ |  −1 2 |




## Solve applied problems by using linear inequalities.

The table gives some common words and phrases you might see in a problem that suggest inequality.

| *Word Expression* | *Interpretation* |
|---|---|
| ***a*** exceeds ***b*** | $a > b$ |
| ***a*** is at least ***b*** | $a \ge b$ |
| ***a*** is no less than ***b*** | $a \ge b$ |
| ***a*** is at most ***b*** | $a \le b$ |
| ***a*** is no more than ***b*** | $a \le b$ |




***CLASSROOM EXAMPLE 8*** **Using a Linear Inequality to Solve a Rental Problem**

A rental company charges $5 to rent a leaf blower, plus $1.75 per hr. Marge Ruhberg can spend no more than $26 to blow leaves from her driveway and pool deck. What is the ***maximum*** amount of time she can use the rented leaf blower?

**Solution:**

***Step 1*** **Read** the problem again. What is to be found?

The maximum time Marge can afford to rent the blower.

What is given?

The flat rate to rent the leaf blower, the additional hourly charge to rent the leaf blower, and the maximum amount that Marge can spend.

***Step 2*** **Assign a variable.**

Let $h$ = the number of hours she can rent the blower.




***CLASSROOM EXAMPLE 8*** **Using a Linear Inequality to Solve a Rental Problem (cont'd)**

***Step 3*** **Write an inequality.**

She must pay $5, plus $1.75 per hour for $h$ hours and no more than $26.

| Cost of renting | is no more than | 26 |
|---|---|---|
| $5 + 1.75h$ | $\le$ | 26 |

***Step 4*** **Solve.** $1.75h \le 21$ **Subtract 5.**

$h \le 12$ **Divide by 1.75.**

***Step 5*** **State the answer.**

She can use the leaf blower from a maximum of 12 hours.

***Step 6*** **Check**.

If she uses the leaf blower for 12 hr, she will spend $5 + 1.75(12) = 26$ dollars, the maximum.




***CLASSROOM EXAMPLE 9*** **Finding an Average Test Score**

Abbey has scores of 92, 90, and 84 on his first three tests. What score must he make on his fourth test in order to keep an average of at least 90?

**Solution:**

**Let $x$ = score on the fourth test.**

To find the average of four numbers, add them and then divide by 4. His average score must be at least 90.

$$\underbrace{\text{Average}} \qquad \underbrace{\text{is at least}} \qquad \underline{90}$$

$$\frac{92+90+84+x}{4} \ge 90$$




***CLASSROOM EXAMPLE 9*** **Finding an Average Test Score (cont'd)**

$$\frac{266+x}{4} \ge 90$$

$$266+x \ge 360$$

$$x \ge 94$$

He must score 94 or more on his fourth test.

**Check:**

$$\frac{92+90+84+94}{4} \ge 90$$

$$\frac{92+90+84+94}{4} = \frac{360}{4} = 90$$

## 2.6 Set Operations and Compound Inequalities

**Objectives**

1 Find the intersection of two sets.

2 Solve compound inequalities with the word ***and.***

3 Find the union of two sets.

4 Solve compound inequalities with the word ***or.***



## Objective 1

## Find the intersection of two sets.



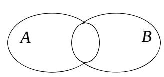

### Find the intersection of two sets.

The **intersection** of two sets is defined with the word ***and.***

**Intersection of Sets**

For any two sets $A$ and $B$, the **intersection** of $A$ and $B$, symbolized $A \cap B$, is defined as follows:

$$A \cap B = \{x \mid x \text{ is an element of } A \text{ and } x \text{ is an element of } B\}.$$







***CLASSROOM EXAMPLE 1*** **Finding the Intersection of Two Sets**

Let $A = \{3, 4, 5, 6\}$ and $B = \{5, 6, 7\}$. Find $A \cap B$.

