5.1 Integer Exponents and Scientific Notation

Objectives

1 Use the product rule for exponents.

2 Define 0 and negative exponents.

3 Use the quotient rule for exponents.

4 Use the power rules for exponents.

5 Simplify exponential expressions.

6 Use the rules for exponents with scientific notation.

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Integer Exponents and Scientific Notation

We use exponents to write products of repeated factors. For example,

2^5 is defined as 2 • 2 • 2 • 2 • 2 = 32.

The number 5, the **exponent,** shows that the **base** 2 appears as a factor five times. The quantity 2^5 is called an **exponential** or a **power.** We read 2^5 as **"2 to the fifth power"** or **"2 to the fifth."**

Use the product rule for exponents.

Product Rule for Exponents

If m and n are natural numbers and a is any real number, then

$$a^m \cdot a^n = a^{m+n}.$$

That is, when multiplying powers of like bases, keep the same base and add the exponents.

CAUTION Be careful not to multiply the bases. ***Keep the same base and add the exponents.***

CLASSROOM EXAMPLE 1 **Using the Product Rule for Exponents**

Apply the product rule, if possible, in each case.

Solution:

$m^8 \cdot m^6 = m^{8+6} = m^{14}$

$m^5 \cdot p^4$ Cannot be simplified further because the bases m and p are not the same. The product rule does not apply.

$(-5p^4)(-9p^5) = (-5)(-9)(p^4p^5) = 45p^{4+5} = 45p^9$

$(-3x^2y^3)(7xy^4) = (-3)(7)\,x^2xy^3y^4 = -21x^{2+1}y^{3+4} = -21x^3y^7$

Define 0 and negative exponents.

Zero Exponent

If a is any nonzero real number, then

$$a^0 = 1.$$

The expression 0^0 is undefined.

CLASSROOM EXAMPLE 2 **Using 0 as an Exponent**

Evaluate.

Solution:

$29^0 = 1$

$(-29)^0 = 1$

$-29^0 = -(29^0) = -1$

$8^0 - 15^0 = 1 - 1 = 0$

Define 0 and negative exponents.

Negative Exponent

For any natural number *n* and any nonzero real number *a*,

$$a^{-n} = \frac{1}{a^n}.$$

A negative exponent does not indicate a negative number; negative exponents lead to reciprocals.

CAUTION $3^{-2} = \frac{1}{3^{2}} = \frac{1}{9}$ Not negative $\quad -3^{-2} = -\frac{1}{3^{2}} = -\frac{1}{9}$ Negative

CLASSROOM EXAMPLE 3 **Using Negative Exponents**

Write with only positive exponents.

Solution:

$6^{-5} \quad = \frac{1}{6^5}$

$(2x)^{-4},\ x \neq 0 \quad = \frac{1}{(2x)^4},\ x \neq 0$

$-7p^{-4},\ p \neq 0 \quad = -7\left(\frac{1}{p^4}\right) \quad = -\frac{7}{p^4},\ p \neq 0$

Evaluate $4^{-1} - 2^{-1}$. $= \frac{1}{4} - \frac{1}{2} \quad = \frac{1}{4} - \frac{2}{4} \quad = -\frac{1}{4}$

CLASSROOM EXAMPLE 4 **Using Negative Exponents**

Evaluate.

Solution:

$$\frac{1}{4^{-3}} = \frac{1}{\frac{1}{4^3}} = 1 \div \frac{1}{4^3} = 1 \cdot \frac{4^3}{1} = 4^3 = 64$$

$$\frac{3^{-3}}{9^{-1}} = \frac{\frac{1}{3^3}}{\frac{1}{9}} = \frac{1}{3^3} \div \frac{1}{9} = \frac{1}{3^3} \cdot \frac{9}{1} = \frac{1}{27} \cdot \frac{9}{1} = \frac{9}{27} = \frac{1}{3}$$

Define 0 and negative exponents.

Special Rules for Negative Exponents

If $a \neq 0$ and $b \neq 0$, then

$$\frac{1}{a^{-n}} = a^n \quad \text{and} \quad \frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}.$$

Use the quotient rule for exponents.

Quotient Rule for Exponents

If *a* is any nonzero real number and *m* and *n* are integers, then

$$\frac{a^m}{a^n} = a^{m-n}.$$

That is, when dividing powers of like bases, keep the same base and subtract the exponent of the denominator from the exponent of the numerator.

CAUTION Be careful when working with quotients that involve negative exponents in the denominator. Write the numerator exponent, then a subtraction symbol, and then the denominator exponent. Use parentheses.

CLASSROOM EXAMPLE 5 **Using the Quotient Rule for Exponents**

Apply the quotient rule, if possible, and write each result with only positive exponents.

Solution:

$$\frac{m^8}{m^{13}} = m^{8-13} = m^{-5} = \frac{1}{m^5},\quad m \neq 0$$

$$\frac{5^{-6}}{5^{-8}} = 5^{-6-(-8)} = 5^{-6+8} = 5^2, \text{ or } 25$$

$\frac{x^3}{y^5},\ y \neq 0$ Cannot be simplified because the bases *x* and *y* are different. The quotient rule does not apply.

