3.2 The Slope of a Line

Objectives

1. Find the slope of a line, given two points on the line.
2. Find the slope of a line, given an equation of the line.
3. Graph a line, given its slope and a point on the line.
4. Use slopes to determine whether two lines are parallel, perpendicular, or neither.
5. Solve problems involving average rate of change.

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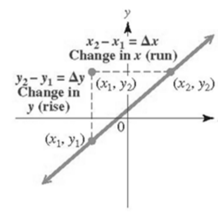
The Slope of a Line

Slope is the ratio of vertical change, or **rise,** to horizontal change, or **run.** A simple way to remember this is to think, **"slope is rise over run."**



Find the slope of a line, given two points on the line.

Slope Formula

The **slope *m*** of the line through the distinct points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_1 \neq x_2).$$

The Greek letter **delta,** Δ, is used in mathematics to denote "change in," so Δy and Δx represent the change in y and the change in x, respectively.

CLASSROOM EXAMPLE 1 **Finding the Slope of a Line**

Find the slope of the line through points (– 6, 9) and (3, – 5).

Solution:

If (– 6, 9) = (x_1, y_1) and (3, –5) = (x_2, y_2), then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-9}{3-(-6)} = \frac{-14}{9} = -\frac{14}{9}$$

Thus, the slope is $-\frac{14}{9}$.

CAUTION ***In calculating slope, be careful to subtract the y-values and the x-values in the same order. The change in y is the numerator and the change in x is the denominator.***

CLASSROOM EXAMPLE 1 **Finding the Slope of a Line (cont'd)**

If the ordered pairs are interchanged so that (–6, 9) = (x_2, y_2), and (3, –5) = (x_1, y_1) in the slope formula, the slope is the same.

Solution:

$$m = \frac{9-(-5)}{-6-3} = \frac{14}{-9} = -\frac{14}{9}$$

***y*-values are in the numerator, *x*-values are in the denominator.**

Objective 2

Find the slope of a line, given an equation of the line.

CLASSROOM EXAMPLE 2 — **Finding the Slope of a Line**

Find the slope of the line $3x - 4y = 12$.

Solution:

The intercepts can be used as two different points needed to find the slope.

Let $y = 0$ to find that the x-intercept is (4, 0).
Then let $x = 0$ to find that the y-intercept is (0, –3).

Use the two points in the slope formula.

$$m = \frac{\text{rise}}{\text{run}} = \frac{-3-0}{0-4} = \frac{-3}{-4} = \frac{3}{4}$$

The slope is $\frac{3}{4}$.

CLASSROOM EXAMPLE 3 — **Finding Slopes of Horizontal and Vertical Lines**

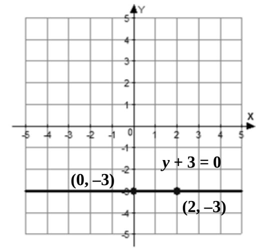
Find the slope of the line.

$y + 3 = 0$

Solution:

To find the slope of the line with equation y + 3 = 0, select two different points on the line such as (0, –3) and (2, –3), and use the slope formula.

$$m = \frac{-3-(-3)}{2-0} = \frac{0}{2} = 0$$



The slope is 0.

CLASSROOM EXAMPLE 3 — **Finding Slopes of Horizontal and Vertical Lines (cont'd)**

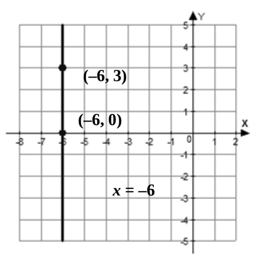
Find the slope of the line.

$x = -6$

Solution:

To find the slope of the line with equation $x = -6$, select two different points on the line such as (– 6, 0) and (– 6, 3), and use the slope formula.

$$m = \frac{3-0}{-6-(-6)} = \frac{3}{0}$$



The slope is undefined.

Find the slope of a line, given an equation of the line.

Horizontal and Vertical Lines

- An equation of the form $y = b$ always intersects the y-axis at the point $(0, b)$.

The line with that equation is horizontal and has slope 0.

- An equation of the form $x = a$ always intersects the x-axis at the point $(a, 0)$.

The line with that equation is vertical and has undefined slope.

CLASSROOM EXAMPLE 4 — **Finding the Slope from an Equation**

Find the slope of the graph of $3x + 4y = 9$.

Solution:

Solve the equation for y.

$$3x + 4y = 9$$
$$4y = -3x + 9$$
$$y = -\frac{3}{4}x + \frac{9}{4}$$

The slope is $-\frac{3}{4}$.

Objective 3

Graph a line, given its slope and a point on the line.

CLASSROOM EXAMPLE 5 — **Using the Slope and a Point to Graph Lines**

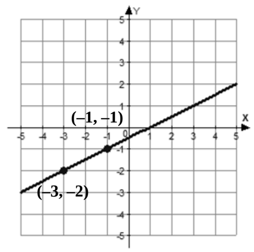
Graph the line passing through (–3, –2) that has slope $\frac{1}{2}$.

