3.1 The Rectangular Coordinate System

Objectives

1 Interpret a line graph.

2 Plot ordered pairs.

3 Find ordered pairs that satisfy a given equation.

4 Graph lines.

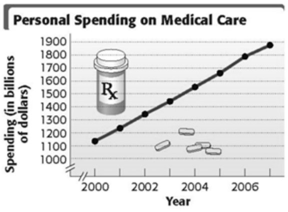
5 Find *x*- and *y*-intercepts.

6 Recognize equations of horizontal and vertical lines and lines passing through the origin.

7 Use the midpoint formula.

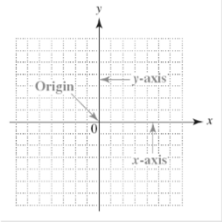
Interpret a line graph.

The line graph in the figure to the right presents information based on a method for locating a point in a plane developed by René Descartes, a 17th-century French mathematician. Today, we still use this method to plot points and graph linear equations in two variables whose graphs are straight lines.



Source: U.S. Department of Commerce.

Plot ordered pairs.



Each of the pair of numbers

(3, 2), (–5, 6), and (4, –1)

is an example of an **ordered pair**.

An ordered pair is a pair of numbers written within parentheses, consisting of a **first component** and a **second component.**

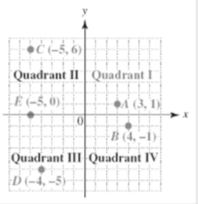
We graph an ordered pair by using two perpendicular number lines that intersect at their 0 points, as shown in the plane in the figure to the right. The common 0 point is called the **origin.**

The first number in the ordered pair indicates the position relative to the *x*-axis, and the second number indicates the position relative to the *y*-axis.

Plot ordered pairs.

The position of any point in this plane is determined by referring to the horizontal number line, or ***x*-axis,** and the vertical number line, or ***y*-axis.** The x-axis and the y-axis make up a **rectangular** (or **Cartesian**) **coordinate system.**

The four regions of the graph, shown below, are called **quadrants I, II, III,** and IV, reading counterclockwise from the upper right quadrant. The points on the *x*-axis and *y*-axis to not belong to any quadrant.



CLASSROOM EXAMPLE 1

Completing Ordered Pairs and Making a Table

Complete the table of ordered pairs for 3*x* – 4*y* = 12.

Solution:

x	*y*
0	–3
	0
	–2
–6	

a. (0, __)

Replace *x* with 0 in the equation to find *y*.

$$3x - 4y = 12$$
$$3(0) - 4y = 12$$
$$0 - 4y = 12$$
$$-4y = 12$$
$$y = -3$$

CLASSROOM EXAMPLE 1

Completing Ordered Pairs and Making a Table (cont'd)

Complete the table of ordered pairs for 3*x* – 4*y* = 12.

Solution:

x	*y*
0	–3
4	0
	–2
–6	

b. (__, 0)

Replace *y* with 0 in the equation to find *x*.

$$3x - 4y = 12$$
$$3x - 4(0) = 12$$
$$3x - 0 = 12$$
$$3x = 12$$
$$x = 4$$

CLASSROOM EXAMPLE 1

Completing Ordered Pairs and Making a Table (cont'd)

Complete the table of ordered pairs for $3x - 4y = 12$.

Solution:

x	y
0	–3
4	0
$\frac{4}{3}$	–2
–6	

c. (__, –2)

Replace y with –2 in the equation to find x.

$$3x - 4y = 12$$
$$3x - 4(-2) = 12$$
$$3x + 8 = 12$$
$$3x = 4$$
$$x = \frac{4}{3}$$

CLASSROOM EXAMPLE 1

Completing Ordered Pairs and Making a Table (cont'd)

Complete the table of ordered pairs for $3x - 4y = 12$.

Solution:

x	y
0	–3
4	0
$\frac{4}{3}$	–2
–6	$\frac{-15}{2}$

d. (–6, __)

Replace x with –6 in the equation to find y.

$$3x - 4y = 12$$
$$3(-6) - 4y = 12$$
$$-18 - 4y = 12$$
$$-4y = 30$$
$$y = \frac{-15}{2}$$

Graph lines.

The **graph of an equation** is the set of points corresponding to ***all*** ordered pairs that satisfy the equation. It gives a "picture" of the equation.

Linear Equation in Two Variables

A **linear equation in two variables** can be written in the form

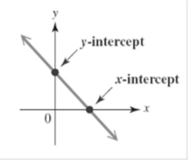
$$Ax + By = C,$$

where A, B, and C are real numbers and A and B not both 0. This form is called **standard form.**

Find x- and y- intercepts.

A straight line is determined if any two different points on a line are known. Therefore, finding two different points is enough to graph the line.

The ***x*-intercept** is the point (if any) where the line intersects the x-axis; likewise, the ***y*-intercept** is the point (if any) where the line intersects the y-axis.



Find x- and y- intercepts.

Finding Intercepts

When graphing the equation of a line, find the intercepts as follows.

Let $y = 0$ to find the x-intercept.

Let $x = 0$ to find the y-intercept.

While two points, such as the two intercepts are sufficient to graph a straight line, ***it is a good idea to use a third point to guard against errors.***

CLASSROOM EXAMPLE 2

Finding Intercepts

Find the x-and y-intercepts and graph the equation $2x - y = 4$.

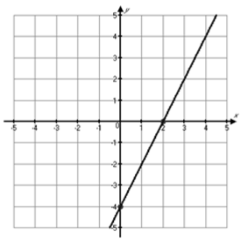
Solution:

***x*-intercept: Let *y* = 0.**

$2x - 0 = 4$
$2x = 4$
$x = 2$ (2, 0)

***y*-intercept: Let *x* = 0.**

$2(0) - y = 4$
$-y = 4$
$y = -4$ (0, –4)



Recognize equations of horizontal and vertical lines and lines passing through the origin.

A line parallel to the x-axis will not have an x-intercept. Similarly, a line parallel to the y-axis will not have a y-intercept.

