3.1 The Rectangular Coordinate System

Objectives

1. Interpret a line graph.
2. Plot ordered pairs.
3. Find ordered pairs that satisfy a given equation.
4. Graph lines.
5. Find *x*- and *y*-intercepts.
6. Recognize equations of horizontal and vertical lines and lines passing through the origin.
7. Use the midpoint formula.

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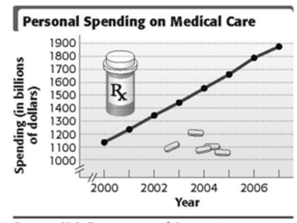
Interpret a line graph.

The line graph in the figure to the right presents information based on a method for locating a point in a plane developed by René Descartes, a 17th-century French mathematician. Today, we still use this method to plot points and graph linear equations in two variables whose graphs are straight lines.



Source: U.S. Department of Commerce.

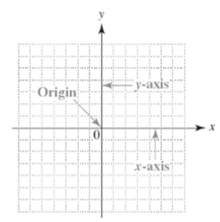
Plot ordered pairs.

Each of the pair of numbers

(3, 2), (–5, 6), and (4, –1)

is an example of an **ordered pair**.

An ordered pair is a pair of numbers written within parentheses, consisting of a **first component** and a **second component.**



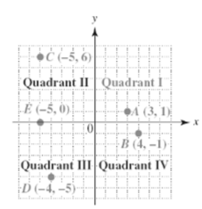
We graph an ordered pair by using two perpendicular number lines that intersect at their 0 points, as shown in the plane in the figure to the right. The common 0 point is called the **origin.**

The first number in the ordered pair indicates the position relative to the *x*-axis, and the second number indicates the position relative to the *y*-axis.

Plot ordered pairs.

The position of any point in this plane is determined by referring to the horizontal number line, or ***x*-axis,** and the vertical number line, or ***y*-axis.** The x-axis and the y-axis make up a **rectangular** (or **Cartesian**) **coordinate system.**

The four regions of the graph, shown below, are called **quadrants I, II, III,** and IV, reading counterclockwise from the upper right quadrant. The points on the *x*-axis and *y*-axis to not belong to any quadrant.



CLASSROOM EXAMPLE 1 **Completing Ordered Pairs and Making a Table**

Complete the table of ordered pairs for $3x - 4y = 12$.

Solution:

x	*y*
0	–3
	0
	–2
–6	

a. (0, __)

Replace *x* with 0 in the equation to find *y*.

$$3x - 4y = 12$$
$$3(0) - 4y = 12$$
$$0 - 4y = 12$$
$$-4y = 12$$
$$y = -3$$

CLASSROOM EXAMPLE 1 **Completing Ordered Pairs and Making a Table (cont'd)**

Complete the table of ordered pairs for $3x - 4y = 12$.

Solution:

x	*y*
0	–3
4	0
	–2
–6	

b. (__, 0)

Replace *y* with 0 in the equation to find *x*.

$$3x - 4y = 12$$
$$3x - 4(0) = 12$$
$$3x - 0 = 12$$
$$3x = 12$$
$$x = 4$$

CLASSROOM EXAMPLE 1 — **Completing Ordered Pairs and Making a Table (cont'd)**

Complete the table of ordered pairs for $3x - 4y = 12$.

Solution:

x	y
0	–3
4	0
$\frac{4}{3}$	–2
–6	

c. (__, –2)

Replace y with –2 in the equation to find x.

$$3x - 4y = 12$$
$$3x - 4(-2) = 12$$
$$3x + 8 = 12$$
$$3x = 4$$
$$x = \frac{4}{3}$$

CLASSROOM EXAMPLE 1 — **Completing Ordered Pairs and Making a Table (cont'd)**

Complete the table of ordered pairs for $3x - 4y = 12$.

Solution:

x	y
0	–3
4	0
$\frac{4}{3}$	–2
–6	$\frac{-15}{2}$

d. (–6, __)

Replace x with –6 in the equation to find y.

$$3x - 4y = 12$$
$$3(-6) - 4y = 12$$
$$-18 - 4y = 12$$
$$-4y = 30$$
$$y = \frac{-15}{2}$$

Graph lines.

The **graph of an equation** is the set of points corresponding to ***all*** ordered pairs that satisfy the equation. It gives a "picture" of the equation.