**Solution:**

**The set $A \cap B$, the intersection of $A$ and $B$, contains those elements that belong to both $A$ and $B$; that is, the number s 5 and 6.**

$$A \cap B = \{3, 4, 5, 6\} \cap \{5, 6, 7\}$$

**Therefore,** $A \cap B = \{5,6\}$.




### Find the intersection of two sets.

A **compound inequality** consists of two inequalities linked by a consecutive word such as ***and*** or ***or.***

Examples of compound inequalities are

$$x + 1 \leq 9 \quad \text{and} \quad x - 2 \geq 3$$

$$2x > 4 \quad \text{or} \quad 3x - 6 < 5.$$




### Solve compound inequalities with the word *and.*

**Solving a Compound Inequality with *and***

***Step 1*** Solve each inequality individually.

***Step 2*** Since the inequalities are joined with ***and,*** the solution set of the compound inequality will include all numbers that satisfy both inequalities in ***Step 1*** (the intersection of the solution sets).

*CLASSROOM EXAMPLE 2* **Solving a Compound Inequality with *and***

Solve the compound inequality, and graph the solution set.
$x + 3 < 1$ and $x - 4 > -12$

**Solution:**

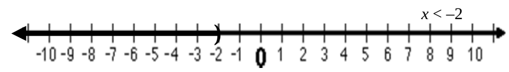
***Step 1*** Solve each inequality individually.

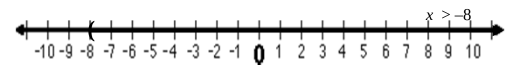
$$x + 3 < 1 \quad \text{and} \quad x - 4 > -12$$
$$x + 3 - 3 < 1 - 3 \qquad x - 4 + 4 > -12 + 4$$
$$x < -2 \qquad x > -8$$



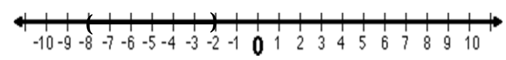

*CLASSROOM EXAMPLE 2* **Solving a Compound Inequality with *and* (cont'd)**

***Step 2*** Because the inequalities are joined with the word and, the solution set will include all numbers that satisfy both inequalities.




The solution set is $(-8, -2)$.







*CLASSROOM EXAMPLE 3* **Solving a Compound Inequality with *and***

Solve the compound inequality and graph the solution set.

$$2x \le 4x + 8 \quad \text{and} \quad 3x \ge -9$$

**Solution:**

***Step 1*** Solve each inequality individually.

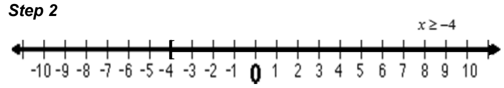
$$2x \le 4x + 8 \quad \text{and} \quad 3x \ge -9$$
$$-2x \le 8 \qquad x \ge -3$$
$$x \ge -4$$

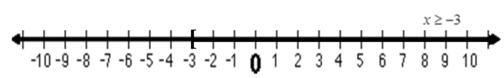
**Remember to reverse the inequality symbol.**




*CLASSROOM EXAMPLE 3* **Solving a Compound Inequality with *and* (cont'd)**

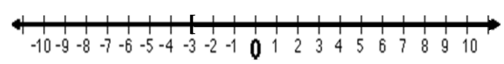
***Step 2***




The overlap of the graphs consists of the numbers that are greater than or equal to – 4 and are also greater than or equal to – 3.

The solution set is $[-3, \infty)$.







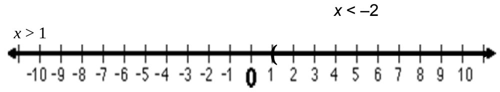
*CLASSROOM EXAMPLE 4* **Solving a Compound Inequality with *and***

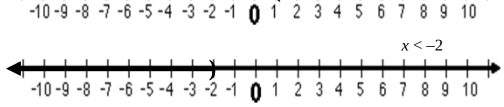
Solve and graph.
$x + 2 > 3$ and $2x + 1 < -3$.