CLASSROOM EXAMPLE 3

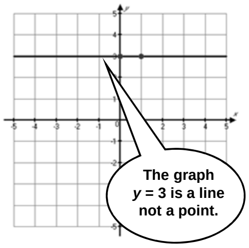
Graphing a Horizontal Line

Graph $y = 3$.

Solution:

Writing $y = 3$ as $0x + 1y = 3$ shows that any value of x, including $x = 0$, gives $y = 3$. Since y is always 3, there is no value of x corresponding to $y = 0$, so the graph has no x-intercepts.

x	y
0	3
1	3



The graph $y = 3$ is a line not a point.

The horizontal line $y = 0$ is the x-axis.

CLASSROOM EXAMPLE 4

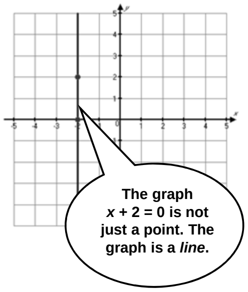
Graphing a Vertical Line

Graph $x + 2 = 0$.

Solution:

$1x + 0y = –2$ shows that any value of y, leads to $x = –2$, making the x-intercept $(–2, 0)$. No value of y makes $x = 0$.

x	y
–2	0
–2	2



The graph $x + 2 = 0$ is not just a point. The graph is a *line*.

The vertical line x = 0 is the y-axis.

CLASSROOM EXAMPLE 5

Graphing a Line That Passes through the Origin

Graph $3x – y = 0$.

Solution:

Find the intercepts.

x-intercept: Let $y = 0$.

$3x – 0 = 0$

$3x = 0$

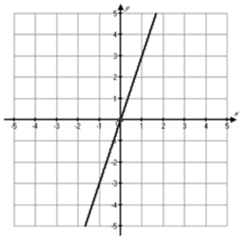
$x = 0$

y-intercept: Let $x = 0$.

$3(0) – y = 0$

$–y = 0$

$y = 0$



The x-intercept is (0, 0).

The y-intercept is (0, 0).

Use the midpoint formula.

Midpoint Formula

If the endpoints of a line segment PQ are (x_1, y_1) and (x_2, y_2), its midpoint M is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

In the midpoint formula, the small numbers 1 and 2 in the ordered pairs are called **subscripts,** read as **"x-sub-one and y-sub-one."**

CLASSROOM EXAMPLE 6

Finding the Coordinates of a Midpoint

Find the coordinates of the midpoint of line segment PQ with endpoints $P(–5, 8)$ and $Q(2, 4)$.

Solution:

Use the midpoint formula with $x_1 = –5$, $x_2 = 2$, $y_1 = 8$, and $y_2 = 4$:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-5+2}{2}, \frac{8+4}{2}\right)$$

$$= \left(\frac{-3}{2}, \frac{12}{2}\right)$$

$$= (-1.5,\ 6)$$

3.2 The Slope of a Line

Objectives

1 Find the slope of a line, given two points on the line.

2 Find the slope of a line, given an equation of the line.

3 Graph a line, given its slope and a point on the line.

4 Use slopes to determine whether two lines are parallel, perpendicular, or neither.

5 Solve problems involving average rate of change.

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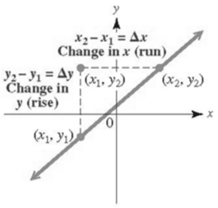
The Slope of a Line

Slope is the ratio of vertical change, or **rise,** to horizontal change, or **run.** A simple way to remember this is to think, **"slope is rise over run."**



Find the slope of a line, given two points on the line.

Slope Formula

The **slope *m*** of the line through the distinct points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_1 \neq x_2).$$

The Greek letter **delta,** Δ, is used in mathematics to denote "change in," so Δy and Δx represent the change in *y* and the change in *x,* respectively.

CLASSROOM EXAMPLE 1

Finding the Slope of a Line

Find the slope of the line through points (– 6, 9) and (3, – 5).

Solution:

If (– 6, 9) = (x_1, y_1) and (3, –5) = (x_2, y_2), then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-9}{3-(-6)} = \frac{-14}{9} = -\frac{14}{9}$$

Thus, the slope is $-\frac{14}{9}$.

CAUTION

In calculating slope, be careful to subtract the y-values and the x-values in the same order. The change in y is the numerator and the change in x is the denominator.

CLASSROOM EXAMPLE 1

Finding the Slope of a Line (cont'd)

If the ordered pairs are interchanged so that (–6, 9) = (x_2, y_2), and (3, –5) = (x_1, y_1) in the slope formula, the slope is the same.

Solution:

$$m = \frac{9-(-5)}{-6-3} = \frac{14}{-9} = -\frac{14}{9}$$

***y*-values are in the numerator, *x*-values are in the denominator.**

Objective 2

Find the slope of a line, given an equation of the line.

CLASSROOM EXAMPLE 2

Finding the Slope of a Line

Find the slope of the line 3*x* – 4*y* = 12.

Solution:

The intercepts can be used as two different points needed to find the slope.

Let *y* = 0 to find that the x-intercept is (4, 0).
Then let *x* = 0 to find that the y-intercept is (0, –3).

Use the two points in the slope formula.

$$m=\frac{\text{rise}}{\text{run}}=\frac{-3-0}{0-4}=\frac{-3}{-4}=\frac{3}{4}$$

The slope is $\frac{3}{4}$.

CLASSROOM EXAMPLE 3

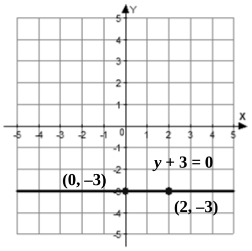
Finding Slopes of Horizontal and Vertical Lines

Find the slope of the line.

$$y+3=0$$

Solution:

To find the slope of the line with equation y + 3 = 0, select two different points on the line such as (0, –3) and (2, –3), and use the slope formula.



$$m=\frac{-3-(-3)}{2-0}=\frac{0}{2}=0$$

The slope is 0.