Linear Equation in Two Variables

A **linear equation in two variables** can be written in the form

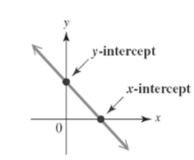
$$Ax + By = C,$$

where A, B, and C are real numbers and A and B not both 0. This form is called **standard form.**

Find x- and y- intercepts.

A straight line is determined if any two different points on a line are known. Therefore, finding two different points is enough to graph the line.

The ***x*-intercept** is the point (if any) where the line intersects the x-axis; likewise, the ***y*-intercept** is the point (if any) where the line intersects the y-axis.



Find x- and y- intercepts.

Finding Intercepts

When graphing the equation of a line, find the intercepts as follows.

Let $y = 0$ to find the x-intercept.

Let $x = 0$ to find the y-intercept.

While two points, such as the two intercepts are sufficient to graph a straight line, ***it is a good idea to use a third point to guard against errors.***

CLASSROOM EXAMPLE 2 — **Finding Intercepts**

Find the x-and y-intercepts and graph the equation $2x - y = 4$.

Solution:

***x*-intercept: Let $y = 0$.**

$2x - 0 = 4$

$2x = 4$

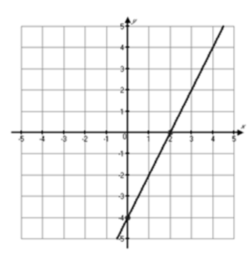
$x = 2$ (2, 0)

***y*-intercept: Let $x = 0$.**

$2(0) - y = 4$

$-y = 4$

$y = -4$ (0, –4)



Recognize equations of horizontal and vertical lines and lines passing through the origin.

A line parallel to the x-axis will not have an x-intercept. Similarly, a line parallel to the y-axis will not have a y-intercept.

CLASSROOM EXAMPLE 3

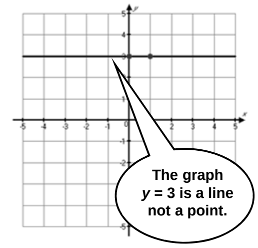
Graphing a Horizontal Line

Graph $y = 3$.

Solution:

Writing $y = 3$ as $0x + 1y = 3$ shows that any value of x, including $x = 0$, gives $y = 3$. Since y is always 3, there is no value of x corresponding to $y = 0$, so the graph has no x-intercepts.

x	y
0	3
1	3



The horizontal line $y = 0$ is the x-axis.

CLASSROOM EXAMPLE 4

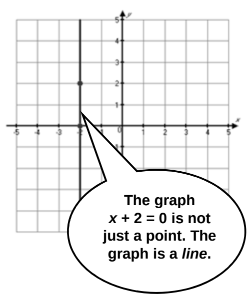
Graphing a Vertical Line

Graph $x + 2 = 0$.

Solution:

$1x + 0y = –2$
shows that any value of y, leads to $x = –2$, making the x-intercept $(–2, 0)$.
No value of y makes $x = 0$.

x	y
–2	0
–2	2



The vertical line $x = 0$ is the y-axis.

CLASSROOM EXAMPLE 5

Graphing a Line That Passes through the Origin

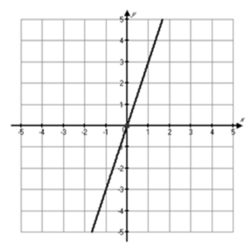
Graph $3x – y = 0$.

Solution:

Find the intercepts.

x-intercept: Let $y = 0$.
$3x – 0 = 0$
$3x = 0$
$x = 0$

y-intercept: Let $x = 0$.
$3(0) – y = 0$
$–y = 0$
$y = 0$



The x-intercept is (0, 0).
The y-intercept is (0, 0).

Use the midpoint formula.

Midpoint Formula

If the endpoints of a line segment PQ are (x_1, y_1) and (x_2, y_2), its midpoint M is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

In the midpoint formula, the small numbers 1 and 2 in the ordered pairs are called **subscripts,** read as **"x-sub-one and y-sub-one."**

CLASSROOM EXAMPLE 6

Finding the Coordinates of a Midpoint

Find the coordinates of the midpoint of line segment PQ with endpoints $P(–5, 8)$ and $Q(2, 4)$.

Solution:

Use the midpoint formula with $x_1 = –5$, $x_2 = 2$, $y_1 = 8$, and $y_2 = 4$:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5+2}{2}, \frac{8+4}{2}\right)$$
$$= \left(\frac{-3}{2}, \frac{12}{2}\right)$$
$$= (-1.5,\ 6)$$