**Solution:**

Solve each inequality individually.

$$x + 2 > 3 \quad \text{and} \quad 2x + 1 < -3$$
$$x > 1 \qquad 2x < -4$$
$$x < -2$$



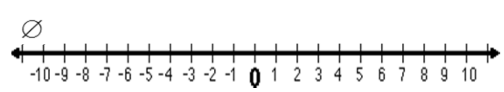




*CLASSROOM EXAMPLE 4* **Solving a Compound Inequality with *and* (cont'd)**

There is no number that is both greater than 1 and less than –2, so the given compound inequality has no solution.

The solution set is $\emptyset$.



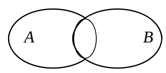
## Find the union of two sets.

The **union** of two sets is defined with the word ***or.***

**Union of Sets**

For any two sets *A* and *B*, the **union** of *A* and *B*, symbolized $A \cup B$, is defined as follows:

$$A \cup B = \{x \mid x \text{ is an element of } A \text{ or } x \text{ is an element of } B\}$$







***CLASSROOM EXAMPLE 5*** **Finding the Union of Two Sets**

Let *A* = {3, 4, 5, 6} and *B* = {5, 6, 7}. Find $A \cup B$.

**Solution:**

The set $A \cup B$, the union of *A* and *B*, consists of all elements in either *A* or *B* (or both).

Start by listing the elements of set *A*: 3,4,5,6.

Then list any additional elements from set *B*. In this case, the elements 5 and 6 are already listed, so the only additional element is 7.

Therefore,

$A \cup B$ = {3, 4, 5, 6, 7}.

Notice that although the elements 5 and 6 appear in both sets A and B, they are written only once in $A \cup B$.




**Objective 4**

## Solve compound inequalities with the word *or*.




## Solve compound inequalities with the word *or.*

**Solving a Compound Inequality with *or***

***Step 1*** Solve each inequality individually.

***Step 2*** Since the inequalities are joined with ***or,*** the solution set of the compound inequality includes all numbers that satisfy either one of the two inequalities in ***Step 1*** (the union of the solution sets).




***CLASSROOM EXAMPLE 6*** **Solving a Compound Inequality with *or***

Solve and graph the solution set.

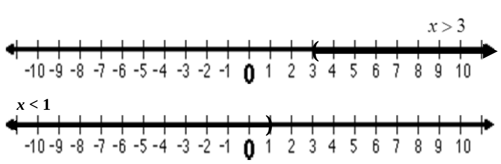
$x - 1 > 2$ or $3x + 5 < 2x + 6$

**Solution:**

***Step 1*** Solve each inequality individually.

$x - 1 > 2$ or $3x + 5 < 2x + 6$

$x > 3$ $\quad$ $x < 1$





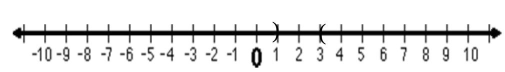


***CLASSROOM EXAMPLE 6*** **Solving a Compound Inequality with *or* (cont'd)**

***Step 2***

The graph of the solution set consists of all numbers greater than 3 ***or*** less than1.

The solution set is $(-\infty, 1) \cup (3, \infty)$.



**CLASSROOM EXAMPLE 7** — **Solving a Compound Inequality with *or***

Solve and graph.

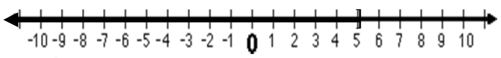
$3x - 2 \le 13$ or $x + 5 \le 7$

**Solution:**

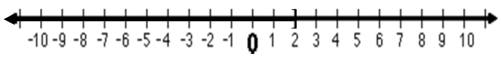
Solve each inequality individually.

$$\begin{array}{ccc} 3x - 2 \le 13 & \text{or} & x + 5 \le 7 \\ 3x \le 15 & & \\ x \le 5 & \text{or} & x \le 2 \end{array}$$

$x \le 5$



$x \le 2$

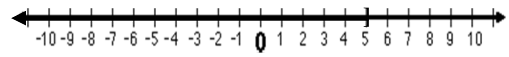





**CLASSROOM EXAMPLE 7** — **Solving a Compound Inequality with *or* (cont'd)**

The solution set is all numbers that are either less than or equal to 5 or less than or equal to 2. All real numbers less than or equal to 5 are included.