Use the power rules for exponents.

Power Rule for Exponents

If *a* and *b* are real numbers and *m* and *n* are integers, then

$$\textbf{a) } (a^m)^n = a^{mn}, \quad \textbf{b) } (ab)^m = a^m b^m,$$

and

$$\textbf{c) } \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (b \neq 0).$$

That is,

a) To raise a power to a power, multiply exponents.
b) To raise a product to a power, raise each factor to that power.
c) To raise a quotient to a power, raise the numerator and the denominator to that power.

CLASSROOM EXAMPLE 6 — Using the Power Rules for Exponents

Simplify, using the power rules.

Solution:

$$(r^5)^4 = (r^5)^4 = r^{5\cdot 4} = r^{20}$$

$$(-3y^5)^2 = (-3)^2(y^5)^2 = 9y^{5\cdot 2} = 9y^{10}$$

$$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$$

Use the power rules for exponents.

Special Rules for Negative Exponents, Continued

If $a \neq 0$ and $b \neq 0$ and *n* is an integer, then

$$a^{-n} = \left(\frac{1}{a}\right)^n \quad \text{and} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n.$$

That is, any nonzero number raised to the negative *n*th power is equal to the reciprocal of that number raised to the *n*th power.

CLASSROOM EXAMPLE 7 — Using Negative Exponents with Fractions

Write with only positive exponents and then evaluate.

Solution:

$$\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}$$

$$\left(\frac{1}{2x}\right)^{-5},\ x \neq 0 = \left(\frac{2x}{1}\right)^5 = 2x^5 = 32x^5$$

Use the power rules for exponents.

Definition and Rules for Exponents

For all integers *m* and *n* and all real numbers *a* and *b*, the following rules apply.

Product Rule $a^m \cdot a^n = a^{m+n}$

Quotient Rule $\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$

Zero Exponent $a^0 = 1 \quad (a \neq 0)$

Use the power rules for exponents.

Definition and Rules for Exponents, Continued

Negative Exponent $a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$

Power Rules $(a^m)^n = a^{mn} \quad (ab)^m = a^m b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (b \neq 0)$

Special Rules $\frac{1}{a^{-n}} = a^n \quad (a \neq 0) \quad \frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n} \quad (a, b \neq 0)$

$a^{-n} = \left(\frac{1}{a}\right)^n \quad (a \neq 0) \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad (a, b \neq 0)$

Objective 5

Simplify exponential expressions.

CLASSROOM EXAMPLE 8 **Using the Definitions and Rules for Exponents**

Simplify. Assume that all variables represent nonzero real numbers.

Solution:

$$\left(4^2\right)^{-5} = 4^{2(-5)} = 4^{-10} = \frac{1}{4^{10}}$$

$$x^{-4} \bullet x^{-6} \bullet x^{8} = x^{-4+(-6)+8} = x^{-2} = \frac{1}{x^2}$$

$$\frac{\left(m^2 n\right)^{-2}}{m^{-3} n} = \frac{\left(m^2\right)^{-2} n^{-2}}{m^{-3} n} = \frac{m^{-4} n^{-2}}{m^{-3} n} = \frac{m^{-4}}{m^{-3}} \bullet \frac{n^{-2}}{n}$$

$$= m^{-4-(-3)} n^{-2-1} = m^{-4+3} n^{-3} = m^{-1} n^{-3}$$

$$= \frac{1}{mn^3}$$

CLASSROOM EXAMPLE 8 **Using the Definitions and Rules for Exponents (cont'd)**

Simplify. Assume that all variables represent nonzero real numbers.

Solution:

$$\left(\frac{2y}{x^3}\right)^2 \left(\frac{4y}{x}\right)^{-1} = \frac{2^2 y^2}{x^6} \bullet \frac{4^{-1} y^{-1}}{x^{-1}}$$

Combination of rules

$$= \frac{2^2 4^{-1} y^1}{x^5}$$

$$= \frac{2^2 y}{4x^5}$$

$$= \frac{y}{x^5}$$

Use the rules for exponents with scientific notation.

In scientific notation, a number is written with the decimal point after the first nonzero digit and multiplied by a power of 10.

This is often a simpler way to express very large or very small numbers.

Use the rules for exponents with scientific notation.

Scientific Notation

A number is written in **scientific notation** when it is expressed in the form

$$a \times 10^n$$

where $1 \leq |a| < 10$ and n is an integer.

Use the rules for exponents with scientific notation.

Converting to Scientific Notation

Step 1 **Position the decimal point.** Place a caret, ^, to the right of the first nonzero digit, where the decimal point will be placed.

Step 2 **Determine the numeral for the exponent.** Count the number of digits from the decimal point to the caret. This number gives the absolute value of the exponent on 10.

