4.3 Applications of Systems of Linear Equations

Objectives

1. Solve geometry problems by using two variables.
2. Solve money problems by using two variables.
3. Solve mixture problems by using two variables.
4. Solve distance–rate–time problems by using two variables.
5. Solve problems with three variables by using a system of three equations.

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Applications of Systems of Linear Equations

PROBLEM-SOLVING HINT

When solving an applied problem using two variables, it is a good idea to pick letters that correspond to the descriptions of the unknown quantities. For example above, we could choose *c* to represent the number of citrons and *w* to represent the number of wood apples.

Applications of Systems of Linear Equations

Solving an Applied Problem by Writing a System of Equations

Step 1 **Read** the problem, several times if necessary. What information is given? What is to be found? This is often stated in the last sentence.

Step 2 **Assign variables** to represent the unknown values. Use a sketch, diagram, or table, as needed.

Step 3 **Write a system of equations** using the variable expressions.

Step 4 **Solve** the system of equations.

Step 5 **State the answer** to the problem. Label it appropriately. Does it seem reasonable?

Step 6 **Check** the answer in the words of the ***original*** problem.

CLASSROOM EXAMPLE 1 **Finding the Dimensions of a Soccer Field**

A rectangular soccer field has perimeter 360 yd. Its length is 20 yd more than its width. What are its dimensions?

Solution:

Step 1 **Read** the problem again. We are asked to find the dimensions of the field.

Step 2 **Assign variables.**

Let L = the length and W = the width.

Step 3 **Write a system of equations.**

The perimeter of a rectangle is given by $2W + 2L = 360$.

Since the length is 20 yd more than the width, $L = W + 20$.

CLASSROOM EXAMPLE 1 **Finding the Dimensions of a Soccer Field (cont'd)**

The system is

$$L = W + 20 \quad (1)$$
$$2W + 2L = 360 \quad (2)$$

Step 4 **Solve.** Substitute $W + 20$ for L in equations (2).

$$2W + 2(W + 20) = 360$$
$$2W + 2W + 40 = 360$$
$$4W = 320$$
$$W = 80$$

Substitute $W = 80$ into equation (1). $L = 80 + 20 = 100$

CLASSROOM EXAMPLE 1 **Finding the Dimensions of a Soccer Field (cont'd)**

Step 5 **State the answer.**

The length of the field is 100 yards and the width is 80 yards.

Step 6 **Check.**

The perimeter of the soccer field is 2(100) + 2(80) = 360 yd, and the length, 100 yards is 20 more than the width, since 100 – 20 = 80.

The answer is correct.

CLASSROOM EXAMPLE 2 — **Solving a Problem about Ticket Prices**

For the 2009 Major League Baseball and National Football League seasons, based on average ticket prices, three baseball tickets and two football tickets would have cost $229.90. Two baseball tickets and one football ticket would have cost $128.27. What were the average ticket prices for the tickets for the two sports?
(*Source:* Team Marketing Report.)

Solution:

Step 1 **Read** the problem again. There are two unknowns.

Step 2 **Assign variables.**

Let x = the average cost of baseball tickets, and y = the average cost of football tickets.

CLASSROOM EXAMPLE 2 — **Solving a Problem about Ticket Prices (cont'd)**

Step 3 **Write a system of equations.**

$$3x + 2y = 229.90 \quad (1)$$
$$2x + y = 128.27 \quad (2)$$

Step 4 **Solve.**

Multiply equation (2) by –2 and add to equation (1).

$$\begin{array}{rcrl} 3x + 2y & = & 229.90 & (1) \\ -4x - 2y & = & -256.54 & -2 \times (2) \\ \hline -x & = & -26.64 & \\ x & = & 26.64 & \end{array}$$

Let x = 26,64 in equation (2).

$$2(26.64) + y = 128.27$$
$$53.28 + y = 128.27$$
$$y = 74.99$$

CLASSROOM EXAMPLE 2 — **Solving a Problem about Ticket Prices (cont'd)**

Step 5 **State the answer.**

The average cost of a baseball ticket is $26.64 and the average cost of a football ticket is $74.99.

Step 6 **Check.**

3(26.64) + 2(74.99) = 229.90

and 2(26.64) + 74.99 = 128.27.

The answer is correct.

CLASSROOM EXAMPLE 3 — **Solving a Mixture Problem**

A grocer has some $4-per-lb coffee and some $8-per-lb coffee that she will mix to make 50 lb of $5.60-per-lb coffee. How many pounds of each should be used?

Solution:

Step 1 **Read** the problem.

Step 2 **Assign variables.**

Let x = number of pounds of the $4-per-lb coffee and y = the number of pounds of the $8-per-pound coffee.

Price per Pound	Number of Pounds	Value of Coffee
$4	x	$4x$
$8	y	$8y$
$5.60	50	5.6(50) = 280

CLASSROOM EXAMPLE 3 — **Solving a Mixture Problem (cont'd)**

Step 3 **Write a system of equations.**

$$x + y = 50 \quad (1)$$
$$4x + 8y = 280 \quad (2)$$

Step 4 **Solve.**

To eliminate x, multiply equation (1) by –4 and add to equation (2).

$$\begin{array}{rcrl} -4x - 4y & = & -200 & -4 \times (1) \\ 4x + 8y & = & 280 & (2) \\ \hline 4y & = & 80 & \\ y & = & 20 & \end{array}$$

Since y = 20 and $x + y$ = 50, x = 30.

CLASSROOM EXAMPLE 3 — **Solving a Mixture Problem (cont'd)**

Step 5 **State the answer.**

To mix the coffee, 30 lb of $4-per-lb coffee and 20 lb of $8-per-lb coffee should be used.