CLASSROOM EXAMPLE 3

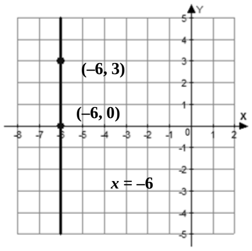
Finding Slopes of Horizontal and Vertical Lines (cont'd)

Find the slope of the line.

$$x=-6$$

Solution:

To find the slope of the line with equation *x* = – 6, select two different points on the line such as (– 6, 0) and (– 6, 3), and use the slope formula.



$$m=\frac{3-0}{-6-(-6)}=\frac{3}{0}$$

The slope is undefined.

Find the slope of a line, given an equation of the line.

Horizontal and Vertical Lines

- ❑An equation of the form *y* = *b* always intersects the *y*-axis at the point (0, *b*).

The line with that equation is horizontal and has slope 0.

- ❑An equation of the form *x* = *a* always intersects the *x*-axis at the point (a, 0).

The line with that equation is vertical and has undefined slope.

CLASSROOM EXAMPLE 4

Finding the Slope from an Equation

Find the slope of the graph of 3*x* + 4*y* = 9.

Solution:

Solve the equation for *y*.

$$3x+4y=9$$
$$4y=-3x+9$$
$$y=-\frac{3}{4}x+\frac{9}{4}$$

The slope is $-\frac{3}{4}$.

Objective 3

Graph a line, given its slope and a point on the line.

CLASSROOM EXAMPLE 5

Using the Slope and a Point to Graph Lines

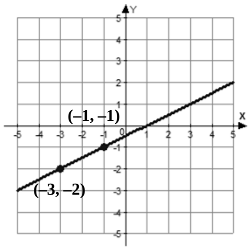
Graph the line passing through (–3, –2) that has slope $\frac{1}{2}$.

Solution:

Locate the point (–3, –2) on the graph. Use the slope formula to find a second point on the line.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{1}{2},$$

From (–3, –2), move up 1 and then 2 units to the right to (–1, –1).



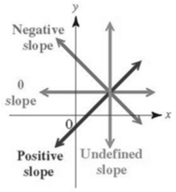
Draw a line through the two points.

Graph a line, given its slope and a point on the line.

Orientation of a Line in the Plane

A positive slope indicates that the line goes ***up* (rises)** from left to right.

A negative slope indicates that the line goes ***down* (falls)** from left to right.



Objective 4

Use slopes to determine whether two lines are parallel, perpendicular, or neither.

Use slopes to determine whether two lines are parallel, perpendicular, or neither.

Slopes of Parallel Lines

Two nonvertical lines with the same slope are parallel.

Two nonvertical parallel lines have the same slope.

CLASSROOM EXAMPLE 6

Determining Whether Two Lines Are Parallel

Determine whether the line through (–1, 2) and (3, 5) is parallel to the line through (4, 7) and (8, 10).

Solution:

The line through (–1, 2) and (3, 5) has slope

$$m_1 = \frac{5-2}{3-(-1)} = \frac{3}{4}.$$

The line through (4, 7) and (8, 10) has slope

$$m_2 = \frac{10-7}{8-4} = \frac{3}{4}.$$

Yes, the slopes are the same, so the lines are ***parallel*.**

Use slopes to determine whether two lines are parallel, perpendicular, or neither.

Slopes of Perpendicular Lines

If neither is vertical, perpendicular lines have slopes that are negative reciprocals—that is, their product is −1. Also, lines with slopes that are negative reciprocals are perpendicular.

A line with 0 slope is perpendicular to a line with undefined slope.

CLASSROOM EXAMPLE 7

Determining Whether Two Lines Are Perpendicular

Are the lines with these equations perpendicular?

$3x + 5y = 6$
$5x – 3y = 2$

Solution:

Find the slope of each line by solving each equation for *y*.

$$\begin{aligned} 3x+5y&=6 \\ 5y&=-3x+6 \\ y&=-\frac{3}{5}x+\frac{6}{5} \end{aligned} \qquad \begin{aligned} 5x-3y&=2 \\ -3y&=-5x+2 \\ y&=\frac{5}{3}x-\frac{2}{3} \end{aligned}$$

Yes, since the product of the slopes of the two lines is $\left(-\frac{3}{5}\right)\left(\frac{5}{3}\right)=-1,$ the lines are ***perpendicular*****.**

CLASSROOM EXAMPLE 8

Determining Whether Two Lines Are Parallel, Perpendicular, or Neither

Determine whether the lines with these equations are ***parallel, perpendicular,*** or ***neither.***
$4x − y = 2$
$x – 4y = −8$

Solution:

Find the slope of each line by solving each equation for *y*.

$$\begin{aligned} 4x-y&=2 \\ -y&=-4x+2 \\ y&=4x-2 \end{aligned} \qquad \begin{aligned} x-4y&=-8 \\ -4y&=-x-8 \\ y&=\frac{1}{4}x+2 \end{aligned}$$

The slopes, 4 and $\frac{1}{4}$, are not equal so the lines are not parallel.

The lines not perpendicular because their product is 1, not −1.

Thus, the two lines are neither parallel nor perpendicular.

Objective 5

Solve problems involving average rate of change.

CLASSROOM EXAMPLE 9

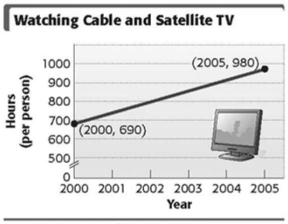
Interpreting Slope as Average Rate of Change

Americans spent an average of 886 hr in 2003 watching cable and satellite TV. Using this number for 2003 and the number for 2000 from the graph, find the average rate of change to the nearest tenth of an hour from 2000 to 2003. How does it compare to the average rate of change found in Example 9?

Solution:

$(x_1 y_1) = (2000, 690)$

$(x_2 y_2) = (2003, 886)$


Watching Cable and Satellite TV

Source: Veronis Suhler Stevenson.

CLASSROOM EXAMPLE 9

Interpreting Slope as Average Rate of Change (cont'd)

Average rate of change =

$$\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{886-690}{2003-2000}$$

$$=\frac{196}{3}=63.5$$

A positive slope indicates an increase.