Step 3 **Determine the sign for the exponent.** Decide whether multiplying by 10^n should make the result of ***Step 1*** greater or less. The exponent should be positive to make the result greater; it should be negative to make the result less.

CLASSROOM EXAMPLE 9 — **Writing Numbers in Scientific Notation**

Write the number in scientific notation.

29,800,000

Solution:

Step 1 Place a caret to the right of the 2 (the first nonzero digit) to mark the new location of the decimal point.

Step 2 Count from the decimal point 7 places, which is understood to be after the caret.

$$29{,}800{,}000 = 2.9{,}800{,}000. \leftarrow$$ **Decimal point moves 7 places to the left**

Step 3 Since 2.98 is to be made greater, the exponent on 10 is positive.

$$29{,}800{,}000 = 2.98 \times 10^{7}$$

CLASSROOM EXAMPLE 9 — **Writing Numbers in Scientific Notation (cont'd)**

Write the number in scientific notation.

0.0000000503

Solution:

Step 1 Place a caret to the right of the 5 (the first nonzero digit) to mark the new location of the decimal point.

Step 2 Count from the decimal point 8 places, which is understood to be after the caret.

$$0.0000000503 = 0.00000005.03 \leftarrow$$ **Decimal point moves 7 places to the left**

Step 3 Since 5.03 is to be made less, the exponent 10 is negative.

$$0.0000000503 = 5.03 \times 10^{-8}$$

Use the rules for exponents with scientific notation.

Converting a Positive Number from Scientific Notation

Multiplying a positive number by a positive power of 10 makes the number greater, so move the decimal point to the right if *n* is positive in 10^n.

Multiplying a positive number by a negative power of 10 makes the number less, so move the decimal point to the left if *n* is negative.

If *n* is 0, leave the decimal point where it is.

CLASSROOM EXAMPLE 10 — **Converting from Scientific Notation to Standard Notation**

Write each number in standard notation.

Solution:

2.51×10^3 = 2510

Move the decimal 3 places to the right.

-6.8×10^{-4} = –0.000068

Move the decimal 4 places to the left.

When converting from scientific notation to standard notation, ***use the exponent to determine the number of places and the direction in which to move the decimal point.***

CLASSROOM EXAMPLE 11 — **Using Scientific Notation in Computation**

Evaluate $\dfrac{200{,}000 \times 0.0003}{0.06}$.

Solution:

$$= \frac{2\times10^{5}\times3\times10^{-4}}{6\times10^{-2}}$$

$$= \frac{2\times3\times10^{5}\times10^{-4}}{6\times10^{-2}}$$

$$= \frac{2\times3\times10^{1}}{6\times10^{-2}}$$

$$= \frac{2\times3}{6}\times10^{3} \quad = 1\times10^{3} \quad = 1000$$

CLASSROOM EXAMPLE 12 — **Using Scientific Notation to Solve Problems**

The distance to the sun is 9.3×10^7 mi. How long would it take a rocket traveling at 3.2×10^3 mph to reach the sun?

Solution:

$d = rt$, so $t = \dfrac{d}{r}$

$$= \frac{9.3\times10^{7}}{3.2\times10^{3}}$$

$$= \frac{9.3}{3.2}\times10^{7-3}$$

$$\approx 2.9\times10^{4}$$

It would take approximately 2.9×10^4 hours.

5.2 Adding and Subtracting Polynomials

Objectives

1 Know the basic definitions for polynomials.

2 Add and subtract polynomials.

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Know the basic definitions for polynomials.

A **term** is a number (constant), a variable, or the product or quotient of a number and one or more variables raised to powers.

$$4x,\quad \frac{1}{2}m^5 \text{ or } \left(\frac{m^5}{2}\right),\quad -7z^9,\quad 6x^2z,\quad \frac{5}{3x^2},\quad \text{and } 9$$

The number in the product is called the **numerical coefficient,** or just the **coefficient**.

A term or a sum of two or more terms is and **algebraic expression.** The simplest kind of algebraic expression is a ***polynomial.***

The number 0 has no degree, since 0 times a variable to any power is 0.

Know the basic definitions for polynomials.

Polynomial

A **polynomial** in x is a term or a finite sum of terms of the form ax^n, where a is a real number and the exponent n is a whole number.

Polynomials

$$3x-5,\quad 4m^3-5m^2p+8,\quad \text{and}\quad -5t^2s^3$$

Not Polynomials

$$x^{-1}+3x^{-2},\quad \sqrt{9-x},\quad \text{and}\quad \frac{1}{x}$$

Know the basic definitions for polynomials.

A polynomial containing only the variable x is called a **polynomial in x.** A polynomial in one variable is written in **descending powers** of the variable if the exponents on the variable decrease from left to right.

$$x^5 - 6x^2 + 12x - 5$$

When written in descending powers of the variable, the greatest-degree term is written first and is called the **leading term** of the polynomial. Its coefficient is the **leading coefficient.**

If a polynomial in a single variable is written in descending powers of that variable, the degree of the polynomial will be the degree of the leading term.