The solution set is $(-\infty, 5]$.






**CLASSROOM EXAMPLE 8** — **Solving a Compound Inequality with *or***

Solve and graph.

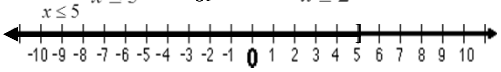
$3x - 2 \le 13$ or $x + 5 \ge 7$

**Solution:**

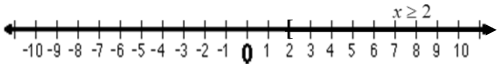
Solve each inequality individually.

$$\begin{array}{ccc} 3x - 2 \le 13 & \text{or} & x + 5 \ge 7 \\ 3x \le 15 & & \\ x \le 5 & \text{or} & x \ge 2 \end{array}$$

$x \le 5$



$x \ge 2$

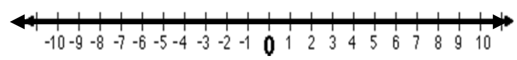





**CLASSROOM EXAMPLE 8** — **Solving a Compound Inequality with *or* (cont'd)**

The solution set is all numbers that are either less than or equal to 5 or greater than or equal to 2. All real numbers are included.

The solution set is $(-\infty, -\infty)$.






**CLASSROOM EXAMPLE 9** — **Applying Intersection and Union**

The five highest grossing domestic films as of 2009 are listed in this table. List the elements that satisfy each set.

Five All-Time Highest-Grossing Domestic Films

| Film | Admissions | Gross Income |
|---|---|---|
| *Gone with the Wind* | 202,044,600 | $1,450,680,400 |
| *Star Wars* | 178,119,600 | $1,278,898,700 |
| *The Sound of Music* | 142,415,400 | $1,022,542,400 |
| *E.T.* | 141,854,300 | $1,018,514,100 |
| *The Ten Commandments* | 131,000,000 | $ 940,580,000 |

*Source:* boxofficemojo.com.





**CLASSROOM EXAMPLE 9** — **Applying Intersection and Union (cont'd)**

The set of films with admissions greater than 130,000,000 ***and*** gross income less than $950,000,000.

**Solution:**

All films had admissions greater than 130,000,000, but only one film had gross income less than $950,000,000.

**The solution set is {*The Ten Commandments*}.**

The set of films with admissions greater than 130,000,000 ***or*** gross income less than $500,000,000.

All films had admissions greater than 130,000,000 but no films had gross income less than $500,000,000. Since we have an ***or*** statement and only one of the conditions has to be met, the second condition doesn't have an effect on the solution set.

**The solution set is {*Gone with the Wind, Star Wars, The Sound of Music, E.T., The Ten Commandments*}.**

## 2.7 Absolute Value Equations and Inequalities

**Objectives**

1. Use the distance definition of absolute value.
2. Solve equations of the form $|ax + b| = k$, for $k > 0$.
3. Solve inequalities of the form $|ax + b| < k$ and of the form $|ax + b| > k$, for $k > 0$.
4. Solve absolute value equations that involve rewriting.
5. Solve equations of the form $|ax + b| = |cx + d|$ .
6. Solve special cases of absolute value equations and inequalities.

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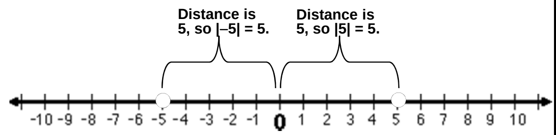
### Use the distance definition of absolute value.

The **absolute value** of a number $x$, written $|x|$, is the distance from $x$ to 0 on the number line.