The average rate of change from 2000 to 2003 is 63.5 hours per year, which is greater than 58 hours per year from 2000 to 2005 as in **Example 9.**

CLASSROOM EXAMPLE 10

Interpreting Slope as Average Rate of Change

In 2000, 942.5 million compact discs were sold in the United States. In 2006, 614.9 million CDs were sold. Find the average rate of change in CDs sold per year.

(*Source:* Recording Industry Association of America.)

Solution:

$(x_1 y_1) = (2000, 942.5) \qquad (x_2 y_2) = (2006, 614.9)$

A negative slope indicates a decrease.

Average rate of change = $\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{614.9-942.5}{2006-2000} \quad =\frac{-327.6}{6} \quad \approx -54.6$$

The average rate of change from 2000 to 2006 was –54.6 million CDs per year.

3.3 Linear Equations in Two Variables

Objectives

1. Write an equation of a line, given its slope and *y*-intercept.
2. Graph a line, using its slope and *y*-intercept.
3. Write an equation of a line, given its slope and a point on the line.
4. Write equations of horizontal and vertical lines.
5. Write an equation of a line, given two points on the line.
6. Write an equation of a line parallel or perpendicular to a given line.
7. Write an equation of a line that models real data.

Write an equation of a line, given its slope and *y*-intercept.

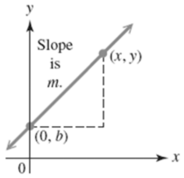
In **Section 3.2,** we found the slope of a line from its equation by solving the equation for y. Once we had isolated *y,* we found that the slope was the coefficient of *x*. However, we still don't know what the last number in the equation represents. That is, the number with no variable in the equation of the form, *y* = 4*x* + 8. To find out, suppose a line has a slope *m* and *y*-intercept (0, *b*).

$$m = \frac{y-b}{x-0} \quad \text{or} \quad m = \frac{y-b}{x}$$

$$mx = y - b$$

$$mx + b = y$$

$$y = mx + b$$



Thus, *b* is the *y*-intercept or 8 in our original equation.

Write an equation of a line, given its slope and *y*-intercept.

Slope-Intercept Form

The **slope-intercept form** of the equation of a line with slope ***m*** and *y*-intercept (0, *b*) is

$$y = mx + b.$$

↑ ↑

Slope *y*-intercept (0, *b*)

CLASSROOM EXAMPLE 1 **Writing an Equation of a Line**

Write an equation of the line with slope 2 and *y*-intercept (0, –3).

Solution:

Let *m* = 2
Let *b* = –3

Substitute these values into the slope-intercept form.

y = *mx* + *b*

y = 2*x* – 3

CLASSROOM EXAMPLE 2 **Graphing Lines Using Slope and *y*- Intercept**

Graph the line, using the slope and *y*-intercept.

x + 2*y* = – 4

Solution:

Write the equation in slope-intercept form by solving for *y*.

$x + 2y = -4$

$2y = -x - 4$ **Subtract *x*.**

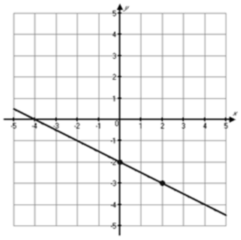
$$y = -\frac{1}{2}x - 2$$

Slope **y-intercept (0, –2)**

CLASSROOM EXAMPLE 2 **Graphing Lines Using Slope and *y*- Intercept (cont'd)**

Graph: $y = -\frac{1}{2}x - 2$

1. Plot the *y*-intercept, (0, –2).
2. The slope is $\frac{-1}{2}$ or $\frac{1}{-2}$.
3. Using $-\frac{1}{2}$, begin at (0, –2) and move 1 unit ***down*** and 2 units ***right***.
4. The line through these two points is the required graph.



Objective 3

Write an equation of a line, given its slope and a point on the line.

Write an equation of a line, given its slope a point on the line.

Point-Slope Form

The **point-slope form** of the equation of a line with slope ***m*** passing through the point (x_1, y_1) is

Slope

$$y - y_1 = m(x - x_1).$$

Given point

CLASSROOM EXAMPLE 3

Writing an Equation of a Line, Given the Slope and a Point

Write an equation of the line with slope $\frac{2}{5}$ and passing through the point (3, – 4).

Solution:

Use the point-slope form with (x_1, y_1) = (3, – 4) and $m = \frac{2}{5}$.

$y - y_1 = m(x - x_1)$ **Point-slope form**

$y - (-4) = \frac{2}{5}(x - 3)$ **Substitute.**

$y + 4 = \frac{2}{5}(x - 3)$

$5y + 20 = 2x - 6$ **Multiply by 5.**

$5y = 2x - 26$ **Subtract 20.**

$y = \frac{2}{5}x - \frac{26}{5}$ **Divide by 5.**

Objective 4

Write equations of horizontal and vertical lines.

Write equations of horizontal and vertical lines.

Equations of Horizontal and Vertical Lines

The horizontal line through the point *(a, b*) has equation *y* = *b*.

The vertical line through the point *(a, b*) has equation *x* = *a*.

CLASSROOM EXAMPLE 4

Writing Equations of Horizontal and Vertical Lines

Write an equation of the line passing through the point (2, − 1) that satisfies the given condition.

Solution:

Undefined slope

This is a vertical line, since the slope is undefined. A vertical line through the point (*a*, *b*) has equation *x* = *a*. Here the *x*-coordinate is 2, so the equation is *x* = 2.

Slope 0

Since the slope is 0, this is a horizontal line. A horizontal line through point (*a*, *b*) has equation *y* = *b*. Here the *y*-coordinate is − 1, so the equation is *y* = − 1.

CLASSROOM EXAMPLE 5

Writing an Equation of a Line, Given Two Points

Write an equation of the line passing through the points (– 2, 6) and (1, 4). Give the final answer in standard form.

Solution:

First find the slope by the slope formula.

$$m=\frac{4-6}{1-(-2)}=\frac{-2}{3}=-\frac{2}{3}$$

Use either point as (x_1, y_1) in the point-slope form of the equation of a line.