CLASSROOM EXAMPLE 1 **Writing Polynomials in Descending Powers**

Write the polynomial in descending powers of the variable. Then give the leading term and the leading coefficient.

$-3z^4 + 2z^3 + z^5 - 6z$

Solution:

$z^5 - 3z^4 + 2z^3 - 6z$

The largest exponent is 5, it would be the first term and its coefficient would be 1.

Know the basic definitions for polynomials.

Some polynomials with a specific number of terms are so common that they are given special names.

Trinomial: has exactly three terms

Binomial: has exactly two terms

Monomial: has only one term

Type of Polynomial	*Examples*
Monomial	$5x$, $7m^9$, -8, x^2y^2
Binomial	$3x^2 - 6$, $11y + 8$, $5a^2b + 3a$
Trinomial	$y^2 + 11y + 6$, $8p^3 - 7p + 2m$, $-3 + 2k^5 + 9z^4$
None of these	$p^3 - 5p^2 + 2p - 5$, $-9z^3 + 5c^2 + 2m^5 + 11r^2 - 7r$

CLASSROOM EXAMPLE 2 — **Classifying Polynomials**

Identify each polynomial as a ***monomial, binomial, trinomial,*** or none of these. Also, give the degree.

$a^4b^2 - ab^6$

Solution:

Binomial of degree of 7

-100

Monomial of degree of 0

Objective 2

Add and subtract polynomials.

CLASSROOM EXAMPLE 3 — **Combining Like Terms**

Combine like terms.

Solution:

$2z^4 + 3x^4 + z^4 - 9x^4$

$= 2z^4 + z^4 + 3x^4 - 9x^4$

$= 3z^4 - 6x^4$

$3t + 4r - 4t - 8r$

$= 3t - 4t + 4r - 8r$

$= -t - 4r$

$5x^2z - 3x^3z^2 + 8x^2z + 12x^3z^2$

$= 5x^2z + 8x^2z - 3x^3z^2 + 12x^3z^2$

$= 13x^2z + 9x^3z^2$

Add and subtract polynomials.

Adding Polynomials

To add two polynomials, combine like terms.

CAUTION ***Only like terms can be combined.***

CLASSROOM EXAMPLE 4 — **Adding Polynomials**

Add.

$(-5p^3 + 6p^2) + (8p^3 - 12p^2)$

Solution:

Use commutative and associative properties to rearrange the polynomials so that like terms are together. Then use the distributive property to combine like terms.

$= -5p^3 + 8p^3 + 6p^2 - 12p^2$

$= 3p^3 - 6p^2$

$$\begin{array}{r} -6r^5 + 2r^3 - r^2 \\ 8r^5 - 2r^3 + 5r^2 \\ \hline 2r^5 \qquad + 4r^2 \end{array}$$

You can add polynomials vertically by placing like terms in columns.

CLASSROOM EXAMPLE 5 — **Subtracting Polynomials**

Subtract

$(p^4 + p^3 + 5) - (3p^4 + 5p^3 + 2)$

Solution:

Change every sign in the second polynomial and add.

$= p^4 + p^3 + 5 - 3p^4 - 5p^3 - 2$

$= p^4 - 3p^4 + p^3 - 5p^3 + 5 - 2$

$= -2p^4 - 4p^3 + 3$

$$\begin{array}{r} 2k^3 - 3k^2 - 2k + 5 \\ 4k^3 + 6k^2 - 5k + 8 \\ \hline \end{array} \qquad \begin{array}{r} 2k^3 - 3k^2 - 2k + 5 \\ -4k^3 - 6k^2 + 5k - 8 \\ \hline -2k^3 - 9k^2 + 3k - 3 \end{array}$$

To subtract vertically, write the first polynomial above the second, lining up like terms in columns.Change all the signs in the second polynomial and add.

5.3 Polynomial Functions, Graphs and Composition

Objectives

1. Recognize and evaluate polynomial functions.
2. Use a polynomial function to model data.
3. Add and subtract polynomial functions.
4. Find the composition of functions.
5. Graph basic polynomial functions.

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Recognize and evaluate polynomial functions.

Polynomial Function

A **polynomial function of degree *n*** is defined by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0,$$

for real numbers a_n, a_{n-1}, $\ldots a_1$, and a_0, where $a_n \neq 0$ and n is a whole number.

CLASSROOM EXAMPLE 1 **Evaluating Polynomial Functions**

Let $f(x) = -x^2 + 5x - 11$. Find $f(-4)$

Solution:

Read this as "*f* of negative 4," not *f* times negative 4."