For example, the solutions of $|x| = 5$ are 5 and $-5$, as shown below. We need to understand the concept of absolute value in order to solve equations or inequalities involving absolute values. We solve them by solving the appropriate compound equation or inequality.





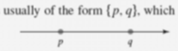


### Use the distance definition of absolute value.

**Solving Absolute Value Equations and Inequalities**

Let $k$ be a positive real number and $p$ and $q$ be real numbers.

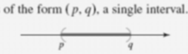
1. To solve $|ax + b| = k$, solve the compound equation
$$ax + b = k \quad \text{or} \quad ax + b = -k.$$
The solution set is usually of the form $\{p, q\}$, which includes two numbers.




2. To solve $|ax + b| > k$, solve the compound inequality
$$ax + b > k \quad \text{or} \quad ax + b < -k.$$
The solution set is of the form $(-\infty, p) \cup (q, \infty)$, which consists of two separate intervals.




3. To solve $|ax + b| < k$, solve the three-part inequality
$$-k < ax + b < k.$$
The solution set is of the form $(p, q)$, a single interval.







### Objective 2

## Solve equations of the form $|ax + b| = k$, for $k > 0$.




### Use the distance definition of absolute value.

***Remember that because absolute value refers to distance from the origin, an absolute value equation will have two parts.***




***CLASSROOM EXAMPLE 1*** **Solving an Absolute Value Equation**

Solve $|3x - 4| = 11$.

**Solution:**

| | | |
|---|---|---|
| $3x - 4 = -11$ | or | $3x - 4 = 11$ |
| $3x - 4 + 4 = -11 + 4$ | | $3x - 4 + 4 = 11 + 4$ |
| $3x = -7$ | | $3x = 15$ |
| $x = -\frac{7}{3}$ | | $x = 5$ |

**Check by substituting** $-\frac{7}{3}$ **and 5 into the original absolute value equation to verify that the solution set is** $\left\{-\frac{7}{3}, 5\right\}$.

## Objective 3

**Solve inequalities of the form $|ax + b| < k$ and of the form $|ax + b| > k$, for $k > 0$.**




***CLASSROOM EXAMPLE 2*** **Solving an Absolute Value Inequality with >**

Solve $|3x - 4| \geq 11$.
**Solution:**

$3x - 4 \leq -11$ or $3x - 4 \geq 11$

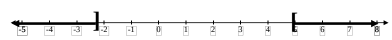
$3x - 4 + 4 \leq -11 + 4$ $\qquad$ $3x - 4 + 4 \geq 11 + 4$

$3x \leq -7$ $\qquad$ $3x \geq 15$

$x \leq -\frac{7}{3}$ $\qquad$ $x \geq 5$

**Check the solution. The solution set is** $\left(-\infty, -\frac{7}{3}\right] \cup [5, \infty)$.

**The graph consists of two intervals.**





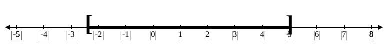

***CLASSROOM EXAMPLE 3*** **Solving an Absolute Value Inequality with <**

Solve $|3x - 4| \leq 11$.
**Solution:**

$$-11 \leq 3x - 4 \leq 11$$
$$-11 + 4 \leq 3x - 4 \leq 11 + 4$$
$$-7 \leq 3x \leq 15$$
$$-\frac{7}{3} \leq x \leq 5$$

**Check the solution. The solution set is** $\left[-\frac{7}{3}, 5\right]$.

The graph consists of a single interval.






**Solve inequalities of the form $|ax + b| < k$ and of the form $|ax + b| > k$, for $k > 0$.**

CAUTION When solving absolute value equations and inequalities of the types in **Examples 1, 2, and 3,** remember the following:

1. The methods describe apply when the constant is alone on one side of the equation or inequality and is ***positive.***
2. Absolute value equations and absolute value inequalities of the form $|ax + b| > k$ translate into **"or"** compound statements.
3. Absolute value inequalities of the form $|ax + b| < k$ translate into **"and"** compound statements, which may be written as three-part inequalities.
4. An **"or"** statement cannot be written in three parts.