We will choose the point (1, 4): x_1 = 1 and y_1 = 4

CLASSROOM EXAMPLE 5

Writing an Equation of a Line, Given Two Points (cont'd)

Using $m=-\frac{2}{3}$; x_1 = 1 and y_1 = 4

$$y-y_1=m(x-x_1)$$

$$y-4=-\frac{2}{3}(x-1)$$ **Substitute.**

$$3y-12=-2x+2$$ **Multiply by 3.**

$$2x+3y=14$$ **Add 2*x* and 12.**

If the other point were used, the same equation would result.

Objective 6

Write an equation of a line parallel or perpendicular to a given line.

CLASSROOM EXAMPLE 6

Writing Equations of Parallel or Perpendicular Lines

Write an equation of the line passing through the point (– 8, 3) and (a) parallel to the line 2*x* – 3*y* = 10; (b) perpendicular to the line 2*x* – 3*y* = 10. Give the final answers in slope-intercept form.

Parallel to the line…

Solution:

Find the slope of the line 2*x* – 3*y* = 10 by solving for *y*.

$$2x-3y=10$$

$$-3y=-2x+10$$

$$y=\frac{2}{3}x-\frac{10}{3}$$

CLASSROOM EXAMPLE 6

Writing Equations of Parallel or Perpendicular Lines (cont'd)

Find an equation of the line passing through the point (–8, 3) when the slope is $\frac{2}{3}$.

Parallel lines have the same slope. Use point slope form and the given point.

$$y-y_1=m(x-x_1)$$

$$y-3=\frac{2}{3}[x-(-8)]$$

$$y-3=\frac{2}{3}(x+8)$$

$$y-3=\frac{2}{3}x+\frac{16}{3}$$

$$y=\frac{2}{3}x+\frac{16}{3}+\frac{9}{3}$$

$$y=\frac{2}{3}x+\frac{25}{3}$$

The fractions were not cleared because we want the equation in slope-intercept form instead of standard form.

CLASSROOM EXAMPLE 6

Writing Equations of Parallel or Perpendicular Lines (cont'd)

Perpendicular to the line…

Solution:

The slope of the perpendicular line would have a slope that is the negative reciprocal of $\frac{2}{3}$, or $-\frac{3}{2}$.

Find an equation of the line passing through the point (–8, 3), when the slope is $-\frac{3}{2}$.

$$y-y_1=m(x-x_1)$$

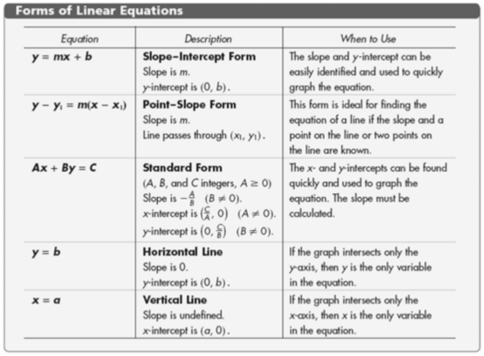
$$y-3=-\frac{3}{2}[x-(-8)]$$

$$y-3=-\frac{3}{2}(x+8)$$

$$y-3=-\frac{3}{2}x-12$$

$$y=-\frac{3}{2}x-9$$

Write an equation of a line parallel or perpendicular to a given line.



Forms of Linear Equations

Equation	*Description*	*When to Use*
$y = mx + b$	**Slope–Intercept Form** Slope is m. y-intercept is $(0, b)$.	The slope and y-intercept can be easily identified and used to quickly graph the equation.
$y - y_1 = m(x - x_1)$	**Point–Slope Form** Slope is m. Line passes through (x_1, y_1).	This form is ideal for finding the equation of a line if the slope and a point on the line or two points on the line are known.
$Ax + By = C$	**Standard Form** (A, B, and C integers, $A \geq 0$) Slope is $-\frac{A}{B}$ $(B \neq 0)$. x-intercept is $\left(\frac{C}{A}, 0\right)$ $(A \neq 0)$. y-intercept is $\left(0, \frac{C}{B}\right)$ $(B \neq 0)$.	The x- and y-intercepts can be found quickly and used to graph the equation. The slope must be calculated.
$y = b$	**Horizontal Line** Slope is 0. y-intercept is $(0, b)$.	If the graph intersects only the y-axis, then y is the only variable in the equation.
$x = a$	**Vertical Line** Slope is undefined. x-intercept is $(a, 0)$.	If the graph intersects only the x-axis, then x is the only variable in the equation.

Objective 7

Write an equation of a line that models real data.

CLASSROOM EXAMPLE 7

Determining a Linear Equation to Describe Real Data

Suppose there is a flat rate of $0.20 plus a charge of $0.10 per minute to make a telephone call. Write an equation that gives the cost y for a call of x minutes.

Solution:

y = $0.20 + 0.10x$

or

y = 0.10x$ + 0.20

CLASSROOM EXAMPLE 8

Finding an Equation of a Line That Models Data

The percentage of the U.S. population 25 yr and older with at least a high school diploma is shown in the table for selected years. (*Source:* U.S. Census Bureau.)

Let x = 0 represent 1950, x = 10 represent 1960, and so on. Use the data for 1950 and 2000 to write an equation that models the data.

Year	*Percent*
1950	34.3
1960	41.4
1970	52.3
1980	66.5
1990	77.6
2000	84.1
2007	85.7

Solution:

We choose two data points (0, 34.3) and (50, 84.1) to find the slope of the line.

$$m = \frac{84.1 - 34.3}{50 - 0} = \frac{49.8}{50} = 0.996$$

The equation is:

y = 0.996x + 34.3

CLASSROOM EXAMPLE 8

Finding an Equation of a Line That Models Data (cont'd)

Use the equation from the first part of the problem to approximate the percentage, to the nearest tenth, of the U.S. population 25 yr and older who were at least high school graduates in 1995.

Solution:

$$y = 0.996x + 34.3$$

$$y = 0.996(45) + 34.3$$

$$y = 79.1 \text{ or } 79.1\ \%$$

Year	*Percent*
1950	34.3
1960	41.4
1970	52.3
1980	66.5
1990	77.6
2000	84.1
2007	85.7

CLASSROOM EXAMPLE 9

Writing an Equation of a Line That Models Data

Use the ordered pairs (4, 220) and (6, 251) to find an equation that models the data in the graph below.

