3.3 Linear Equations in Two Variables

Objectives

1. Write an equation of a line, given its slope and *y*-intercept.
2. Graph a line, using its slope and *y*-intercept.
3. Write an equation of a line, given its slope and a point on the line.
4. Write equations of horizontal and vertical lines.
5. Write an equation of a line, given two points on the line.
6. Write an equation of a line parallel or perpendicular to a given line.
7. Write an equation of a line that models real data.

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Write an equation of a line, given its slope and *y*-intercept.

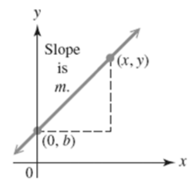
In **Section 3.2,** we found the slope of a line from its equation by solving the equation for y. Once we had isolated *y,* we found that the slope was the coefficient of *x*. However, we still don't know what the last number in the equation represents. That is, the number with no variable in the equation of the form, *y* = 4*x* + 8. To find out, suppose a line has a slope *m* and *y*-intercept (0, *b*).

$$m = \frac{y-b}{x-0} \quad \text{or} \quad m = \frac{y-b}{x}$$

$$mx = y - b$$

$$mx + b = y$$

$$y = mx + b$$



Thus, *b* is the *y*-intercept or 8 in our original equation.

Write an equation of a line, given its slope and *y*-intercept.

Slope-Intercept Form

The **slope-intercept form** of the equation of a line with slope ***m*** and *y*-intercept (0, *b*) is

$$\underbrace{y = mx + b.}$$

Slope   *y*-intercept (0, *b*)

CLASSROOM EXAMPLE 1 **Writing an Equation of a Line**

Write an equation of the line with slope 2 and *y*-intercept (0, –3).

Solution:

Let *m* = 2
Let *b* = –3

Substitute these values into the slope-intercept form.

y = *mx* + *b*

y = 2*x* – 3

CLASSROOM EXAMPLE 2 **Graphing Lines Using Slope and *y*- Intercept**

Graph the line, using the slope and *y*-intercept.

x + 2*y* = – 4

Solution:

Write the equation in slope-intercept form by solving for *y*.

x + 2*y* = – 4
2*y* = – *x* – 4   **Subtract *x*.**

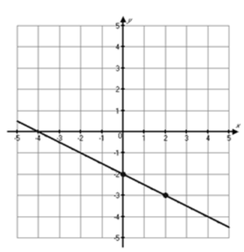
$$y = -\frac{1}{2}x - 2$$

Slope   ***y*-intercept (0, –2)**

CLASSROOM EXAMPLE 2 **Graphing Lines Using Slope and *y*- Intercept (cont'd)**

Graph: $y = -\frac{1}{2}x - 2$

1. Plot the *y*-intercept, (0, –2).
2. The slope is $\frac{-1}{2}$ or $\frac{1}{-2}$.
3. Using $-\frac{1}{2}$, begin at (0, –2) and move 1 unit ***down*** and 2 units ***right***.
4. The line through these two points is the required graph.



Objective 3

Write an equation of a line, given its slope and a point on the line.

Write an equation of a line, given its slope a point on the line.

Point-Slope Form

The **point-slope form** of the equation of a line with slope ***m*** passing through the point (x_1, y_1) is

Slope

$$y - y_1 = m(x - x_1).$$

Given point

CLASSROOM EXAMPLE 3 **Writing an Equation of a Line, Given the Slope and a Point**

Write an equation of the line with slope $\frac{2}{5}$ and passing through the point (3, – 4).

Solution:

Use the point-slope form with (x_1, y_1) = (3, – 4) and m = $\frac{2}{5}$.

$y - y_1 = m(x - x_1)$ **Point-slope form**

$y - (-4) = \frac{2}{5}(x - 3)$ **Substitute.**

$y + 4 = \frac{2}{5}(x - 3)$

$5y + 20 = 2x - 6$ **Multiply by 5.**

$5y = 2x - 26$ **Subtract 20.**

$y = \frac{2}{5}x - \frac{26}{5}$ **Divide by 5.**

Objective 4

Write equations of horizontal and vertical lines.

