6.1 Greatest Common Factors and Factoring by Grouping

Objectives

1. Factor out the greatest common factor.
2. Factor by grouping.

PEARSON

Greatest Common Factors and Factoring by Grouping

Writing a polynomial as the product of two or more simpler polynomials is called **factoring** the polynomial.

$3x(5x - 2) = 15x^2 - 6x$ **Multiplying**

$15x^2 - 6x = 3x(5x - 2)$ **Factoring**

Factoring "undoes" or reverses, multiplying.

Factor out the greatest common factor.

The first step in factoring a polynomial is to find the ***greatest common factor*** for the terms of the polynomial.

The **greatest common factor** (**GCF**) is the largest term that is a factor of all terms in the polynomial.

CLASSROOM EXAMPLE 1 **Factoring Out the Greatest Common Factor**

Factor out the greatest common factor.

$7k + 28$

Solution:

Since 7 is the GCF; factor 7 from each term.

$= 7 \cdot k + 7 \cdot 4$

$= 7(k + 4)$

Check:

$7(k + 4) = 7k + 28$ (Original polynomial)

CLASSROOM EXAMPLE 1 **Factoring Out the Greatest Common Factor (cont'd)**

Factor out the greatest common factor.

Solution:

$32m + 24$

$= 8 \cdot 4m + 8 \cdot 3$

$= 8(4m + 3)$

$8a - 9$

There is no common factor other than 1.

$5z + 5$

$= 5 \cdot z + 5 \cdot 1$

$= 5(z + 1)$

CLASSROOM EXAMPLE 2 **Factoring Out the Greatest Common Factor**

Factor out the greatest common factor.

$100m^5 - 50m^4 + 25m^3$

Solution:

The numerical part of the GCF is 25.

The variable parts are m^5, m^4, and m^3, use the least exponent that appears on *m.*

The GCF is $25m^3$.

$= 25m^3 \cdot 4m^2 - 25m^3 \cdot 2m + 25m^3 \cdot 1$

$= 25m^3(4m^2 - 2m + 1)$

CLASSROOM EXAMPLE 2

Factoring Out the Greatest Common Factor (cont'd)

Factor out the greatest common factor.

$5m^4x^3 + 15m^5x^6 - 20m^4x^6$

Solution:

The numerical part of the GCF is 5.

The least exponent that occurs on m is m^4.

The least exponent that appears on x is x^3.

The GCF is $5m^4x^3$.

$= 5m^4x^3 \cdot 1 + 5m^4x^3 \cdot 3mx^3 - 5m^4x^3 \cdot 4x^3$

$= 5m^4x^3(1 + 3mx^3 - 4x^3)$

CLASSROOM EXAMPLE 3

Factoring Out a Binomial Factor

Factor out the greatest common factor.

$(a + 2)(a - 3) + (a + 2)(a + 6)$

Solution:

The GCF is the binomial $a + 2$.

$= (a + 2)[(a - 3)+(a + 6)]$
$= (a + 2)(a - 3 + a + 6)$
$= (a + 2)(2a + 3)$

$(y - 1)(y + 3) - (y - 1)(y + 4)$

$= (y - 1)[(y + 3) - (y + 4)]$
$= (y - 1)(y + 3 - y - 4)$
$= (y - 1)(-1)$ or $-y + 1$

CLASSROOM EXAMPLE 3

Factoring Out a Binomial Factor (cont'd)

Factor out the greatest common factor.

$k^2(a + 5b) + m^2(a + 5b)^2$

Solution:

The GCF is the binomial $a + 5b$.

$= k^2(a + 5b) + m^2(a + 5b)^2$

$= (a + 5b)[k^2 + m^2(a + 5b)]$

$= (a + 5b)(k^2 + m^2a + 5m^2b)$

CLASSROOM EXAMPLE 4

Factoring Out a Negative Common Factort

Factor $-6r^2 + 5r$ in two ways.

Solution:

r could be used as the common factor giving

$= r \cdot -6r + r \cdot 5$

$= r(-6r + 5)$

Because of the negative sign, $-r$ could also be used as the common factor.

$= -r(6r) + (-r)(5)$

$= -r(6r - 5)$

Factoring by grouping.

Sometimes ***individual terms*** of a polynomial have a greatest common factor of 1, but it still may be possible to factor the polynomial by using a process called ***factoring by grouping.***

We usually factor by grouping when a polynomial has more than three terms.

CLASSROOM EXAMPLE 5

Factoring by Grouping

Factor.

$6p - 6q + rp - rq$.

Solution:

Group the terms as follows:

Terms with a common factor of p — Terms with a common factor of q.

$(6p + rp) + (-6q - rq)$

Factor $(6p + rp)$ as $p(6 + r)$ and factor $(-6q - rq)$ as $-q(6 + r)$

$= (6p + rp) + (-6q - rq)$
$= p(6 + r) - q(6 + r)$
$= (6 + r)(p - q)$

CLASSROOM EXAMPLE 6

Factoring by Grouping

Factor.

$xy - 2y - 4x + 8$.

Solution:

Grouping gives: $xy - 4x - 2y + 8$

$= xy - 4x - 2y + 8$

$= x(y - 4) - 2(y - 4)$

$= (y - 4)(x - 2)$

Factoring by grouping.

Factoring by Grouping

Step 1 **Group terms.** Collect the terms into groups so that each group has a common factor.

Step 2 **Factor within groups.** Factor out the common factor in each group.

Step 3 **Factor the entire polynomial.** If each group now has a common factor, factor it out. If not, try a different grouping.

Always check the factored form by multiplying.

CLASSROOM EXAMPLE 7

Factoring by Grouping

Factor.

$kn + mn - k - m$

Solution:

Group the terms:

$= (kn + mn) + (-k - m)$

$= n(k + m) + (-1)(k + m)$

$= (k + m)(n - 1)$

CLASSROOM EXAMPLE 8

Rearranging Terms before Factoring by Grouping

Factor.

$10x^2y^2 - 18 + 15y^2 - 12x^2$.

Solution:

Group the terms so that there is a common factor in the first two terms and a common factor in the last two terms.

$= (10x^2y^2 + 15y^2) + (-12x^2 - 18)$

$= 5y^2(2x^2 + 3) - 6(2x^2 + 3)$

$= (2x^2 + 3)(5y^2 - 6)$