Solution:

Find the slope through the two points.

$$m = \frac{251 - 220}{6 - 4} = \frac{31}{2} = 15.5$$

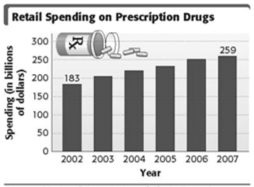
Then, we substitute one of the points into the point-slope form of the equation.

$$y - y_1 = m(x - x_1)$$

$$y - 220 = 15.5(x - 4)$$

$$y - 220 = 15.5x - 62$$

$$y = 15.5x + 158$$



Source: National Association of Chain Drug Stores.

3.4 Linear Inequalities in Two Variables

Objectives

1 Graph linear inequalities in two variables.

2 Graph the intersection of two linear inequalities.

3 Graph the union of two linear inequalities.

PEARSON

Graph linear inequalities in two variables.

Linear Inequalities in Two Variables

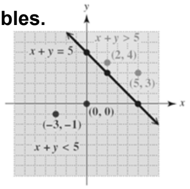
An inequality that can be written as

Ax + By < C, Ax + By ≤ C, Ax + By > C, or Ax + By ≥ C,

where *A*, *B*, and *C* are real numbers and *A* and *B* are not both 0, is a **linear inequality in two variables.**

Graph linear inequalities in two variables.

Consider the graph. The graph of the line $x + y = 5$ divides the points in the rectangular coordinate system into three sets:



1. Those points that lie on the line itself and satisfy the equation **$x + y = 5$** [like (0, 5), (2, 3), and (5, 0)]. This line, called the **boundary line,** divides the two ***regions*** in the plane that are graphed by the following two inequalities.

2. Those that lie in the half-plane above the line and satisfy the inequality **$x + y > 5$** [like (5, 3) and (2, 4)];

3. Those that lie in the half-plane below the line and satisfy the inequality **$x + y < 5$** [like (0, 0) and (–3, –1)].

Graph linear inequalities in two variables.

Graphing a Linear Inequality

Step 1 **Draw the graph of the straight line that is the boundary**. Make the line solid if the inequality involves **≤** or **≥.** Make the line dashed if the inequality involves < and >.

Step 2 **Choose a test point.** Choose any point not on the line, and substitute the coordinates of that point in the inequality.

Step 3 **Shade the appropriate region.** Shade the region that includes the test point of it satisfies the original inequality. Otherwise, shade the region on the other side of the boundary line.

CAUTION When drawing the boundary line in ***Step 1,*** be careful to draw a solid line if the inequality includes equality (≤, ≥) or a dashed line if equality is not included (<, >).

CLASSROOM EXAMPLE 1 **Graphing a Linear Inequality**

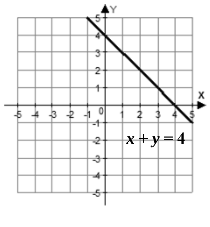
Graph $x + y \le 4$.

Solution:

Step 1 Graph the line $x + y = 4$, which has intercepts (4,0) and (0, 4), as a solid line since the inequality involves "≤".

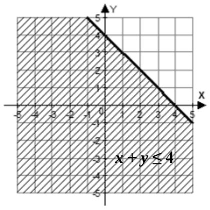
Step 2 Test (0, 0).

$$x + y \le 4$$
$$0 + 0 \le 4 \quad ? \quad \textbf{True}$$
$$0 \le 4$$



CLASSROOM EXAMPLE 1 **Graphing a Linear Inequality (cont'd)**

Step 3 Since the result is true, shade the region that contains (0, 0).



Graph linear inequalities in two variables.

If the inequality is written in the form $y > mx + b$ or $y < mx + b$, then the inequality symbol indicates which half-plane to shade.

If $y > mx + b$, then shade **above** the boundary line;

If $y < mx + b$, then shade below the boundary line;

This method works only if the inequality is solved for y.

A common error in using the method just described is to use the original inequality symbol when deciding which half-plane to shade. Be sure to use the inequality symbol found in the inequality ***after*** it is solved for *y*.

CLASSROOM EXAMPLE 2

Graphing a Linear Inequality

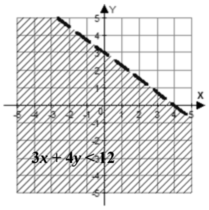
Graph $3x + 4y < 12$.

Solution:

Solve the inequality for *y*.

$$4y < -3x + 12$$

$$y < -\frac{3}{4}x + 3$$



Graph the boundary line $y = -\frac{3}{4}x + 3$ as a dashed line because the inequality symbol is <.

Since the **inequality is solved for *y*** and the inequality symbol is <, we shade the half-plane ***below*** the boundary line.

CLASSROOM EXAMPLE 2

Graphing a Linear Inequality (cont'd)

As a check, choose a test point not on the line, say (0, 0), and substitute for *x* and *y* in the original inequality.

$$3x + 4y < 12$$

$$3(0) + 4(0) < 12$$

$$0 < 12$$ **True**

This decision agrees with the decision to shade below the line.

Graph the intersection of two linear inequalities.

A pair of inequalities joined with the word ***and*** is interpreted as the intersection of the solution sets of the inequalities. ***The graph of the intersection of two or more inequalities is the region of the plane where all points satisfy all of the inequalities at the same time.***

CLASSROOM EXAMPLE 3

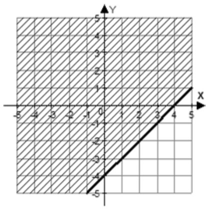
Graphing the Intersection of Two Inequalities

Graph $x - y \le 4$ and $x \ge -2$.

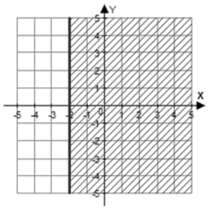
Solution:

To begin graph each inequality separately.