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Equations of Horizontal and Vertical Lines

The horizontal line through the point *(a, b*) has equation *y* = *b*.

The vertical line through the point *(a, b*) has equation *x* = *a*.

CLASSROOM EXAMPLE 4 **Writing Equations of Horizontal and Vertical Lines**

Write an equation of the line passing through the point (2, − 1) that satisfies the given condition.

Solution:

Undefined slope

This is a vertical line, since the slope is undefined. A vertical line through the point (*a*, *b*) has equation *x* = *a*. Here the *x*-coordinate is 2, so the equation is *x* = 2.

Slope 0

Since the slope is 0, this is a horizontal line. A horizontal line through point (*a*, *b*) has equation *y* = *b*. Here the *y*-coordinate is − 1, so the equation is *y* = − 1.

CLASSROOM EXAMPLE 5 — **Writing an Equation of a Line, Given Two Points**

Write an equation of the line passing through the points (– 2, 6) and (1, 4). Give the final answer in standard form.

Solution:

First find the slope by the slope formula.

$$m = \frac{4-6}{1-(-2)} = \frac{-2}{3} = -\frac{2}{3}$$

Use either point as (x_1, y_1) in the point-slope form of the equation of a line.

We will choose the point (1, 4): $x_1 = 1$ and $y_1 = 4$

CLASSROOM EXAMPLE 5 — **Writing an Equation of a Line, Given Two Points (cont'd)**

Using $m = -\frac{2}{3}$; $x_1 = 1$ and $y_1 = 4$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{2}{3}(x - 1)$$ **Substitute.**

$$3y - 12 = -2x + 2$$ **Multiply by 3.**

$$2x + 3y = 14$$ **Add 2*x* and 12.**

If the other point were used, the same equation would result.

Objective 6

Write an equation of a line parallel or perpendicular to a given line.

CLASSROOM EXAMPLE 6 — **Writing Equations of Parallel or Perpendicular Lines**

Write an equation of the line passing through the point (– 8, 3) and (a) parallel to the line $2x - 3y = 10$; (b) perpendicular to the line $2x - 3y = 10$. Give the final answers in slope-intercept form.

Parallel to the line…

Solution:

Find the slope of the line $2x - 3y = 10$ by solving for y.

$$2x - 3y = 10$$

$$-3y = -2x + 10$$

$$y = \frac{2}{3}x - \frac{10}{3}$$

CLASSROOM EXAMPLE 6 — **Writing Equations of Parallel or Perpendicular Lines (cont'd)**

Find an equation of the line passing through the point (–8, 3) when the slope is $\frac{2}{3}$.

Parallel lines have the same slope. Use point slope form and the given point.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{2}{3}[x - (-8)]$$

$$y - 3 = \frac{2}{3}(x + 8)$$

$$y - 3 = \frac{2}{3}x + \frac{16}{3}$$

$$y = \frac{2}{3}x + \frac{16}{3} + \frac{9}{3}$$

$$y = \frac{2}{3}x + \frac{25}{3}$$

The fractions were not cleared because we want the equation in slope-intercept form instead of standard form.

CLASSROOM EXAMPLE 6 — **Writing Equations of Parallel or Perpendicular Lines (cont'd)**

Perpendicular to the line…

Solution:

The slope of the perpendicular line would have a slope that is the negative reciprocal of $\frac{2}{3}$, or $-\frac{3}{2}$.

Find an equation of the line passing through the point (–8, 3), when the slope is $-\frac{3}{2}$.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{3}{2}[x - (-8)]$$

$$y - 3 = -\frac{3}{2}(x + 8)$$

$$y - 3 = -\frac{3}{2}x - 12$$

$$y = -\frac{3}{2}x - 9$$

Write an equation of a line parallel or perpendicular to a given line.