Step 6 **Check.**

30 + 20 = 50

and 4(30) + 8(20) = 280.

The answer is correct.

CLASSROOM EXAMPLE 4 **Solving a Motion Problem**

A train travels 600 mi in the same time that a truck travels 520 mi. Find the speed of each vehicle if the train's average speed is 8 mph faster than the truck's.

Solution:

***Step 1* Read** the problem.
We need to find the speed of each vehicle.

***Step 2* Assign variables.**

Let x = the train's speed and y = the truck's speed.

	Distance	Rate	Time
Train	600	x	$600/x$
Truck	520	y	$520/y$

The times must be equal.

CLASSROOM EXAMPLE 4 **Solving a Motion Problem (cont'd)**

***Step 3* Write a system of equations.**

$$\frac{600}{x} = \frac{520}{y}$$
$$600y = 520x$$
$$-520x + 600y = 0 \qquad (1)$$
$$x = y + 8. \qquad (2)$$

***Step 4* Solve.**
Substitute $y + 8$ for x in equation (1) to find y.

$$-520x + 600y = 0 \qquad (1)$$
$$-520(y + 8) + 600y = 0$$
$$-520y - 4160 + 600y = 0$$
$$80y = 4160$$
$$y = 52$$

Since $y = 52$ and $x = y + 8$, $x = 60$.

CLASSROOM EXAMPLE 4 **Solving a Motion Problem (cont'd)**

***Step 5* State the answer.**

The train's speed is 60 mph, the truck's speed is 52 mph.

***Step 6* Check.**

$$60 = 52 + 8$$

It would take the train 10 hours to travel 600 miles at 60 mph, which is the same amount of time it would take the truck to travel 520 miles at 52 mph.

The answer is correct.

Objective 5

Solve problems with three variables by using a system of three equations.

Solve problems with three variables by using a system of three equations.

PROBLEM-SOLVING HINT

If an application requires finding ***three*** unknown quantities, we can use a system of ***three*** equations to solve it. We extend the method used for two unknowns.

CLASSROOM EXAMPLE 5 **Solving a Problem Involving Prices**

A department store display features three kinds of perfume: Felice, Vivid, and Joy. There are 10 more bottles of Felice than Vivid, and 3 fewer bottles of Joy than Vivid. Each bottle of Felice costs $8, Vivid costs $15, and Joy costs $32. The total value of the all the perfume is $589. How many bottles of each are there?

Solution:

***Step 1* Read** the problem.
There are 3 unknowns.

***Step 2* Assign variables.**

Let x = the number of bottles of Felice at $8
y = the number of bottles of Vivid at $15, and
z = the number of bottles of Joy at $32.

CLASSROOM EXAMPLE 5 — **Solving a Problem Involving Prices (cont'd)**

Step 3 **Write a system of equations.**

There are 10 more bottles of Felice, so $x = y + 10$. (1)

There are 3 fewer bottles of Joy than Vivid, so $z = y - 3$. (2)

The total value is \$589, so $8x + 15y + 32z = 589$. (3)

CLASSROOM EXAMPLE 5 — **Solving a Problem Involving Prices (cont'd)**

Step 4 **Solve.**

Substitute $y + 10$ for x and $y - 3$ for z in equation (3) to find y.

$$8(y + 10) + 15y + 32(y - 3) = 589$$
$$8y + 80 + 15y + 32y - 96 = 589$$
$$55y - 16 = 589$$
$$55y = 605$$
$$y = 11$$

Since $y = 11$, $x = y + 10 = 21$ and $z = y - 3 = 8$.

CLASSROOM EXAMPLE 5 — **Solving a Problem Involving Prices (cont'd)**

Step 5 **State the answer.**

There are 21 bottles Felice, 11 of Vivid, and 8 of Joy.

Step 6 **Check.**

$$21(8) + 11(15) + 8(32) = 589$$

The answer is correct.

CLASSROOM EXAMPLE 6 — **Solving a Business Production Problem**

A paper mill makes newsprint, bond, and copy machine paper.

- Each ton of newsprint requires 3 tons of recycled paper and 1 ton of wood pulp.
- Each ton on bond requires 2 tons of recycled paper, and 4 tons of wood pulp, and 3 tons of rags.
- Each ton of copy machine paper requires 2 tons of recycled paper, 3 tons of wood pulp, and 2 tons of rags.

The mill has 4200 tons of recycled paper, 5800 tons of wood pulp, and 3900 tons of rags. How much of each kind of paper can be made from these supplies?

Solution:

Step 1 **Read** the problem.

CLASSROOM EXAMPLE 6 — **Solving a Business Production Problem (cont'd)**

Step 2 **Assign variables.**

Let x = the number of tons of newsprint
y = the number of tons of bond, and
z = the number of tons of copy machine paper.

Step 3 **Write a system of equations.**

$$3x + 2y + 2z = 4200 \quad (1)$$
$$x + 4y + 3z = 5800 \quad (2)$$
$$3y + 2z = 3900 \quad (3)$$

CLASSROOM EXAMPLE 6 — **Solving a Business Production Problem (cont'd)**

Step 4 **Solve** the system to find $x = 400$, $y = 900$, and $z = 600$.

$$3x + 2y + 2z = 4200$$
$$x + 4y + 3z = 5800$$
$$3y + 2z = 3900$$

Step 5 **State the answer.**

The paper mill can make 400 tons of newsprint, 900 tons of bond, and 600 tons of copy machine paper.

Step 6 **Check** that these values satisfy the conditions of the problem.

The answer is correct.