$x - y \le 4$



$x \ge -2$

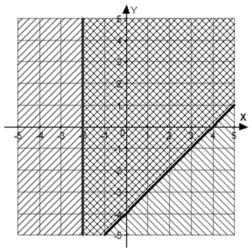


CLASSROOM EXAMPLE 3

Graphing the Intersection of Two Inequalities (cont'd)

Then we use shading to identify the intersection of the graphs.

$x - y \le 4$ **and** $x \ge -2$



Graph the union of two linear inequalities.

When two inequalities are joined by the word ***or,*** we must find the union of the graphs of the inequalities. ***The graph of the union of two inequalities includes all points satisfy either inequality.***

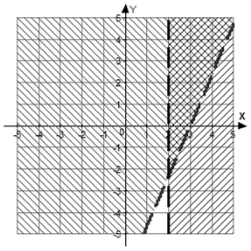
CLASSROOM EXAMPLE 4 **Graphing the Union of Two Inequalities**

Graph $7x - 3y < 21$ or $x > 2$.

Solution:

Graph each inequality with a dashed line.

The graph of the union is the region that includes all points on both graphs.



$7x - 3y < 21$ or $x > 2$

3.5 Introduction to Relations and Functions

Objectives

1 Distinguish between independent and dependent variables.

2 Define and identify relations and functions.

3 Find the domain and range.

4 Identify functions defined by graphs and equations.

Objective 1

Distinguish between independent and dependent variables.

Distinguish between independent and dependent variables.

We often describe one quantity in terms of another:

The amount of your paycheck if you are paid hourly depends on the number of hours you worked.

The cost at the gas station depends on the number of gallons of gas you pumped into your car.

The distance traveled by a car moving at a constant speed depends on the time traveled.

If the value of the variable *y* depends on the value of the variable *x*, then *y* is the **dependent variable** and *x* is the **independent variable.**

Independent Variable **Dependent Variable**

(*x*, *y*)

Define and identify relations and functions.

Relation

A **relation** is a set of ordered pairs.

Function

A **function** is a relation in which, for each value of the first component of the ordered pairs, there is ***exactly one value*** of the second component.

CLASSROOM EXAMPLE 1 **Determining Whether Relations are Functions**

Determine whether each relation defines a function.

Solution:

{(0, 3), (–1, 2), (–1, 3)}

No, the same *x*-value is paired with a different *y*-value.

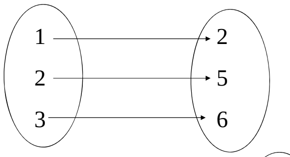
In a function, no two ordered pairs can have the same first component and different second components.

{(5, 4), (6, 4), (7, 4)}

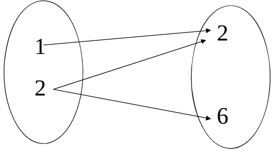
Yes, each different *x*-value is paired with a *y*-value. This does not violate the definition of a function.

Define and identify relations and functions.

Relations and functions can also be expressed as a correspondence or ***mapping*** from one set to another.



Function



Not a function

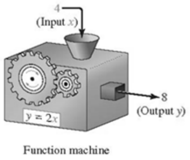
Define and identify relations and functions.

Relations and functions can be defined in many ways.

- ❑As a set of **ordered pairs** (as in **Example 1**).
- ❑As a **correspondence** or ***mapping*** (as previously illustrated).
- ❑As a **table.**
- ❑As a **graph.**
- ❑As an **equation** (or rule).

Another way to think of a function relationship is to think of the independent variable as an input and the dependent variable as an output. This is illustrated by the input-output (function) machine for the function defined by

$y = 2x$.



Function machine

Find the domain and range.

Domain and Range

In a relation, the set of all values of the independent variable (*x*) is the **domain.**

The set of all values of the dependent variable (*y*) is the **range.**

CLASSROOM EXAMPLE 2

Finding Domains and Ranges of Relations

Give the domain and range of the relation represented by the table below. Does it define a function?

Solution:

Number of Gallons Pumped	*Cost of This Number of Gallons*
0	0($3.20) = $ 0.00
1	1($3.20) = $ 3.20
2	2($3.20) = $ 6.40
3	3($3.20) = $ 9.60
4	4($3.20) = $12.80

Domain: {0, 1, 2, 3, 4}

Range: {$0, $3.20, $6.40, $9.60, $12.80}

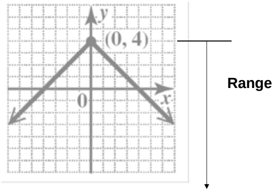
Yes, the relation defines a function.

CLASSROOM EXAMPLE 3

Finding Domains and Ranges from Graphs

Give the domain and range of the relation.

Solution:



The arrowheads indicate that the line extends indefinitely left and right.

Domain: (–∞**,** ∞)

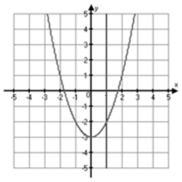
Range: (–∞, 4]

Identify functions defined by graphs and equations.

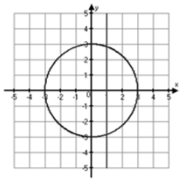
Vertical Line Test

If every vertical line intersects the graph of a relation in no more than one point, then the relation is a function.

Identify functions defined by graphs and equations.



Function



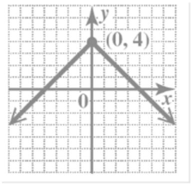
Not a Function

CLASSROOM EXAMPLE 4

Using the Vertical Line Test

Use the vertical line test to decide whether the relation shown below is a function.

Solution:



Yes, the relation is a function.

Identify functions defined by graphs and equations.

Relations are often defined by equations. If a relation is defined by an equation, keep the following guidelines in mind when finding its domain.

- ❑***Exclude from the domain any values that make the denominator of a fraction equal to 0.***
- ❑***Exclude from the domain any values that result in an even root of a negative number.***

Graphs that do not represent functions are still relations. All equations and graphs represent relations, and all relations have a domain and range.

Identify functions defined by graphs and equations.

Agreement on Domain

Unless specified otherwise, the domain of a relation is assumed to be all real numbers that produce real numbers when substituted for the independent variable.