$f(-4) = -x^2 + 5x - 11$

$= -(-4)^2 + 5(-4) - 11$ **Substitute –4 for *x*.**

$= -16 - 20 - 11$ **Order of operations**

$= -47$ **Subtract.**

CLASSROOM EXAMPLE 2 **Using a Polynomial Model to Approximate Data**

The number of students enrolled in public schools (grades pre-K-12) in the United States during the years 1990 through 2006 can be modeled by the polynomial function defined by

$$P(x) = -0.01774x^2 + 0.7871x + 41.26,$$

where $x = 0$ corresponds to the year 1990, $x = 1$ corresponds to 1991, and so on, and $P(x)$ is in millions. Use this function to approximate the number of public school students in 2000.

(*Source:* Department of Education.)

Solution:

$P(x) = -0.01774x^2 + 0.7871x + 41.26$

$P(10) = -0.01774(10)^2 + 0.7871(10) + 41.26$

$P(10) = -1.774 + 7.87 + 41.26$

$= 47.4$ million students

Add and subtract polynomial functions.

The operations of addition, subtraction, multiplication, and division are also defined for functions.

For example, businesses use the equation "profit equals revenue minus cost," written in function notation as

$$P(x) = R(x) - C(x)$$

Profit function — Revenue function — Cost function

where x is the number of items produced and sold.

Add and subtract polynomial functions.

Adding and Subtracting Functions

If $f(x)$ and $g(x)$ define functions, then

$$(f + g)(x) = f(x) + g(x)$$ **Sum function**

and

$$(f - g)(x) = f(x) - g(x).$$ **Difference function**

In each case, the domain of the new function is the intersection of the domains of $f(x)$ and $g(x)$.

CLASSROOM EXAMPLE 3 | **Adding and Subtracting Functions**

For $f(x) = 3x^2 + 8x - 6$ and $g(x) = -4x^2 + 4x - 8$, find each of the following.

Solution:

$(f + g)(x)$

$= f(x) + g(x)$
$= (3x^2 + 8x - 6) + (-4x^2 + 4x - 8)$
$= -x^2 + 12x - 14$

$(f - g)(x)$

$= f(x) - g(x)$
$= (3x^2 + 8x - 6) - (-4x^2 + 4x - 8)$
$= 3x^2 + 8x - 6 + 4x^2 - 4x + 8$
$= 7x^2 + 4x + 2$

CLASSROOM EXAMPLE 4 | **Adding and Subtracting Functions**

For $f(x) = 18x^2 - 24x$ and $g(x) = 3x$, find each of the following.

$(f + g)(x)$ and $(f + g)(-1)$ $(f - g)(x)$ and $(f - g)(1)$

Solution:

$(f + g)(x)$
$= f(x) + g(x)$
$= 18x^2 - 24x + 3x$
$= 18x^2 - 21x$

$(f - g)(x)$
$= f(x) - g(x)$
$= 18x^2 - 24x - 3x$
$= 18x^2 - 27x$

$(f + g)(-1)$
$= f(-1) + g(-1)$
$= [18(-1)^2 - 24(-1)] + 3(-1)$
$= [18 + 24] - 3$
$= 39$

$(f - g)(1)$
$= f(1) - g(1)$
$= [18(1)^2 - 24(1)] - 3(1)$
$= [18 - 24] - 3$
$= -9$

Find the composition of functions.

Composition of Functions

If f and g are functions, then the **composite function,** or **composition,** of g and f is defined by

$$(g \circ f)(x) = g(f(x))$$

for all x in the domain of f such that $f(x)$ is in the domain of g.

CLASSROOM EXAMPLE 5 | **Evaluating a Composite Function**

Let $f(x) = x - 4$ and $g(x) = x^2$. Find $(f \circ g)(3)$.

Solution:

Evaluate the "inside" function value first.

$(f \circ g)(3) = f(g(3))$
$= f(3^2)$
$= f(9)$
$= 9 - 4$
$= 5$

Now evaluate the "outside" function.

CLASSROOM EXAMPLE 6 | **Finding Composite Functions**

Let $f(x) = 3x + 6$ and $g(x) = x^3$. Find the following.

Solution:

$(f \circ g)(2)$ $(g \circ f)(x)$

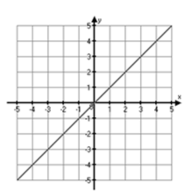
$(f \circ g)(2) = f(2^3)$
$= f(8)$
$= 3(8) + 6$
$= 30$

$(g \circ f)(x) = g(3x + 6)$
$= (3x + 6)^3$

Graph basic polynomial functions.

The simplest polynomial function is the **identity function,** defined by $f(x) = x$ and graphed below. This function pairs each real number with itself.

x	$f(x) = x$
–2	–2
–1	–1
0	0
1	1
2	2

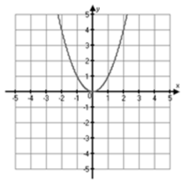


The domain (set of x-values) is all real numbers, $(-\infty, \infty)$.
The range (set of y-values) is also $(-\infty, \infty)$.

Graph basic polynomial functions.