Solution:

Locate the point (–3, –2) on the graph. Use the slope formula to find a second point on the line.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{1}{2},$$

From (–3, –2), move up 1 and then 2 units to the right to (–1, –1).



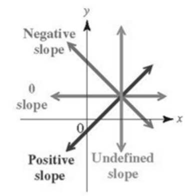
Draw a line through the two points.

Graph a line, given its slope and a point on the line.

Orientation of a Line in the Plane

A positive slope indicates that the line goes ***up*** **(rises)** from left to right.

A negative slope indicates that the line goes ***down*** **(falls)** from left to right.



Objective 4

Use slopes to determine whether two lines are parallel, perpendicular, or neither.

Use slopes to determine whether two lines are parallel, perpendicular, or neither.

Slopes of Parallel Lines

Two nonvertical lines with the same slope are parallel.

Two nonvertical parallel lines have the same slope.

CLASSROOM EXAMPLE 6 — **Determining Whether Two Lines Are Parallel**

Determine whether the line through (–1, 2) and (3, 5) is parallel to the line through (4, 7) and (8, 10).

Solution:

The line through (–1, 2) and (3, 5) has slope

$$m_1 = \frac{5-2}{3-(-1)} = \frac{3}{4}.$$

The line through (4, 7) and (8, 10) has slope

$$m_2 = \frac{10-7}{8-4} = \frac{3}{4}.$$

Yes, the slopes are the same, so the lines are ***parallel*****.**

Use slopes to determine whether two lines are parallel, perpendicular, or neither.

Slopes of Perpendicular Lines

If neither is vertical, perpendicular lines have slopes that are negative reciprocals—that is, their product is −1. Also, lines with slopes that are negative reciprocals are perpendicular.

A line with 0 slope is perpendicular to a line with undefined slope.

CLASSROOM EXAMPLE 7 — **Determining Whether Two Lines Are Perpendicular**

Are the lines with these equations perpendicular?

$3x + 5y = 6$
$5x - 3y = 2$

Solution:

Find the slope of each line by solving each equation for *y*.

$$3x + 5y = 6 \qquad 5y = -3x + 6 \qquad y = -\frac{3}{5}x + \frac{6}{5}$$

$$5x - 3y = 2 \qquad -3y = -5x + 2 \qquad y = \frac{5}{3}x - \frac{2}{3}$$

Yes, since the product of the slopes of the two lines is $\left(-\frac{3}{5}\right)\left(\frac{5}{3}\right) = -1,$ the lines are ***perpendicular*.**

CLASSROOM EXAMPLE 8 — **Determining Whether Two Lines Are Parallel, Perpendicular, or Neither**

Determine whether the lines with these equations are ***parallel, perpendicular,*** or ***neither.***
$4x - y = 2$
$x - 4y = -8$

Solution:

Find the slope of each line by solving each equation for *y*.

$$4x - y = 2 \qquad -y = -4x + 2 \qquad y = 4x - 2$$

$$x - 4y = -8 \qquad -4y = -x - 8 \qquad y = \frac{1}{4}x + 2$$

The slopes, 4 and $\frac{1}{4}$, are not equal so the lines are not parallel.

The lines not perpendicular because their product is 1, not −1.

Thus, the two lines are neither parallel nor perpendicular.

Objective 5

Solve problems involving average rate of change.

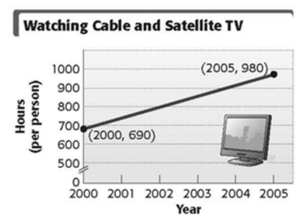
CLASSROOM EXAMPLE 9 — **Interpreting Slope as Average Rate of Change**

Americans spent an average of 886 hr in 2003 watching cable and satellite TV. Using this number for 2003 and the number for 2000 from the graph, find the average rate of change to the nearest tenth of an hour from 2000 to 2003. How does it compare to the average rate of change found in Example 9?

Solution:

$$(x_1, y_1) = (2000, 690)$$

$$(x_2, y_2) = (2003, 886)$$



Source: Veronis Suhler Stevenson.

CLASSROOM EXAMPLE 9 — **Interpreting Slope as Average Rate of Change (cont'd)**

Average rate of change =

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{886 - 690}{2003 - 2000} = \frac{196}{3} = 63.5$$

A positive slope indicates an increase.

The average rate of change from 2000 to 2003 is 63.5 hours per year, which is greater than 58 hours per year from 2000 to 2005 as in **Example 9.**

CLASSROOM EXAMPLE 10 — **Interpreting Slope as Average Rate of Change**

In 2000, 942.5 million compact discs were sold in the United States. In 2006, 614.9 million CDs were sold. Find the average rate of change in CDs sold per year.

(*Source:* Recording Industry Association of America.)

Solution:

$$(x_1, y_1) = (2000, 942.5) \qquad (x_2, y_2) = (2006, 614.9)$$

Average rate of change = $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{614.9 - 942.5}{2006 - 2000} = \frac{-327.6}{6} \approx -54.6$$

A negative slope indicates a decrease.

The average rate of change from 2000 to 2006 was –54.6 million CDs per year.