CLASSROOM EXAMPLE 5

Identifying Functions from Their Equations

Decide whether each relation defines y as a function of x, and give the domain.

Solution:

$y = -2x + 7$

y is always found by multiplying by negative two and adding 7. Each value of x corresponds to just one value of y.

Yes, (–∞, ∞)

$y = \sqrt{5x-6}$

$$5x - 6 \geq 0$$
$$5x \geq 6$$
$$x \geq \frac{6}{5}$$

Yes, $\left[\frac{6}{5}, \infty\right)$

CLASSROOM EXAMPLE 5

Identifying Functions from Their Equations (cont'd)

$y^4 = x$

Solution: **No, [0, ∞)**

$y \geq 4x + 2$

$$4x + 2 \geq 0$$
$$4x \geq -2$$
$$x \geq -\frac{1}{2}$$

No, (–∞, ∞)

$y = \frac{6}{5+3x}$

$$0 = 5 + 3x$$
$$-5 = 3x$$
$$-\frac{5}{3} = x$$

Yes $\left(-\infty, -\frac{5}{3}\right) \cup \left(-\frac{5}{3}, \infty\right)$

Identify functions defined by graphs and equations.

Variations of the Definition of a Function

1. A **function** is a relation in which, for each value of the first component of the ordered pairs, there is exactly one value of the second component.

2. A **function** is a set of distinct ordered pairs in which no first component is repeated.

3. A **function** is a rule or correspondence that assigns exactly one range value to each domain value.

3.6 Function Notation and Linear Functions

Objectives

1 Use function notation.

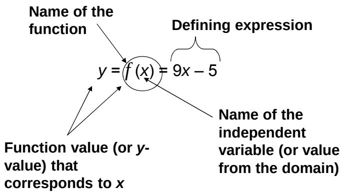
2 Graph linear and constant functions.

Use function notation.

When a function f is defined with a rule or an equation using x and y for the independent and dependent variables, we say, "y is a function of x" to emphasize that ***y* depends on *x*.** We use the notation

$$y = f(x),$$

called **function notation,** to express this and read $f(x)$ as "***f* of *x*.**"



Name of the function

Defining expression

$$y = f(x) = \underbrace{9x - 5}$$

Name of the independent variable (or value from the domain)

Function value (or *y*-value) that corresponds to *x*

CLASSROOM EXAMPLE 1

Evaluating a Function

Let $f(x) = 6x - 2.$ Find the value of the function f for x = −3.

Solution:

$$f(-3) = 6(-3) - 2$$

$$f(-3) = -18 - 2$$

$$f(x) = -20$$

CLASSROOM EXAMPLE 2

Evaluating a Function

Let $f(x) = \dfrac{-3x+5}{2}.$ Find the following.

$f(-3)$

$f(t)$

Solution:

$$f(-3) = \frac{-3(-3)+5}{2}$$

$$= \frac{9+5}{2}$$

$$= 7$$

$$f(t) = \frac{-3(t)+5}{2}$$

CLASSROOM EXAMPLE 3

Evaluating a Function

Let $g(x) = 5x - 1$. Find and simplify $g(m + 2)$.

Solution:

$g(x) = 5x - 1$

$g(m + 2) = 5(m + 2) - 1$

$= 5m + 10 - 1$

$= 5m + 9$

CLASSROOM EXAMPLE 4

Evaluating Functions

Find $f(2)$ for each function.

$f = \{(2, 6), (4, 2)\}$

$f(x) = -x^2$

Solution:

x	f(x)
2	6
4	2
0	0

$f(2) = 6$

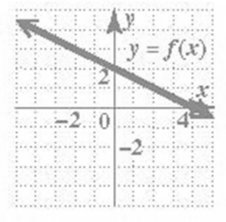
$f(2) = -2^2$

$f(2) = -4$

CLASSROOM EXAMPLE 5

Finding Function Values from a Graph

Refer to the graph of the function.



Solution:

Find *f* (2).

$$f(2) = 1$$

Find *f* (−2).

$$f(-2) = 3$$

For what value of x is *f* (*x*) = 0?

$$f(4) = 0$$

Use function notation.

Finding an Expression for *f* (*x*)

Step 1 Solve the equation for *y*.

Step 2 Replace *y* with *f* (*x*).

CLASSROOM EXAMPLE 6

Writing Equations Using Function Notation

Rewrite the equation using function notation *f* (*x*). Then find *f* (1) and *f* (*a*).

$x^2 - 4y = 3$

Solution:

Step 1 Solve for *y*.

$$-4y = -x^2 + 3$$

$$y = \frac{-x^2}{-4} + \frac{3}{-4}$$

$$y = \frac{x^2}{4} - \frac{3}{4}$$

$$f(x) = \frac{x^2}{4} - \frac{3}{4}$$

CLASSROOM EXAMPLE 6

Writing Equations Using Function Notation (cont'd)

Find *f* (1) and *f* (*a*).

Solution:

Step 2 Replace *y with f* (*x*).

$$f(1) = \frac{x^2}{4} - \frac{3}{4}$$

$$f(1) = \frac{(1)^2}{4} - \frac{3}{4}$$

$$= \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$$

$$f(a) = \frac{x^2}{4} - \frac{3}{4}$$

$$f(a) = \frac{(a)^2}{4} - \frac{3}{4}$$

$$= \frac{a^2 - 3}{4}$$

Graph linear and constant functions.

Linear Function

A function that can be defined by

f(x) = ax + b

for real numbers *a* and *b* is a **linear function**. The value of *a* is the slope *m* of the graph of the function. The domain of any linear function is (−∞, ∞).

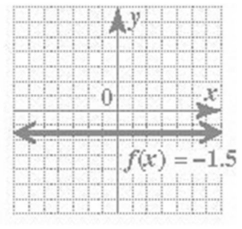
CLASSROOM EXAMPLE 7

Graphing Linear and Constant Functions

Graph the function. Give the domain and range.

f (*x*) = −1.5

Solution:



Domain: (−∞, ∞)

Range: {−1.5}