Forms of Linear Equations

Equation	Description	When to Use
$y = mx + b$	**Slope-Intercept Form** Slope is m. y-intercept is $(0, b)$.	The slope and y-intercept can be easily identified and used to quickly graph the equation.
$y - y_1 = m(x - x_1)$	**Point-Slope Form** Slope is m. Line passes through (x_1, y_1).	This form is ideal for finding the equation of a line if the slope and a point on the line or two points on the line are known.
$Ax + By = C$	**Standard Form** (A, B, and C integers, $A \geq 0$) Slope is $-\frac{A}{B}$ $(B \neq 0)$. x-intercept is $\left(\frac{C}{A}, 0\right)$ $(A \neq 0)$. y-intercept is $\left(0, \frac{C}{B}\right)$ $(B \neq 0)$.	The x- and y-intercepts can be found quickly and used to graph the equation. The slope must be calculated.
$y = b$	**Horizontal Line** Slope is 0. y-intercept is $(0, b)$.	If the graph intersects only the y-axis, then y is the only variable in the equation.
$x = a$	**Vertical Line** Slope is undefined. x-intercept is $(a, 0)$.	If the graph intersects only the x-axis, then x is the only variable in the equation.

Objective 7

Write an equation of a line that models real data.

CLASSROOM EXAMPLE 7 **Determining a Linear Equation to Describe Real Data**

Suppose there is a flat rate of \$0.20 plus a charge of \$0.10 per minute to make a telephone call. Write an equation that gives the cost *y* for a call of *x* minutes.

Solution:

y = \$0.20 + \$0.10*x*

or

y = \$0.10*x* + 0.20

CLASSROOM EXAMPLE 8 **Finding an Equation of a Line That Models Data**

The percentage of the U.S. population 25 yr and older with at least a high school diploma is shown in the table for selected years. (*Source:* U.S. Census Bureau.)

Let *x* = 0 represent 1950, *x* = 10 represent 1960, and so on. Use the data for 1950 and 2000 to write an equation that models the data.

Solution:

We choose two data points (0, 34.3) and (50, 84.1) to find the slope of the line.

$$m = \frac{84.1 - 34.3}{50 - 0} = \frac{49.8}{50} = 0.996$$

The equation is:

y = 0.996*x* + 34.3

Year	*Percent*
1950	34.3
1960	41.4
1970	52.3
1980	66.5
1990	77.6
2000	84.1
2007	85.7

CLASSROOM EXAMPLE 8 **Finding an Equation of a Line That Models Data (cont'd)**

Use the equation from the first part of the problem to approximate the percentage, to the nearest tenth, of the U.S. population 25 yr and older who were at least high school graduates in 1995.

Solution:

$$y = 0.996x + 34.3$$

$$y = 0.996(45) + 34.3$$

$$y = 79.1 \text{ or } 79.1\ \%$$

Year	*Percent*
1950	34.3
1960	41.4
1970	52.3
1980	66.5
1990	77.6
2000	84.1
2007	85.7

CLASSROOM EXAMPLE 9 **Writing an Equation of a Line That Models Data**

Use the ordered pairs (4, 220) and (6, 251) to find an equation that models the data in the graph below.

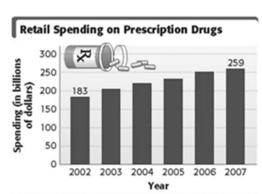
Solution:

Find the slope through the two points.

$$m = \frac{251 - 220}{6 - 4} = \frac{31}{2} = 15.5$$

Then, we substitute one of the points into the point-slope form of the equation.

$$y - y_1 = m(x - x_1)$$
$$y - 220 = 15.5(x - 4)$$
$$y - 220 = 15.5x - 62$$
$$y = 15.5x + 158$$



Source: National Association of Chain Drug Stores.