***CLASSROOM EXAMPLE 4*** **Solving an Absolute Value Equation That Requires Rewriting**

Solve $|3x + 2| + 4 = 15$.
**Solution:**

First get the absolute value alone on one side of the equals sign.

$$|3x + 2| + 4 = 15$$
$$|3x + 2| + 4 - 4 = 15 - 4$$
$$|3a + 2| = 11$$

$3x + 2 = -11$ or $3x + 2 = 11$

$3x = -13$ $\qquad$ $3x = 9$

$x = -\frac{13}{3}$ $\qquad$ $x = 3$

The solution set is $\left\{-\frac{13}{3}, 3\right\}$.




***CLASSROOM EXAMPLE 5*** **Solving Absolute Value Inequalities That Require Rewriting**

Solve the inequality.

$|x + 2| - 3 > 2$

**Solution:**

$$|x + 2| - 3 > 2$$
$$|x + 2| > 5$$

$x + 2 > 5$ or $x + 2 < -5$

$x > 3$ $\qquad$ $x < -7$

Solution set: $(-\infty, -7) \cup (3, \infty)$

**CLASSROOM EXAMPLE 5** Solving Absolute Value Inequalities That Require Rewriting (cont'd)

Solve the inequality.

$|x + 2| - 3 < 2$

**Solution:**

$$|x + 2| < 5$$

$$-5 < x + 2 < 5$$

$$-7 < x < 3$$

Solution set: $(-7, 3)$




## Solve equations of the form $|ax + b| = |cx + d|$.

**Solving $|ax + b| = |cx + d|$**

To solve an absolute value equation of the form

$$|ax + b| = |cx + d|,$$

solve the compound equation

$$ax + b = cx + d \quad \text{or} \quad ax + b = -(cx + d).$$




**CLASSROOM EXAMPLE 6** Solving an Equation with Two Absolute Values

Solve $|4x - 1| = |3x + 5|$.

**Solution:**

$4x - 1 = 3x + 5$ or $4x - 1 = -(3x + 5)$

$4x - 6 = 3x$ or $4x - 1 = -3x - 5$

$-6 = -x$ or $7x = -4$

$x = 6$ or $x = -\frac{4}{7}$

Check that the solution set is $\left\{-\frac{4}{7}, 6\right\}$.




## Solve special cases of absolute value equations and inequalities.

**Special Cases of Absolute Value**

1. The absolute value of an expression can never be negative; that is, $|a| \geq 0$ for all real numbers $a$.
2. The absolute value of an expression equals 0 only when the expression is equal to 0.




**CLASSROOM EXAMPLE 7** Solving Special Cases of Absolute Value Equations

Solve each equation.

$|6x + 7| = -5$

**Solution:**

***The absolute value of an expression can never be negative,*** *so there are no solutions for this equation.* The solution set is $\emptyset$.

$$\left|\frac{1}{4}x - 3\right| = 0$$

The expression $\frac{1}{4}x - 3$ will equal 0 *only* if $\frac{1}{4}x = 3$

$$x = 12.$$

The solution of the equation is 12.

The solution set is {12}, with just one element.




**CLASSROOM EXAMPLE 8** Solving Special Cases of Absolute Value Inequalities

Solve each inequality.

$|x| > -1$

**Solution:**

***The absolute value of a number is always greater than or equal to 0.*** The solution set is $(-\infty, \infty)$.

$|x - 10| - 2 \leq -3$

$|x - 10| \leq -1$ **Add 2 to each side.**

***There is no number whose absolute value is less than –1, so the inequality has no solution.*** The solution set is $\emptyset$.

$|x + 2| \leq 0$

***The value of $|x + 2|$ will never be less than 0. $|x + 2|$ will equal 0 when $x = -2$.*** The solution set is $\{-2\}$.