Another polynomial function, defined by $f(x) = x^2$ and graphed below, is the **squaring function.** For this function, every real number is paired with its square. The graph of the squaring function is a ***parabola.***

x	$f(x) = x^2$
–2	4
–1	1
0	0
1	1
2	4



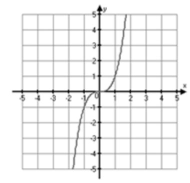
The domain is all real numbers, $(-\infty, \infty)$.

The range is $[0, \infty)$.

Graph basic polynomial functions.

The **cubing function** is defined by $f(x) = x^3$ and graphed below. This function pairs every real number with its cube.

x	$f(x) = x^3$
–2	–8
–1	–1
0	0
1	1
2	8



The domain and range are both $(-\infty, \infty)$.

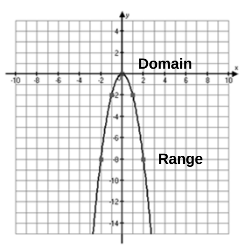
CLASSROOM EXAMPLE 7

Graphing Variations of Polynomial Functions

Graph $f(x) = -2x^2$. Give the domain and range.

Solution:

x	$f(x) = -2x^2$
–2	–8
–1	–2
0	0
1	–2
2	–8



The domain is $(-\infty, \infty)$.

The range is $(-\infty, 0]$.

5.4 Multiplying Polynomials

Objectives

1 Multiply terms.

2 Multiply any two polynomials.

3 Multiply binomials.

4 Find the product of the sum and difference of two terms.

5 Find the square of a binomial.

6 Multiply polynomial functions.

CLASSROOM EXAMPLE 1 — **Multiplying Monomials**

Find the product.

$8k^3y(9ky)$

Solution:

$= (8)(9)k^3 \bullet k^1 \bullet y^1 \bullet y^1$

$= 72k^{3+1}y^{1+1}$

$= 72k^4y^2$

Objective 2

Multiply any two polynomials.

CLASSROOM EXAMPLE 2 — **Multiplying Polynomials**

Find the product.

$-2r(9r - 5)$

Solution:

$= -2r(9r) - 2r(-5)$

$= -18r^2 + 10r$

CLASSROOM EXAMPLE 2 — **Multiplying Polynomials (cont'd)**

Find the product.

$(2k - 5m)(3k + 2m)$

Solution:

$= (2k - 5m)(3k) + (2k - 5m)(2m)$

$= 2k(3k) + (-5m)(3k) + (2k)(2m) + (-5m)(2m)$

$= 6k^2 - 15km + 4km - 10m^2$

$= 6k^2 - 11km - 10m^2$

CLASSROOM EXAMPLE 3 — **Multiplying Polynomials Vertically**

Find the product.

$(4x - 3y)(3x - y)$

Solution:

$4x - 3y$	
$3x - y$	
$-4xy + 3y^2$	**Multiply –*y* (4*x* – 3*y*)**
$12x^2 - 9xy$	**Multiply 3*x* (4*x* – 3*y*)**
$12x^2 - 13xy + 3y^2$	**Combine like terms.**

CLASSROOM EXAMPLE 3 — **Multiplying Polynomials Vertically (cont'd)**

Find the product.

$(5a^3 - 6a^2 + 2a - 3)(2a - 5)$

Solution:

$$\begin{array}{rrrrr} & 5a^3 & -6a^2 & +2a & -3 \\ & & & 2a & -5 \\ \hline & -25a^3 & +30a^2 & -10a & +15 \\ 10a^4 & -12a^3 & +4a^2 & -6a & \\ \hline 10a^4 & -37a^3 & +34a^2 & -16a & +15 \end{array}$$

Combine like terms.

Multiply binomials.

When working with polynomials, the products of two binomials occurs repeatedly. There is a shortcut method for finding these products.

First Terms
Outer Terms
Inner Terms
Last Terms

CAUTION: The FOIL method is an extension of the distributive property, and the acronym ***"FOIL" applies only to multiplying two binomials.***

CLASSROOM EXAMPLE 4 — **Using the FOIL Method**

Use the FOIL method to find each product.

Solution:

$(5r - 3)(2r - 5)$

F O I L

$= (5r)(2r) + (5r)(-5) + (-3)(2r) + (-3)(-5)$

$= 10r^2 \quad - 25r \quad - 6r + 15$

$= 10r^2 - 31r + 15$

$(4y - z)(2y + 3z)$

F O I L

$= (4y)(2y) + (4y)(3z) + (-z)(2y) + (-z)(3z)$

$= 8y^2 + 12yz - 2yz - 3z^2$

$= 8y^2 + 10yz - 3z^2$

Objective 4

Find the product of the sum and difference of two terms.

Find the product of the sum and difference of two terms.

Product of the Sum and Difference of Two Terms

The **product of the sum and difference of the two terms *x* and *y*** is the difference of the squares of the terms.

$$(x + y)(x - y) = x^2 - y^2$$

CLASSROOM EXAMPLE 5 — **Multiplying the Sum and Difference of Two Terms**

Find each product.

Solution:

$(m + 5)(m - 5)$ $= m^2 - 5^2$

$= m^2 - 25$

$(x - 4y)(x + 4y)$ $= x^2 - (4y)^2$

$= x^2 - 4^2y^2$

$= x^2 - 16y^2$

$4y^2(y + 7)(y - 7)$ $= 4y^2\,(y^2 - 49)$

$= 4y^4 - 196y^2$

Find the square of a binomial.

Square of a Binomial

The **square of a binomial** is the sum of the square of the first term, twice the product of the two terms, and the square of the last term.

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

CLASSROOM EXAMPLE 6 **Squaring Binomials**

Find each product.

Solution:

$(t + 9)^2$ $= t^2 + 2 \cdot t \cdot 9 + 9^2$

$= t^2 + 18t + 81$

$(2m + 5)^2$ $= (2m)^2 + 2(2m)(5) + 5^2$

$= 4m^2 + 20m + 25$

$(3k - 2n)^2$ $= (3k)^2 - 2(3k)(2n) + (2n)^2$

$= 9k^2 - 12kn + 4n^2$

CLASSROOM EXAMPLE 7 **Multiplying More Complicated Binomials**

Find each product.

Solution:

$[(x - y) + z][(x - y) - z]$ $= (x - y)^2 - z^2$

$= x^2 - 2(x)(y) + y^2 - z^2$

$= x^2 - 2xy + y^2 - z^2$

$(p + 2q)^3$ $= (p + 2q)^2(p + 2q)$

$= (p^2 + 4pq + 4q^2)(p + 2q)$

$= p^3 + 4p^2q + 4pq^2 + 2p^2q + 8pq^2 + 8q^3$

$= p^3 + 6p^2q + 12pq^2 + 8q^3$

$(x + 2)^4$ $= (x + 2)^2 (x + 2)^2$

$= (x^2 + 4x + 4) (x^2 + 4x + 4)$

$= x^4 + 4x^3 + 4x^2 + 4x^3 + 16x^2 + 16x + 4x^2 + 16x + 16$

$= x^4 + 8x^3 + 24x^2 + 32x + 16$

Objective 6

Multiply polynomial functions.

Multiply polynomial functions.

Multiplying Functions

If $f(x)$ and $g(x)$ define functions, then

$$(fg)(x) = f(x) \cdot g(x).$$ **Product function**

The domain of the product function is the intersection of the domains of $f(x)$ and $g(x)$.

CLASSROOM EXAMPLE 8 **Multiplying Polynomial Functions**

For $f(x) = 3x + 1$ and $g(x) = 2x - 5$, find $(fg)(x)$ and $(fg)(2)$.

Solution:

$(fg)(x) = f(x) \cdot g(x).$

$= (3x + 1)(2x - 5)$

$= 6x^2 - 15x + 2x - 5$

$= 6x^2 - 13x - 5$

$(fg)(2) = 6(2)^2 - 13(2) - 5$

$= 24 - 26 - 5$

$= -7$

5.5 Dividing Polynomials

Objectives

1. Divide a polynomial by a monomial.
2. Divide a polynomial by a polynomial of two or more terms.
3. Divide polynomial functions.

PEARSON

Divide a polynomial by a monomial.

Dividing a Polynomial by a Monomial

To divide a polynomial by a monomial, divide each term in the polynomial by the monomial, and then write each quotient in lowest terms.

CLASSROOM EXAMPLE 1 **Dividing a Polynomial by a Monomial**

Divide.

$$\frac{10x^2-25x+35}{5}.$$

Solution:

$$=\frac{10x^2}{5}-\frac{25x}{5}+\frac{35}{5}$$

$$=2x^2-5x+7$$

Check this answer by multiplying it by the divisor, 5.

$$5\left(2x^2-5x+7\right)=10x^2-25x+35$$

Divisor **Quotient** **Original polynomial**

CLASSROOM EXAMPLE 1 **Dividing a Polynomial by a Monomial (cont'd)**

Divide.

$$\frac{4x^4-7x^3+12x^2}{4x^3}$$

Solution:

$$=\frac{4x^4}{4x^3}-\frac{7x^3}{4x^3}+\frac{12x^2}{4x^3}$$

$$=x-\frac{7}{4}+\frac{3}{x}$$

Check: $4x^3\left(x-\frac{7}{4}+\frac{3}{x}\right)=4x^4-7x^3+12x^2$

Divisor Quotient Original polynomial

CLASSROOM EXAMPLE 1 **Dividing a Polynomial by a Monomial (cont'd)**

Divide.

$$\frac{6a^2b^4-9a^3b^3+4a^3b^4}{a^3b^4}$$

Solution:

$$=\frac{6a^2b^4}{a^3b^4}-\frac{9a^3b^3}{a^3b^4}+\frac{4a^3b^4}{a^3b^4}$$

$$=\frac{6}{a}-\frac{9}{b}+4$$

CLASSROOM EXAMPLE 2 **Dividing a Polynomial by a Polynomial**

Divide.

$$\frac{2k^2+17k+30}{k+6}$$

Solution:

Write the problem as if dividing whole numbers, make sure that both polynomials are written in descending powers of the variables.

$$k+6\overline{)2k^2+17k+30}$$

$$\begin{array}{r} 2k \\ k+6\overline{)2k^2+17k+30} \end{array}$$

Divide the first term of $2k^2$ by the first term of k + 6. Write the result above the division line.

CLASSROOM EXAMPLE 2 — **Dividing a Polynomial by a Polynomial (cont'd)**

$$\begin{array}{r} 2k+5 \\ k+6\overline{)2k^2+17k+30} \\ 2k^2+12k \\ \hline 5k+30 \\ 5k+30 \\ \hline 0 \end{array}$$

Multiply and write the result below. **$2k(k + 6)$**

Subtract. Do this mentally by changing the signs on $2k^2 + 12k$ and adding.

Bring down 30 and continue dividing $5k$ by k.

Subtract.

You can check the result, $2k + 5$, by multiplying $k + 6$ and $2k + 5$.

CLASSROOM EXAMPLE 3 — **Dividing a Polynomial with a Missing Term**

Divide $4x^3 + 3x – 8$ by $x + 2$.

Solution:

Write the polynomials in descending order of the powers of the variables.

Add a term with 0 coefficient as a placeholder for the missing x^2 term.

Missing term

$$x+2\overline{)4x^3+0x^2+3x-8}$$

Remember to include $\frac{\text{remainder}}{\text{divisor}}$ as part of the answer. ***Don't forget to insert a plus sign between the polynomial quotient and this fraction.***

CLASSROOM EXAMPLE 3 — **Dividing a Polynomial with a Missing Term (cont'd)**

$$\begin{array}{r} 4x^2-8x+19 \\ x+2\overline{)4x^3+0x^2+3x-8} \\ 4x^3+8x^2 \\ \hline -8x^2+3x \\ -8x^2-16x \\ \hline 19x-8 \\ 19x+38 \\ \hline -46 \end{array}$$

Remainder → -46

Start with $\frac{4x^3}{x} = 4x^2$

Subtract by mentally by changing the signs on $4x^3 + 8x^2$ and adding.

Bring down the next term.

Next, $\frac{-8x^2}{x} = -8x$

Multiply then subtract.

Bring down the next term.

$19x/19 = 19$

Multiply then subtract.

The solution is: $4x^2-8x+19-\frac{46}{x+2}$ and you can check the result by multiplying.

CLASSROOM EXAMPLE 4 — **Dividing a Polynomial with a Missing Term**

Divide $4m^4 – 23m^3 + 16m^2 – 4m – 1$ by $m^2 – 5m$.

Solution:

Write the polynomial $m^2 – 5m$ as $m^2 – 5m + 0$.

$$m^2+5m+0\overline{)4m^4-23m^3+16m^2-4m-1}$$

Missing term

Since the missing term is the last term it does not need to be written.

CLASSROOM EXAMPLE 4 — **Dividing a Polynomial with a Missing Term (cont'd)**

$$\begin{array}{r} 4m^2-3m+1 \\ m^2-5m\overline{)4m^4-23m^3+16m^2-4m-1} \\ 4m^4-20m^3 \\ \hline -3m^3+16m^2 \\ -3m^3+15m^2 \\ \hline m^2-4m \\ m^2-5m \\ \hline m-1 \end{array}$$

Remainder → $m – 1$

$$4m^2-3m+1+\frac{m-1}{m^2-5m}$$

CLASSROOM EXAMPLE 5 — **Finding a Quotient with a Fractional Coefficient**

Divide $8x^3 + 21x^2 – 2x – 24$ by $4x + 8$.

Solution:

$$\begin{array}{r} 2x^2+\frac{5}{4}x-3 \\ 4x+8\overline{)8x^3+21x^2-2x-24} \\ 8x^3+16x^2 \\ \hline 5x^2-2x \\ 5x^2+10x \\ \hline -12x-24 \\ -12x-24 \\ \hline 0 \end{array}$$

The solution is:

$$2x^2+\frac{5}{4}x-3$$

Divide polynomial functions.

Dividing Functions

If $f(x)$ and $g(x)$ define functions, then

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}.$$ **Quotient function**

The domain of the quotient function is the intersection of the domains of $f(x)$ and $g(x)$, excluding any values of x for which $g(x) = 0$.

CLASSROOM EXAMPLE 6 **Dividing Polynomial Functions**

For $f(x) = 2x^2 + 17x + 30$ and $g(x) = 2x + 5$,

find $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{f}{g}\right)(-1)$.

Solution:

From previous **Example 2,** we conclude that $(f/g)(x) = x + 6$, provided the denominator $2x + 5$, is ***not*** equal to zero.

$$x \neq -\frac{5}{2}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2 + 17x + 30}{2x + 5} = x + 6$$

$$\left(\frac{f}{g}\right)(-1) = -1 + 6 = 5$$